Finite Element Modeling and Simulation of Ultrasonic Guided Wave Propagation using Frequency Response Analysis

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\textbf{Abstract}
COMSOL Multiphysics was successfully utilised for modelling of ultrasonic guided wave propagation in frequency domain analysis. The time domain incident ultrasonic wave was transformed to frequency domain displacement signal which was utilised as the incident signal for frequency domain analysis. The displacement signal obtained by the frequency domain analysis is converted to time domain signal by using Fourier transform. Normalisation of the obtained signal and proper zero padding was necessary to get the desired signal. The Perfectly Matched Layer (PML) was also used to absorb the wave for generating non-reflecting boundary. The effect of various modelling parameters to get a proper signal was also discussed.

\textbf{Introduction}
Modeling and simulation of ultrasonic wave propagation \cite{1} is an important aspect for better understanding of interaction of wave with flaws in material. Time domain analysis \cite{2} using both implicit and explicit code are widely utilised for this purpose. However modeling of wave propagation using time domain analysis is comparatively tricky and needs careful choice of modeling parameters. Simulation in time domain analysis takes considerable time and computational resource. In addition, modeling of visco-elasticity of the wave propagating medium is difficult to be incorporated in time domain analysis.

In contrast to the time domain analysis, modeling of wave propagation in frequency response analysis offers many advantages like requirement of less computational resource, less time of computation, capability of modeling visco-elasticity, utilization of perfectly matched layers (PMLs) etc.

In view of that, modeling and simulation of propagation of ultrasonic guided wave with Finite Element Method (FEM) \cite{3,4} using frequency response analysis was undertaken using COMSOL \cite{5}. Like any other modeling tasks, the correctness of results with frequency response analysis also requires understanding of effects of various modeling parameters. In this paper, modeling of ultrasonic guided wave propagation using frequency response analysis is described. The study also describes the effect of various modeling parameters on the result of wave shape, amplitude. The paper also describes in short about the utilization of PMLs for reflecting and non-reflecting boundaries.

\textbf{Two Dimensional FEM Model in frequency domain}
Commercially available COMSOL Multiphysics software was used to solve the differential equations by Finite Element Method (FEM). The propagation of ultrasonic guided wave in a plate structure was simulated with frequency response analysis.

\textbf{Material & Ultrasonic wave properties}
For the purpose of demonstration of capability of frequency domain analysis for simulation of guided wave propagation, Aluminium was considered as the material with following properties.
<table>
<thead>
<tr>
<th>Material properties</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus (E)</td>
<td>70×10⁹</td>
<td>Pa</td>
</tr>
<tr>
<td>Poisson’s ratio (ν)</td>
<td>0.33</td>
<td>--</td>
</tr>
<tr>
<td>Density (ρ)</td>
<td>2700</td>
<td>Kg/m³</td>
</tr>
</tbody>
</table>

Model Geometry

A plate of 3 mm thickness was considered for the modeling and simulation study. The length of the plate was considered to be 300 mm. The global X-axis is aligned to the length of the plate whereas the width of the plate is aligned along the global y-axis. The excitation was given from the left end side of the plate. Accordingly to the need, the excitation only along the x-direction as well only along y-direction was given. Combined excitation both in x-direction and y-direction was also given.

Application Modes

For the modeling of the 2-D geometry, plane strain conditions was considered because of the dimension of the material considered to be large enough in the third dimension (z-direction) as compared to the x and y directions. The independent variables for the model are the displacements along x-direction and displacement along y-direction. Frequency response analysis was used for the analysis of the model in frequency domain. Lagrange quadratic types of elements were chosen for the simulation. Parameteric solver UMFPACK was used for the solution of the problem.

Frequency Domain Modelling

Acoustic wave propagation which is modelled by the equations from solid dynamics is basically time dependent equations. But the harmonic excitation with time dependence of the form \( f = \hat{f} e^{i\omega t} \) gives rise to an equal harmonic response with the same frequency. In that case the time is completely eliminated and angular frequency became the parameter for solution. This process of solving the acoustic wave propagation equations is known as the frequency response analysis or frequency domain analysis as opposed to time domain analysis. Frequency dependent equation is basically the Fourier transform of the time dependent equation. The solution of the frequency domain analysis is the function of angular frequency whereas solution of time domain analysis is the function of time. The solution in frequency domain analysis as function of angular frequency \( \omega \) is basically the Fourier transform of the full time domain solution obtained by transient analysis. For modelling the problem of ultrasonic guided wave propagation in frequency domain the PDEs are solved with frequency response analysis with parametric solver.

To avoid undesired reflections from the end boundaries, absorbing boundary conditions are required to be set. This is achieved by setting Perfectly Matched Layer (PML) at boundaries. In fact the PML is not exactly a boundary conditions rather an additional domain that absorbs the incident radiations without producing reflections. For each orthogonal absorbing coordinate direction, various parameters like scaled PML width, coordinate of inner PML boundary, and width of PML are need to be set for the required result.

Incident ultrasonic wave

In case of the time domain analysis, the excitation ultrasonic pulse as the incident pulse is given as a function of time which is easy to visualise. However when the problem is solved in frequency domain, the excitation signal used in time domain has to be converted to the amplitude signal as function of frequency before utilised as the input ultrasonic excitation. This is done with the Fourier transform of the time domain signal. The three cycles hanning window operated Tone burst signal of central frequency of 200 KHz is used as the time domain ultrasonic signal utilised for the incident ultrasonic excitation signal. The time domain signal and its corresponding frequency domain excitation signal are shown in figure 1 and 2 respectively.
The frequency domain signal as shown in figure 2 is used as the input signal for the ultrasonic excitation wave for generation of ultrasonic guided wave. The discrete amplitudes of the available signal were given as the required displacements to the model. The linear interpolation was utilised to compute the amplitude signal for the undefined frequencies. The proper PML has to be set at the boundary to receive the desired signal in frequency domain signal. The equation is then solved with the frequency sweep as the input independent parameter. The step of the frequency plays a very important role for generation of proper time domain signal from the frequency domain analysis. The direction of input excitation was given according to the generation of desired guided wave mode. The frequency of 200 KHz was chosen such as to have only Ao and So modes of guided wave in the plate.

Generation of time domain signal

The solution obtained from the frequency domain analysis is the displacements along two mutually perpendicular axes i.e., along x and y direction which is a function of frequency. This frequency domain signal was then transformed to the time domain using inverse Fourier transform.

Simulation results for effect of different modelling parameters

Effect of PMLs in the boundary

Figure 3 shows the simulated ultrasonic signal with frequency domain analysis with use of PML at the boundary. The PML is set to give non reflecting boundary conditions. The signal was observed at a distance of 0.2 m from the excitation end. Because of non-reflecting PML conditions, only one signal packet of Ao mode is seen to be present. Figure 4 shows the Ao mode signals with reflections from boundaries. In this case the PML was set to give reflected signal from the boundaries. The solution obtained by frequency domain analysis indicates the importance of setting PMLs for the desired signal. Figure 5 shows the frequency domain analysis result without any PMLs at the boundary.
Effect of various parameters during reconstruction of frequency domain signal to time domain

The parameters like numbers of zero padding and normalisation of the frequency domain signal for reconstruction to time domain signal plays very important role. The improper choice of these parameters results in change in desired signal as shown in figure 6 & figure 7.

Comparison of Simulation result with Frequency and Time domain analysis

Figure 8 & 9 shows the time domain signal obtained by time domain analysis as well as reconstructed from frequency domain analysis. The signals are observed at a distance of 140 mm (figure 8) and at 200 mm (figure 9) from the end of excitation. The Ao and So modes are seen to be present in figure 8 & 9 respectively.
The time domain signal was generated with the time domain analysis with input signal as shown in figure 1, whereas the signal as shown in figure 2 was used for solution in frequency domain analysis. The solution was obtained for time duration of 400 microseconds. The solution from both time domain and frequency domain analysis matches exactly on the time scale which indicates the capability of frequency domain analysis. The time domain analysis signal was simulated by following the proper modelling parameters [1]. In this case of frequency domain analysis, PML was set to get the reflected signal from the boundary.

![Figure 8: Ao mode signal (Normalised) generated with time domain and frequency domain analysis](image1)

![Figure 9: So mode guided wave signal (Normalised) generated with time & Frequency Domain Analysis](image2)

To solve the given problem in frequency domain analysis takes about 10 times lesser computational time than to solve in time domain analysis. Moreover, frequency domain analysis can be utilised for simulation using complex moduli for modelling of viscoelasticity. The higher attenuation of Ao mode as compared to So mode is also seen to be better modelled by frequency domain analysis.

Conclusion

The finite element numerical method available in COMSOL can be easily utilised to solve ultrasonic guided wave propagation using frequency domain analysis. The time domain problem can be easily solved in frequency domain analysis with requirement of less computational resources. The problems in frequency domain analysis can be solved with parametric solver with different frequencies. The time domain signal can then be generated by using Fourier transforms of the available displacement data with frequency. The frequency domain analysis is not only the faster method but also can incorporate material damping in the model easily.

References