Wave Propagation Correlations between Finite Element Simulations and Tests for Enhanced Structural Health Monitoring

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ABSTRACT

Ultrasonic Lamb waves are frequently used in non-destructive testing of elastic thin plate-like structures. Modern real-time Structure Health Monitoring (SHM) systems take advantage of their long travel distance of propagation and their capacity to detect surface and also internal defects. Signal interpretation involved in the damage diagnosis process becomes a very difficult task due to dispersive and multimode characteristics of these guided waves, making numerical simulation a valuable component to understand wave generation and propagation issues.

The work here presented is concerned with implementation of an efficient analysis methodology through the finite element method in order to study Lamb wave propagation in homogeneous elastic plates using phased array piezoelectric transducers operating simultaneously as wave transmitters and receivers. A detailed study of the influence on the pulse-echo response of different aspects has been conducted in order to provide modelling guidelines. Additionally, a specific postprocessing procedure has been implemented which allows to selectively study and display each propagating mode separately. Experiments have been performed on aluminium plates, using different excitation frequencies, and PZT voltage variation was found to be in good correlation with that computed from numerical simulations. The developed methodology includes the wave generation using PZTs, wave propagation in an isotropic and homogeneous plate, interaction with the plate boundaries, and wave signals in panel surface captured by PZT elements including the generation of image sequences for both fundamental symmetric and anti-symmetric Lamb wave modes.

INTRODUCTION

Modelling is a critical part in the SHM process. The measured data obtained through a SHM system does not bring information about structure boundaries or number, location and type of flaws. These data need post-processing through robust mathematical tools that convert the measured data into meaningful quantities that help not only to locate the part boundaries, but also detect flaws in the structure and quantify their severity.

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Numerical simulations of guided wave propagation are widely used to identify different Lamb wave characteristics, for example time of flight, mode conversion and generation, change in amplitude and attenuation, velocity, etc..., since it helps in understanding the physics of the phenomenon. There are different numerical methods that are commonly applied to analyse Lamb wave propagation problems, like the Finite Difference Method-FDM, Finite Element Method-FEM, Spectral Finite Element Method-SFEM or Boundary Element Method-BEM. Among these numerical methods, FEM stands out as the most versatile and powerful method due to its ability to model complex geometries [1] and accuracy.

FE simulations have been widely used in the analysis of Lamb wave propagation, in isotropic and orthotropic materials, and interaction with damage problems [2], [3]. When the details to be analyzed are small (e.g. piezoelectric actuators), it is necessary to use short wavelengths and therefore very high frequency excitation signals. Typical modeling recommendations suggest the use of at least 8–10 elements per wavelength, hence FEM requires the use of very refined meshes for modeling wave propagation problems, which leads on huge computational effort to get accurate results.

DESCRIPTION OF THE METHODOLOGY

The proposed simulation methodology for the numerical analysis of wave propagation in isotropic and homogeneous plates by means of FEM includes the wave generation, using piezoelectric transducers, the wave propagation using a resolution method based on an implicit-explicit integration scheme, and postprocessing techniques to identify and extract fundamental symmetric and anti-symmetric Lamb wave modes.

The simulation of a SHM system in a host structure is a multi-physics problem (involving mechanical-piezoelectric coupling) that cannot be solved using an explicit code since these types of codes are able to simulate wave propagation, but cannot perform piezoelectric analyses. Those analyses are available in implicit solvers only. A possible solution to overcome this limitation is to solve the multi-physics dynamic analysis directly by using an implicit code alone, but the problem then is that computational cost increases dramatically with the number of elements (directly related to the size of the analysed geometry or the wavelength involved in the analysis). Reducing the implicit domain to a minimum area around the transducers (see Figure 1), while the rest of the model is analysed using an explicit resolution method, reduces the computational effort to solve the propagation analysis since the computational cost of an explicit simulation increases linearly with the number of elements.

![Figure 1. FE model showing the different integration domains used in the simulations.](image)
The use of a coupled explicit-implicit simulation scheme allows to incorporate in the FE simulation the piezoelectric transducers as well as to analyse large FE models, but the voltage signal interpretation is still a problem. Typical results of FE simulations are contour maps for the displacement, strain or stress fields but those results provide limited information about the wave propagation phenomenon or the identification of individual propagation modes.

The proposed zero-order modes visualization methodology allows to identify the contribution of the symmetrical ($s_0$) and anti-symmetric ($a_0$) modes to the output voltage signal of the different sensors. This procedure recovers valuable information that is embedded in typical simulation results, but that is not directly accessible as an output variable of the commercial finite element codes. The postprocessing methodology is based on the calculation of two displacement based indexes, corresponding to the plate thickness variation ($s_0$ mode indicator) and the out of plane displacements of the mid-plate surface ($a_0$ mode indicator). The Figure 2 summarizes the expressions used to evaluate both indexes, as well as the symmetrical and anti-symmetric mode shapes.

$$s_0 \text{ index} = U^B_z - U^A_z$$
$$a_0 \text{ index} = \frac{1}{2} (U^B_z + U^A_z)$$

In the previous expressions $U_z$, corresponds to the out of plane displacement of the top (point B) and bottom surface (point A) of the plate. These indexes are directly related to the physical interpretation of the $s_0$ and $a_0$ modes, as it is shown in Figure 2.

**VALIDATION OF THE METHODOLOGY**

The simulation methodology is validated through a numerical-experimental comparison, analysing the influence of the coupled implicit-explicit resolution scheme and the correlation between the voltage peaks in the transducer output signal and the arrival times of the $a_0$ and $s_0$ wave fronts. Experimental tests have been carried out on a simplified geometry (circular aluminium plate, diameter: 200 mm, thickness: 1 mm), using two transducer elements bonded in the top and bottom surfaces, as it is shown in Figure 3. The transducer in the top surface is excited by means of a 12 volt 3-cycle sinusoidal signal with frequencies ranging from 150 to 400 kHz, whereas the output voltage of the transducer in the bottom surface is recorded in order to set up a comparison with the simulation results. All the experimental tests have been performed by AERNNOVA by use of their PAMELA SHM™ system [4].
Several simulations have been carried out, analysing the resolution scheme, pure implicit or coupled implicit-explicit scheme as well as the meshing densities and computational time steps in order to provide modelling guidelines. Figure 4 shows the results obtained for an excitation signal of 400 kHz, comparing the coupled explicit-implicit resolution scheme and the pure implicit one.

![Figure 4. Comparison of the output voltage for the implicit and implicit-explicit resolution schemes.](image)

Considering the results of the implicit method as a benchmark, a perfect agreement in the voltage output for the bottom transducer is obtained using the implicit-explicit resolution method, although the computational cost is reduced ten times. Figure 5 shows the results, in terms of voltage output, obtained through a FE simulation, using an implicit-explicit resolution technique and an excitation frequency of 400 kHz with those obtained by means of experimental acquisition.

![Figure 5. Simulation results compared with experimental test.](image)

According to the results enclosed in the Figure 5, the voltage output obtained in the simulation shows a good agreement with the experimental one, including attenuation effect. The simulation model takes into account that effect including the material damping by means of a Rayleigh model.

**FE ANALYSES ON DIFFERENT OPERATION MODES OF A PHASED ARRAY**

The developed simulation methodology has been applied to analyse the Lamb wave propagation behaviour on a flat rectangular panel with a set of piezoelectric transducers bonded to the top surface of the plate. The plate is made of aluminium with a thickness of 1 mm. and the array consists of twelve piezoelectric elements operating as sensor and actuators simultaneously.
Four operating modes for the transducers have been considered, named as simple mode, plane front, focusing and beam forming sequences. The transducers are excited by means of a 12 volt 3-cycle sinusoidal excitation at 350 kHz in all cases but considering different activation time sequences to generate the four wave fronts selected. For the single sequence simulation, only first transducer in the array is excited while for the plane front wave generation the full array is excited. In the case of beam forming the full array is excited sequentially with fixed time increments among transducer activation whereas for the focusing operating mode, the time increment between transducers activation is variable.

The results of the simulations carried out are presented in terms of plots of the plate surface displacement for different time points along the simulation, as well as the results of the postprocessing methodology in order to visualise the \( a_0 \) and \( s_0 \) propagation modes.

The graphical results for the simple sequence operation are shown in Figure 6, while those corresponding to a plane front are shown in Figure 7. The results of the beam forming simulation results are enclosed in Figures 8 and 9. Finally, graphical results for the focusing operation mode are summarised in Figures 10 and 11.

![Figure 6. Simulation of a simple sequence operating mode. Displacements field, \( s_0 \) and \( a_0 \) propagation modes at 0, 36, 72, 108, 144 and 180 \( \mu s \).](attachment:image_url)
Figure 7. Simulation of a plane sequence operating mode. Displacements field, $s_0$ and $a_0$ propagation modes at 45, 90, 135, 180 and 225 μs.

Figure 8. Simulation of a beam forming sequence operating mode. Displacements field, $s_0$ and $a_0$ propagation modes at 37.5 μs.
Figure 9. Simulation of a beam forming sequence operating mode. Displacements field, $s_0$ and $a_0$ propagation modes at 75, 112.5, 150 and 187.5 μs.

Figure 10. Simulation of a focusing sequence operating mode. Displacements field, $s_0$ and $a_0$ propagation modes at 22.5 and 45 μs.
CONCLUSIONS

Despite of the main drawback of FEM (computational cost), the proposed simulation methodology includes the generation of the excitation signal by means of piezoelectric actuators, the propagation analysis using an implicit-explicit integration scheme in order to reduce the computational effort, and postprocessing techniques to visualize the symmetric and anti-symmetric propagation modes of a Lamb wave. Experimental results are found to be in good correlation with that computed from numerical simulations. Additionally, modelling guidelines have been extracted to accurate capture the propagation behaviour of $s_0$ and $a_0$ modes. The meshing requirements are determined by the $a_0$ wavelength, imposing at least 10–20 elements per wavelength and 3 elements through the plate thickness.

REFERENCES