ACOUSTIC EMISSION SOURCE LOCATION ON AN ARBITRARY SURFACE BY GEODESIC CURVE EVOLUTION

G. PRASANNA, M. R. BHAT and C. R. L. MURTHY
Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560 012, India

Abstract

Location of an acoustic emission (AE) source is generally obtained by assuming simplified velocity models on surfaces and solids, for which a definite parametric representation exists. These attempts employ triangulation to form distance equations using time-differences obtained from experimental data, which are solved analytically or numerically to get to the location of source. This approach may not be suitable for complex geometry components. Also, the problem gets compounded if the material of the structure is anisotropic warranting complex analytical velocity models. Hence, there has been a need to obtain a practicable source location solution in a more general setup on any arbitrary surface containing finite discontinuities. The approach suggested here is based on the fact that the wave takes minimum energy path to travel from a source to any other point in the connected domain. An AE signal takes this path from the source to an AE sensor array. By propagating the waves in reverse virtually from these sensors along the minimum energy path and by locating the first intersection point of these waves, one can get the source location.

Keywords: Source location, geodesics, geodesic evolution

Introduction

One of the major advantages of AE technique as an on-line monitoring tool is its capability to locate active defects in larger structural components without having to physically scan them. Different methods used for source location include zonal location, computed location, and continuous location. These approaches are affected by the signal attenuation and dispersion due to inhomogeneity and geometry of the material [1]. An alternative location technique uses the concept of ‘the first sensor hit by an AE event’ to identify a more generalized region around each sensor, from which the AE signal likely originated. In this case, one can determine which one of the several sensor regions on the test specimen has more concentrated AE activity [2]. Some AE systems determine signal arrival times using fixed threshold techniques [3] and because of the aforementioned complications, such AE systems measure arrival times for signals using various portions of the AE signal, which travel at different velocities.

The approach suggested here is based on the fact that the wave takes minimum energy path to travel from a source to any other point in the connected domain. In isotropic media, minimum energy path gets reduced to shortest distance path, which can be seen mathematically as shortest geodesics. Hence, by allowing geodesic waves to propagate from multiple sensors and identifying their point of meet, one can obtain the source location. In an object with complex geometry, a graph-theory based concept can be employed to determine the shortest path using Dijkstra’s algorithm [4] for finding discrete geodesics, which are propagated from each sensor location till the source is reached.
Our approach

This conceptual view can be visualized as in Fig. 1, which shows waves propagating in all possible directions from a defect location, along the minimum energy path. With imposed material limitations, this path is generated by geodesics. Once the geodesic paths are extracted in a given geometry, the defect location is reached by back-propagating along those paths, from the sensor locations, as shown in Fig. 2. To start with, it can be assumed that AE sensors detect only Rayleigh waves, which in turn means that the study is restricted to 2-D surfaces.

It is to be noted that even the triangulation method utilizes the same approach, only that it has an inherent assumption that the geometry is a 3-D continuum resulting in simple distance equation based on Pythagorean theorem, which is solved analytically or numerically.

Since our approach builds on the fact that a wave takes minimum energy path, when the assumption of isotropy in the media is imposed, the shortest energy path reduces to shortest distance path, which is provided by the evaluating the geodesics. Mathematically, the energy along a path is seen as weighing function defined along the path. Hence, the minimum energy path is given by

\[ \text{Energy along path, } L = \int w(x) \, dx \]

\[ \text{Min. Energy, } \min(L) = \min \left( \int w(x) \, dx \right) \]

\[ \text{If isotropic, Min Energy path, } \min(L) = w \cdot \min \left( \int dx \right) \]

where, \( \min(\int dx) \) is the required geodesic.

So, the minimum-energy-path problem is equivalent to shortest-distance-path problem. The intersection of geodesic wave fronts from multiple sources gives the location of the source.

Formulation - Wave Propagation Approach:

This approach involves discretizing the domain as curved or planar simplicial-complex chains, followed by finding local geodesics in each of the simplex and finally gluing them together to get the required global geodesic. There are many suggested methods to calculate
discrete geodesics and it is still an active area of research investigation [5-7]. Dijkstra [4] proposed an algorithm for the same and most of the present techniques is built over it.

The source location formulation proceeds as follows:

$$D(S1 - S) - D(S2 - S) = V dt$$

which is recast into the implicit form,

$$\Phi (D, V, dt) = 0$$

The geodesics ‘D’ in above formulation is arrived from ‘wave-propagation’ perspective using graph-theory based Dijkstra algorithm.

The governing equation is subsequently further recast as

$$D(S1 - S) \pm V dt = D(S2 - S)$$

This leads to the view that the solution lies in the boundary of the Voronoi diagram and the exact location is the intersection of two or more boundaries.

With above observation and understanding, the algorithm was implemented and tested for shortest path extraction over some simple cases and also over Stanford Bunny (Fig. 3), proving convincingly the wave-propagation based construction.

![Fig. 3: (a) Surface model. (b) Generated by geodesic propagation.](image)

The bunny in Fig. 3(b) was generated by flow of wave along the shortest cost path from an arbitrary vertex in the mesh. The thick lines in the figure are samples of shortest paths between two points on the bunny surface (which are the required ‘geodesics’).

This proves the ability to extract discrete geodesics on arbitrary surface, which is the first part of geodesic evolution approach. The second part of solution is the construction of Voronoi-like diagram to locate the intersection of wavefronts, which is discussed in the following section.
**Voronoi Construction:**

Taking a case of 3-sensor setup provides 3 sets of time-difference equations. Using the re-cast formulation, all the points that are equi-distant from sensor locations are found (using distance map calculated based on Dijkstra algorithm along with V.dt corrections).

When information from only two sensors is available, then only one equation is formulated and hence there exist multiple solutions meeting the distance criteria. This is depicted in Fig. 4(a) with the jagged thick line passing between sensors S1 and S2 indicating all points that are equi-distant (which is the Voronoi diagram). An important observation is that this line passes through the defect location (AE source) and hence we need to search only along this line for getting to the source.

When information from one more sensor (S3) is considered, then 3 equations are formulated, from which the other lines in Voronoi diagram are constructed. All these lines intersect exactly at the AE source, as in Fig. 4(b). The geodesic lines joining the 3 sensors give the Delaunay triangle, which is the dual of conventional Voronoi diagram.

![Fig. 4: Voronoi diagram considering (a) 2 sensors; (b) 3 sensors.](image)

Above construction is trivial to implement in form of set operations. For a given mesh,
- Let nk be the kth node in the mesh and
- D(nkSi) be the distance between kth node and ith sensor and
- Dk(Sij) be the difference in distances of a node k from sensors Si and Sj, i.e.,
  \[ Dk(S_{ij}) = D(nkSi) - D(nkSj), \]
- the Voronoi line between any two sensors Si and Sj is formed by nodes which satisfy the condition that Dk(S_{ij}) = V.dt_{ij}
  where, dt_{ij} is the hit arrival-time difference between the sensor Si and Sj and D’s are geodesic distances and the corresponding line can be seen as set of these nodes which is given by
  \[ L_{ij} = \{ nk \mid Dk(S_{ij}) = V.dt_{ij} \} \]
- Hence for 3 sensors we get, L_{12}, L_{13} & L_{23} as shown in Fig. 4. The intersection point is the intersection node in the set L_{ij} given by
Source, $S = \{ n \mid (L_{12} \cap L_{23} \cap L_{13}) \}$

- For surfaces, which are intrinsically 2-dimensional in parametric space, only two of the above sets are to be included for getting the source node.

**Experimental Evaluation**

The trial is initially made for a curved planar structure. Source location was attempted using both numerical-continuous and numerical-discrete (wave-propagation approach). It was followed by experimentation on an odd geometry component containing sharp changes and discontinuities. The AE setup that is used (MISTRAS) has an auto-sensor-test (AST) mode where each sensor acts as source and emits an AE pulse, which is received by other sensors. With known sensor locations, time-difference equations are formed and solved to get the velocity of the AE in the test component. AE sources were also simulated by pencil-lead breaks at known locations and the AE data was continuously recorded. The extracted data is sent to source location algorithm (coded in MATLAB) for evaluation.

**Cylindrical Geometry:**

The setup is shown in Fig. 5(a). Based on the dimensions, a sector of the object was meshed using ANSYS (Fig. 5b) and mesh information was imported to MATLAB. Wave propagation algorithm was applied over the mesh to find the location of artificially created sources. The program output shows a deviation of approximately 7.5% from the actual, as shown in Table I.

![Fig 5 (a) Cylinder - AE setup (b) ANSYS mesh of a sector.](image)

**Table I: Source location in cylindrical geometry.**

<table>
<thead>
<tr>
<th>Sensor Location</th>
<th>Measured Value</th>
<th>Program output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>S-1</td>
<td>105</td>
<td>0</td>
</tr>
<tr>
<td>S-2</td>
<td>-80.43</td>
<td>0</td>
</tr>
<tr>
<td>S-3</td>
<td>-80.43</td>
<td>360</td>
</tr>
</tbody>
</table>
Figure 6 shows the geometry as 3D model, the test setup, the ANSYS mesh and the MATLAB output (with a thick line showing the path from sensor location to the source). See Table II for the results. Error in this case is less than 10%.

Table II: Source location in clamp.

<table>
<thead>
<tr>
<th>Location</th>
<th>Measured Value</th>
<th>Program output</th>
<th>% Error (Euclidean)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
<td>Z</td>
</tr>
<tr>
<td>L-1</td>
<td>31</td>
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<tr>
<td>L-2</td>
<td>45</td>
<td>8</td>
<td>39</td>
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<tr>
<td>L-3</td>
<td>73</td>
<td>33</td>
<td>31.5</td>
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<td>L-4</td>
<td>114</td>
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<td>28</td>
</tr>
<tr>
<td>L-5</td>
<td>55</td>
<td>0</td>
<td>30</td>
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</table>

Conclusion

The approach presented in the study used a property of wave by which it tends to take minimum-energy path to travel between points on domain. This minimum-energy path was proved to be equivalent to shortest-distance path marking the birth of geodesics. The relation of geodesics to source location problem was established by proving that location of source is the first intersection point of multiple geodesics. This was implemented and proved by using Voronoi like diagram construction. The approach was experimentally validated on curved planar and odd geometry component. The solution based on two methods – conventional Numerical method and
Geodesic Evolution method was presented. It can be asserted that the geodesic curve-evolution method hold great promise for versatile implementation catering to non-conventional geometries. By the very nature of approach, extension of it to inhomogeneous and anisotropic geometry appears feasible.

References


