

# **Application of fracture mechanics-based methodologies for failure predictions in composite structures**

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## **Abstract**

This paper describes analytical determination of the strain energy release rates for interlaminar fracture of composite materials by means of fracture mechanics. Expressions for crack tip element methodology and virtual crack closure technique are briefly reviewed. As an application example, empirical data from a double cantilever beam test on a carbon/epoxy laminate are used to determine Mode I fracture toughness employing these methodologies. Results obtained from finite element analysis yielded very good correlation with material specification, indicating the validity of this approach. Fracture mechanics methodologies and relevant mode mix failure criterion were additionally employed to predict delamination onset and growth on flat rectangular laminates with an embedded delamination. Comparison of the numerical estimations with experimental data showed good agreement.

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# 1 Introduction

Advanced carbon fibre reinforced plastics are being used extensively in the aerospace industry. Due to their high stiffness to weight ratio, fatigue and corrosion resistance the use of carbon fibre composites in military and large civil aircraft is steadily increasing. One of the most commonly observed failure modes in composite structures are interlaminar delamination or interfacial fracture (debond) [1]. Delamination can occur as a result of material degradation caused by impact damage, manufacturing defects, or at free edges due to a mismatch in stiffness of the individual layers. Relatively low through the thickness strength and high shear and peel stresses can initiate delamination failure which may propagate in unstable manner. Delaminations are typically a mixed mode failure phenomenon, consisting of all three fracture modes. The use of fracture mechanics has become a common methodology to determine the onset and growth of delaminations in composite structures.

This paper describes a study to assess the accuracy of two fracture mechanics-based methodologies. The crack tip element methodology (CTE) and the virtual crack closure technique (VCCT) were employed as an alternative approaches to estimate interlaminar Mode I fracture toughness. Experimental results were furthermore obtained for flat rectangular laminates containing an embedded delamination. Three-dimensional finite element (FE) analyses were performed to compare predicted failure loads, onset and growth of delamination with those empirically measured.

# 2 Theoretical Background

## 2.1 Overview of the crack tip element methodology (CTE)

The CTE approach is a computationally efficient methodology to predict debond/delamination growth developed primarily by Davidson [2-6]. It is based on a set of equations that yields total strain energy release rate and the mode mix from the knowledge of the forces and moments in the vicinity of the crack tip [2]. The generic three-dimensional CTE with force and moment resultants is shown in Figure 1. The coordinate system origin is defined at the crack tip at the mid-plane of the uncracked region. The z-axis of the coordinate system has been inverted compared to the original proposal presented in reference [6]. By this modification, plies of the laminate are numbered from the bottom surface which is consistent with the numbering of the MSC.Nastran layered composite element property (PCOMP) card definition.

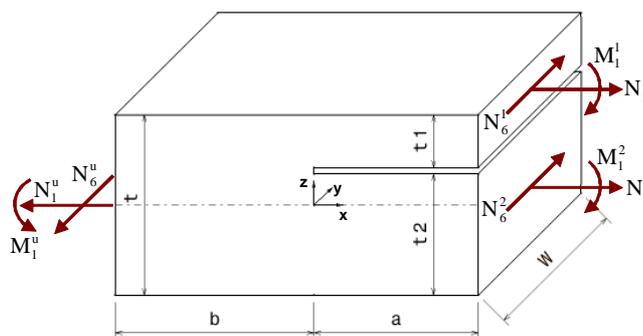


Figure 1. Definition of the 3D CTE

The total energy release rate and the three fracture modes are determined as [5]:

$$G_t = \frac{1}{2} \left[ \left( \Delta N_i \Delta \varepsilon_i^0 + \Delta M_i \Delta \kappa_i \right)_{p=1} + \left( \Delta N_i \Delta \varepsilon_i^0 + \Delta M_i \Delta \kappa_i \right)_{p=2} \right], \quad i = 1, 2, 6 \quad (1)$$

$$G_I = \frac{1}{2} \left[ -\sqrt{c_{11}} N_c \sin \Omega + \sqrt{c_{22}} M_c \cos(\Omega + \Gamma) \right]^2 \quad (2)$$

$$G_{II} = \frac{1}{2} \left[ \sqrt{c_{11}} N_c \cos \Omega + \sqrt{c_{22}} M_c \sin(\Omega + \Gamma) \right]^2 \quad (3)$$

$$G_{III} = G_t - G_I - G_{II} \quad (4)$$

For detailed derivation of Eqs. (1) – (4), refer to [2]. To calculate the total energy release rate,  $G_t$ , at any location along the delamination front plate theory forces,  $\{N_1, N_2, N_6\}_p$ , and moments,  $\{M_1, M_2, M_6\}_p$ , are obtained for the upper sub-laminate (denoted with a subscript  $p=1$ ) and the lower sub-laminate (denoted with a subscript  $p=2$ ). These are taken from the centroidal points of all quadrilateral elements located directly ahead and behind the delamination or debond front. The Mode I and Mode II components are derived from the calculated crack tip force,  $N_c$ , and moment,  $M_c$ . A non-classical mode mix definition, designated as  $\Omega$  in Eqs. (2) and (3), is dependent on the thickness ratio of the upper and lower sub-laminates and is given by [2]:

$$\Omega = \begin{cases} -24 & \eta < -0.468 \\ 60.409\eta - 41.738\eta^3 & \text{if } -0.468 < \eta < 0.468 \\ 24 & \eta > 0.468 \end{cases} \quad (5)$$

where

$$\eta = \log_{10} \frac{t_2}{t_1} \quad (6)$$

The CTE numerical methodology has been integrated into a post-processing tool developed at Cooperative Research Centre for Advanced Composite Structures Limited (CRC-ACS). The tool is incorporated into MSC.Patran pre- and post-processing software. It is an analytical semi-automatic Patran Command Language (PCL) program, based on global-local modelling philosophy, which has the capability to identify critical failure regions in composite structures and perform an assessment of composite bonds and delaminations.

## 2.2 Overview of the virtual crack closure technique (VCCT)

Probably the most commonly employed fracture mechanics based approach in predictive failure analysis of composite structures is the virtual crack closure technique. The technique and its applications are extensively covered by Krueger [7]. This method is originated from the finite crack extension method and the virtual crack extension method firstly discussed in mid 70' [8]. The VCCT methodology, which requires only single FE analysis, is based on the assumption that the energy released when a crack is extended is identical to the energy required to close the crack. Additionally, the single-step method

also presumes that a crack extension of  $\Delta a$  does not significantly alter the state of the crack tip.

The technique uses nodal forces at the delamination front and the displacements behind the delamination front to determine the strain energy release rate (SERR) components. Derivation of the equations to calculate these magnitudes for three-dimensional plate shell problems employing two different FE modelling approaches is presented by Wang and Raju [9]. In the first approach, the nodes at the upper and lower sub-laminate at the crack front are constrained in all six degrees of freedom. The second technique allows the crack front nodes to have independent rotation; restraining only translational degrees of freedom. The later technique proved to realistically simulate behaviour at the delamination front and therefore yielded more accurate strain energy release rates [9]. The SERR components for four-noded shell elements using this modelling approach are determined as:

$$G_I = -\frac{1}{2\Delta A} Z_{Li} (w_{Li} - w_{Li}^*) \quad (7)$$

$$G_{II} = -\frac{1}{2\Delta A} X_{Li} (u_{Li} - u_{Li}^*) \quad (8)$$

$$G_{III} = -\frac{1}{2\Delta A} Y_{Li} (v_{Li} - v_{Li}^*) \quad (9)$$

where  $\Delta A$  is the virtually closed area;  $X_{Li}, Y_{Li}$  and  $Z_{Li}$  are the forces at the crack front in global x, y and z-directions, respectively. Corresponding global displacements behind the crack front at the upper sub-laminate are denoted as  $u_{Li}, v_{Li}$  and  $w_{Li}$ , whereas displacement at the lower sub-laminate are designated as  $u_{Li}^*, v_{Li}^*$  and  $w_{Li}^*$ . For geometric non-linear problem these forces and displacement must be transformed to a local coordinate system.

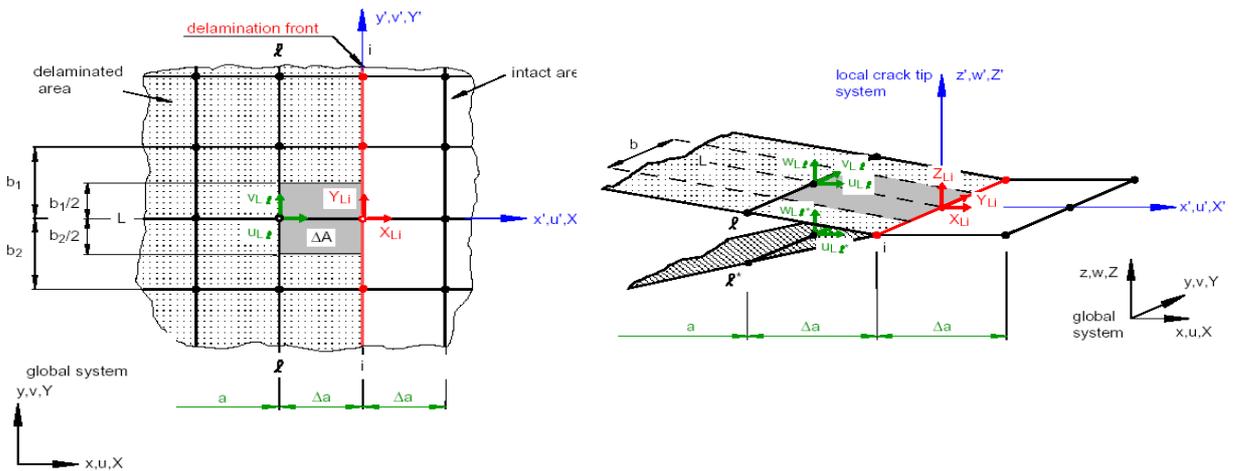


Figure 2. VCCT for shell models [7]

The total strain energy release rate is calculated as a summation of three fracture mechanics components:

$$G_t = G_I + G_{II} + G_{III} \quad (10)$$

The VCCT is applicable to both three-dimensional plate/shell models and solid models. Krueger and O'Brien [10] presented a hybrid shell/3D modeling technique for which solid brick elements are employed only in the close vicinity of the crack tip. This combines the accuracy of the full three-dimensional solution with the computational efficiency of a plate/shell FE model.

The VCCT has been implemented into Abaqus general purpose FE code. A new add-on module, called VCCT for Abaqus, provides simulation capabilities for prediction of the onset and propagation of interlaminar fracture in laminated composite materials.

### 3 Applications

#### 3.1 Application of fracture mechanics for the predictions of Mode I fracture toughness

Conventionally, determination of Mode I fracture toughness of the unidirectional carbon fibre reinforced polymers is standardised by ASTM D 5528 – 94a. The standard uses test data obtained from the double cantilever beam (DCB) test to calculate Mode I fracture toughness,  $G_{IC}$ . Three data reduction methods for calculating  $G_{IC}$  are proposed, namely: modified beam theory, compliance calibration method and modified compliance calibration method.

An alternative approach, based on fracture mechanics, is presented in this section and was used to evaluate  $G_{IC}$  of Hexcel W3G282-F593 carbon/epoxy pre-impregnated (prepreg) fabric. The basic philosophy of this method is to perform a standard DCB test to determine the crack initiation load; the second step involves non-linear FE simulations without crack propagation to precisely capture the empirical load-displacement behaviour for a given crack length and to calculate the SERR at each load step. Finally, Mode I fracture toughness is estimated as the magnitude of the SERR at the load step which yields total reaction force of the FE model equal to the empirical crack initiation load.

Both CTE and VCCT methodologies were employed in the FE analysis of the DCB as shown in Figure 3. The dimension of the FE model represented average values determined from test coupon specimens. CTE methodology was applied using MSC.Nastran, whereas VCCT analysis was conducted using Abaqus. Both FE models were meshed uniformly by linear strain 4-node shell elements with global edge length of 0.5 mm. The load was introduced through an enforced displacement on the upper sub-laminate while the corresponding nodes on the lower sub-laminate were constrained with a pinned joint.

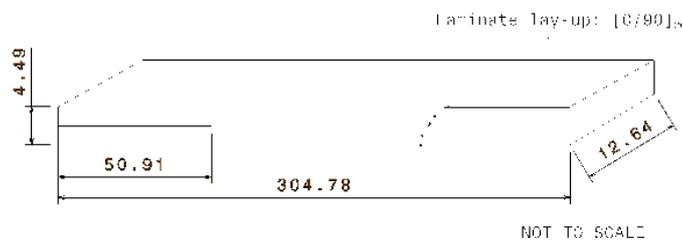


Figure 3. Dimensions of the DCB specimen

The classical Mode I test was performed on five DCB specimens. Based on the experimental observations, it was found that the onset of delamination consistently occurred at approximately 29.5 N. The experimental data were used to estimate Mode I

fracture toughness by fracture mechanics means and the values compared to the material specification as shown in Table 1. Material specification data was obtained by DCB test using the method described in the standard. Theoretical predictions were very accurate; the discrepancies made by these predictions were within 10 per cent. The estimate yielded by VCCT was almost identical to the material specification.

Table 1. Mode I fracture toughness of carbon/epoxy laminate

Material data / Analytical methodology	Mode I fracture toughness (kJ/m <sup>2</sup> )
Material specification	0.273
Crack tip element approach	0.247
Virtual crack closure technique	0.276

Figure 4 provides a comparison of the test results and analytical predictions using VCCT for Abaqus with the estimated  $G_{IC}$ . The general trend of crack propagation was very well captured. Reaction force increased linearly with opening displacement until the critical energy level was achieved. At this point the crack tip propagated by releasing contact constraints at failed node couples, resulting in unloading at constant energy release rate equal to material fracture toughness. It is important to note that the experimentally observed non-linear initial stiffness was caused by a slippage of triangular grips used during the test.

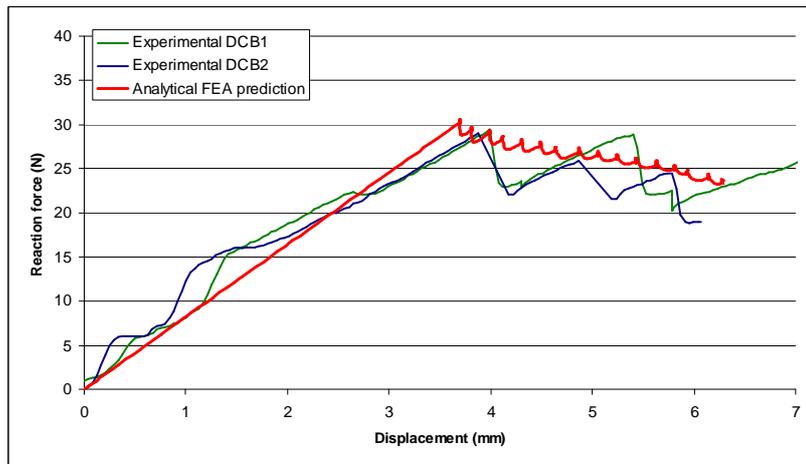


Figure 4. Load versus displacement curve comparisons

### 3.2 Prediction of two-dimensional propagation of delaminations

Moving from one-dimensional to two-dimensional crack propagation, an analytical model and test matrix was developed for analysing the response of laminated composite panels containing rectangular internal delaminations. Flat panels were fabricated from Hexcel W3G282-F593 carbon/epoxy prepreg whose material properties are indicated in Table 2. Test laminates were 130 mm square with a delamination of 70 mm square. Artificial delamination was introduced with thin Airtech Wrightlon 5200 high performance release film placed at the designated location and the interface during lay-up, as shown in Figure 5. The specimens were designed to have a total of 20 layers giving laminates a nominal

thickness of 4.4 mm. The panels were manufactured by hand lay-up and cured using the vacuum bag method with the manufacturers recommended cure cycle.

Table 2. Material properties of Hexcel W3G282-F593 [11]

Material property	Value	Unit
Longitudinal Young's modulus, $E_{11}$	55840	MPa
Transverse Young's modulus, $E_{22}$	55840	MPa
Poisson's ratio, $\nu_{12}$	0.06	
In-plane shear modulus, $G_{12}$	3650	MPa
Out-of plane shear modulus, $G_{13}$	3650	MPa
Out-of plane shear modulus, $G_{23}$	3650	MPa

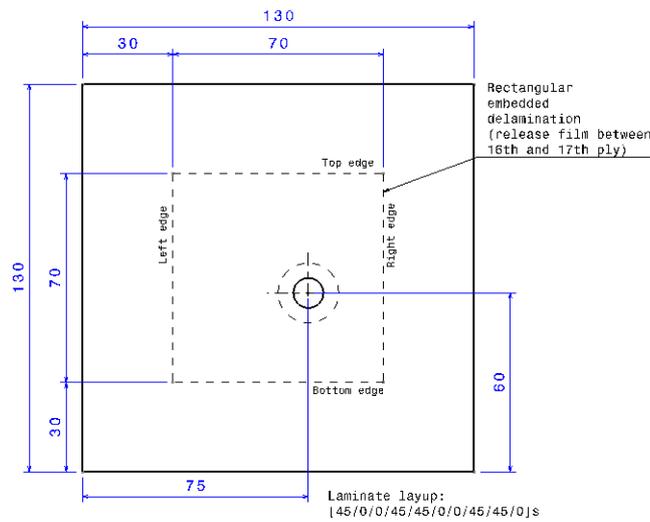


Figure 5. Geometry and layup of the flat panel specimens

The test specimen was subjected to bending load using specially design test fixture as presented in Figure 6. Out-of-plane displacement was introduced by a M10 fastener to enforce separation of sub-laminates at the delamination interface between 16<sup>th</sup> and 17<sup>th</sup> plies. Testing was carried out using an INSTRON 8504 universal testing machine in displacement control at a rate of 0.5 mm/min. A data acquisition system was used to record load and displacement histories from the initial loading to the final failure. The panel was observed carefully during testing for any audible cracking.

Finite element analysis was conducted to give prediction of the onset of the embedded delamination. Similarly to the previous analysis, both MSC.Nastran and Abaqus FE systems were used. The MSC.Nastran FE model was created using double plate modelling technique. Two layers of 4-node shell elements were joined at the interface using rigid links except for the delamination area. Even though contact was not expected, gap elements were used to connect delaminated sub-laminates to prevent physically inadmissible interpenetration of the elements during analysis. Layered material properties were assigned to the elements according to the lay-up shown in Figure 5. The Abaqus FE model was generated using continuum shell elements. Unlike conventional shells, continuum shell elements discretise an entire three-dimensional model. They appear like

solid elements but their kinetic and constitutive behaviour is similar to conventional shell elements. Continuum shell elements simplify contact definition between two bodies. The FE models were constrained to represent the boundary conditions provided by the test fixture. Loading in MSC.Nastran was simulated by direct application of nodal displacements at the appropriate locations. The Abaqus model incorporated rigid-to-deformable body contact to precisely simulate interaction between the loading bolt and laminate during testing.

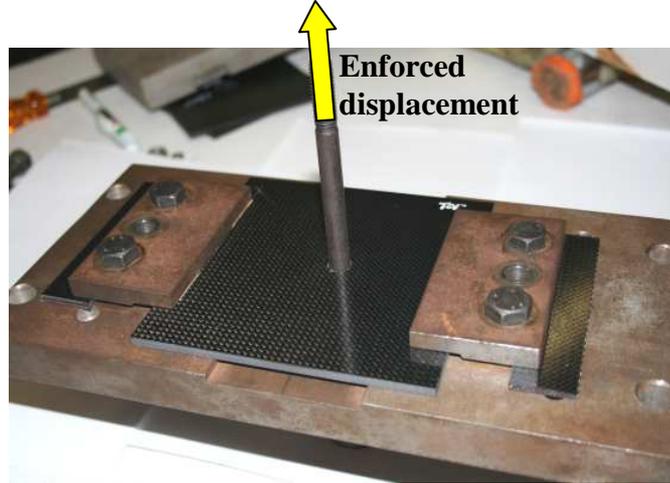


Figure 6. Specimen loaded in testing fixture

Computational estimation of damage initiation and propagation was based on the power law criterion as:

$$\left(\frac{G_I}{G_{IC}}\right)^\alpha + \left(\frac{G_{II}}{G_{IIC}}\right)^\beta + \left(\frac{G_{III}}{G_{IIIC}}\right)^\gamma = 1 \quad (11)$$

The left hand side of the Eqn. (11) calculates a failure index coefficient (FI). Failure initiation is predicted when the FI is equal to or greater than unity. The exponents  $\alpha$ ,  $\beta$  and  $\gamma$  are empirically obtained parameters to precisely capture the contribution of each fracture mode. Based on previous experimental results, these were set to 1.0, 1.5 and 1.0, respectively. Mode II fracture toughness,  $G_{IIC}$ , determined by end notched flexure test, was calculated to be 1.05 kJ/m<sup>2</sup>. Mode III fracture toughness tests have not been conducted since no single test method has been accepted as a standard due to involved complexity and large scatter of results. Therefore Mode III fracture toughness,  $G_{IIIC}$ , was assumed to equal to  $G_{IIC}$ .

Comparison of load-displacement histories between FE analyses and empirical data is presented in Figure 7(a). A dotted vertical line represents failure initiation point predicted by CTE/MSC.Nastran approach. Both numerical models exhibited slightly more compliance than the test specimen. Better correlation was obtained using MSC.Nastran. The MSC.Nastran model, however, had no capability to simulate crack propagation, which explains the stiffness disagreement at load levels beyond the crack initiation point. The results obtained from the FE analyses indicated that the primary onset of embedded delamination occurred at the right hand edge, followed by secondary initiation at the bottom edge (refer to Figure 5). Crack initiation load predictions, summarised in Table 3, were conservative compared to the two test results. The average test initiation load was

determined to be 835 N; numerical methods yielded prediction of about 600 N for CTE and 611 N for VCCT, giving a discrepancy between predictions and test of approximately 30%. It should be noted that during testing, small audible cracks were recorded at approximately 550 N which was assumed to be caused by separation of inserted release film and sub-laminate. Since it was not clear if these were the evidence of crack initiation, additional experiments are planned to increase assurance in obtained test results.

Table 3. Crack initiation predictions

Test results / Analytical methodology	Failure initiation load (N)
Test specimen #1	850
Test specimen #2	820
Crack tip element approach	600
Virtual crack closure technique	611

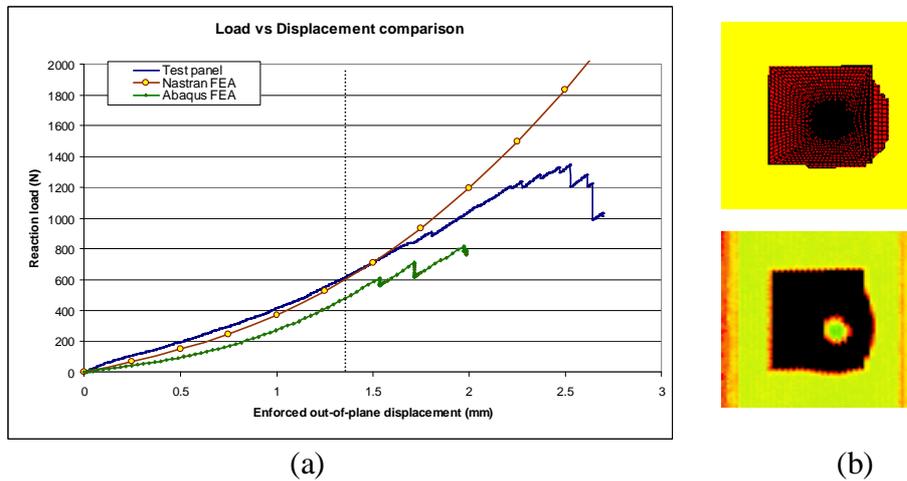


Figure 7. Comparison of FE predictions with test results

The FE prediction of crack propagation shape showed excellent correlation with non-destructive inspection using ultrasonic C-scan, as presented in Figure 7(b). The upper plot shows results of the FE simulation with the red region representing the delamination state. The black area on the bottom plot shows the extent of delamination from test as determined by C-scan.

The investigation of mode mixity at the critical delamination edge revealed that the crack initiation was driven by opening Mode I. Contribution of this mode was 66.5% of the total SERR, followed by in-plane shear Mode II with 32.5%. A variation of the SERR along the crack front at crack initiation load is shown in Figure 8. The presented trend was similarly captured by both numerical methodologies.

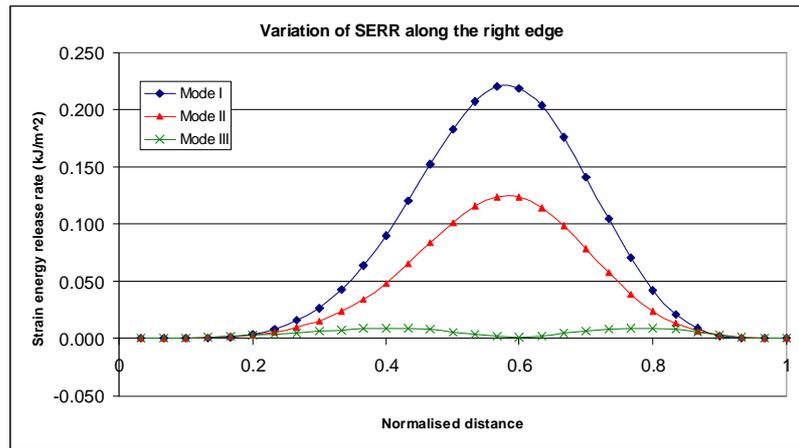


Figure 8. SERR variation along the delamination edge

#### 4 Conclusion

FE analyses were conducted on composite test specimens to demonstrate predictive capabilities of two fracture mechanics-based failure methodologies; namely, CTE methodology and VCCT. The former approach was integrated into the composite failure assessment tool which was developed at CRC-ACS. The VCCT was evaluated using the commercial FE application, VCCT for Abaqus.

A methodology to determine the Mode I interlaminar fracture toughness numerically by fracture mechanics means was described. This investigation indicated that fracture mechanics can be used as an alternative approach to evaluate  $G_{IC}$  of laminated composites. Test and analysis on DCB specimens showed that CTE and VCCT estimated  $G_{IC}$  of the carbon/epoxy laminate with a high degree of accuracy. Some further evaluations are necessary to validate applicability range of this methodology on various material systems and delamination interface orientations.

Mixed mode crack initiation and growth was simulated on a flat rectangular laminate containing an embedded delamination. Primary and secondary crack initiation locations were precisely predicted by both methodologies. Estimated damage growth was also confirmed by non-destructive inspection. Generally, fracture mechanics proved to be reliable approach for the investigation of flaws in composite structures. Both VCCT and CTE numerical methodologies yielded similar accuracy in terms of predicted failure initiation loads and mode mixities. Additional experiments will be conducted to obtain more experimental data for the described test setup in order to improve correlation. Analytical studies will also be extended to the analysis of multiple delaminations.

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