Keywords: Ultrasonics, Flaw Scattering, Planar Flaw, Kirchhoff & GTD.

ABSTRACT
Simulation is helpful for evaluating the performances of inspection techniques and requires the modeling of waves scattering from defects.

Two classical flaw scattering models have been previously evaluated and implemented in the CIVA platform developed by CEA/LIST to deal with planar defects: the geometrical theory of diffraction (GTD) and the Kirchhoff approximation (KA). These two approaches appear to be complementary. Combining them so as to retain only their advantages, we have developed a combined model (the so-called Kirchhoff & GTD) using a procedure similar to the physical theory of diffraction (PTD).

Both theoretical and experimental validations of the Kirchhoff & GTD model have been carried out in various practical NDE (pulse echo and TOFD) configurations studying both direct and corner echo modes. Theoretical validations have consisted in comparisons between this new model and other scattering models (GTD, KA and a finite-element method).

Whereas the previously existing models were notably useful to respectively simulate specular reflection echoes for Kirchhoff and edges diffractions for GTD, the performed validations have shown that the Kirchhoff & GTD model provides a generic modeling of both the two main scattering phenomena arising from a planar flaw: specular reflection and edges diffraction.

INTRODUCTION: AVAILABLE SCATTERING MODELS IN CIVA
The CIVA software platform is developed at CEA-LIST and partners in the aim of simulating non-destructive evaluation [1]. Most of the developed models are based on semi-analytical methods.

The ultrasonic simulation tools in CIVA allow one to fully predict the results of an ultrasonic inspection in a range of applications which requires the computation of the beam propagated, as well as its interaction with flaws [2]. The transducer field is calculated using a pencil method derived from the Rayleigh integral for acoustic radiation [3]. Indeed, inside the coupling material, the Rayleigh integral is computed directly by summing the contributions to the field of each source point of the discretized transducer surface [2]. In the specimen the pencil method which is an extension of ray theory [4] is applied.

The beam to flaw interaction is dealt with different modelling approaches depending on the defect and the inspection characteristics. Three kinds of models for the scattering of ultrasound by flaws have been integrated: approximate analytical solutions, exact analytical solutions and numerical modelling methods. In the last release CIVA 11, numerous improvements of these methods have been added as described hereafter.

The developed approximate analytical solutions are respectively:
- the Kirchhoff approximation [5] to deal with specular reflections from volumetric voids (spherical or hemispherical holes, SDHs) and cracks (rectangular, CAD or elliptical planar, FBHs, multifaceted, branched). This approximation is mostly valid if the observation direction is close to reflection and is particularly suitable to simulate specular reflection, corner effects, etc. The corresponding integrated model requires the meshing of the defect surface. The Kirchhoff model has been extended in Civa11 to deal with anisotropy and impedant (non-rigid) interfaces [6].
- the geometrical theory of diffraction (GTD) to treat scattering from crack edges [7,8]. This approximation is valid away from specular angles and forward paths. The corresponding integrated model requires the meshing of the flaw contour. In Civa 11, the GTD model has been extended to deal with indirect echoes (diffraction echoes after reflection on the specimen surfaces) and has become a 3D GTD model which uses 3D GTD diffractions coefficients.

- the modified Born approximation to deal with solid inclusions. It provides an analytical solution for some flaw geometries (spherical, cylindrical and ellipsoidal) without any meshing of the flaw [9]. The modified Born model has been extended in Civa11 to deal with any incidence on inclusions.

An exact analytical solution for the scattering from a cylindrical cavity, based on the Separation Of Variables (SOV) method, has been used since Civa 10 to simulate the response of a side drilled hole [5]. This model is in addition available in CIVA 11 for solid spherical inclusions.

The main general assumptions applied to deal with the application of the semi-analytical models are described in [2,5].

These previous methods have been experimentally validated in the most commonly used configurations [10].

Since CIVA10.1 release, it is also possible to use a 2D numerical method to model the beam/flaw interaction especially in some complex configurations. Indeed, the hybrid model CIVA/ATHENA [11] is available for simulating the 2D response of SDHs and cracks (rectangular, CAD, multifaceted, branched) and uses the following principle. The pencil method used for CIVA beam calculations is applied to deal with most of the propagation, while intricate interaction phenomena located in a small region surrounding the defects are computed numerically by the finite elements (FEM) code ATHENA developed by EDF.

In Civa 11, another approximate analytical solution has led to the development of a new model called Kirchhoff & GTD (as shown in Figure 1) which is the subject of this paper.

![Figure 1: choice of the Kirchhoff & GTD in CIVA 11.](image)

**PRINCIPLE OF THE KIRCHHOFF & GTD MODEL**

The Kirchhoff & GTD model [12] is devoted to the simulation of both reflection and diffraction echoes from crack-like flaws.

The two previous approaches (Kirchhoff and GTD) appear to be complementary. Combining them so as to retain only their advantages, we have developed a hybrid model (the so-called Kirchhoff & GTD) using a procedure [12] similar to the physical theory of diffraction (PTD)[13].

Indeed, the Kirchhoff model is useful for the modelling of echoes due to specular reflections but is less accurate for observation directions far from the specular one since it doesn’t model correctly and quantitatively edges diffraction. On the other hand, contrary to Kirchhoff, the GTD model is not valid for specular observation direction since the GTD coefficient diverges but GTD is very effective to predict edge diffractions echoes in most configurations.
In the PTD formalism, an approximation for the Kirchhoff scattered field in far field from the flaw is done. It assumes that the Kirchhoff scattered field can be decomposed in an approximate manner in two parts: a geometrical field which includes the specularly reflected field and a contribution arising from the flaw edges corresponding to the edges diffraction field. This diffraction field contribution at the observation point \( x \) has the same form as the GTD field but a different edge diffraction coefficient (depending on the \( \alpha \) incidence and \( \beta \) observation directions and polarizations):

\[
U^{KA\text{Diff}}(x) = D^{KA}_{\alpha\beta}(x) \frac{e^{i\lambda r}}{\sqrt{kr}} \tag{1}
\]

Note that this coefficient characterizes the directivity of the Kirchhoff edge diffraction contribution.

The physical theory of diffraction (PTD) consists in correcting the Kirchhoff edge diffraction field by that modelled by GTD.

This correction leads to add a corrective term to the KA scattered field (without far-field approximation). This corrective term is the difference of wave amplitudes diffracted by the edge given by GTD and KA.

\[
U^{PTD}(x) = U^{KA}(x) + \left[D^{GTD}_{\alpha\beta}(x) - D^{KA}_{\alpha\beta}(x)\right] \frac{e^{i\lambda r}}{\sqrt{kr}} \tag{2}
\]

The PTD field is the sum of the Kirchhoff field and a GTD modified field in which the GTD coefficient has been replaced by the difference between GTD and Kirchhoff edge diffraction coefficients.

At the specular observation direction, the Kirchhoff field (without far-field approximation) is finite leading to an effective prediction of specular reflection. But the KA diffraction coefficient \( D^{KA}_{\alpha\beta}(x) \) for edge diffraction contribution (previously obtained from a far field approximation of the Kirchhoff field) diverges and has the same singularity as the GTD edge diffraction coefficient \( D^{GTD}_{\alpha\beta}(x) \). When making the difference of the two coefficients, their singularities cancel each other and the diffraction coefficients difference \( D^{GTD}_{\alpha\beta}(x) - D^{KA}_{\alpha\beta}(x) \) is finite. Consequently the PTD scattered field is spatially uniform and presents no singularity at the specular observation direction unlike GTD.

When the observation direction is close to the specular direction, the Kirchhoff field is predominant compared to the edge diffraction contribution and the Kirchhoff & GTD model leads to similar results than the Kirchhoff model:

\[
U^{PTD}(x) \approx U^{KA}(x) \tag{3}
\]

When the observation direction is far from to the specular direction, edge diffraction effects are predominant compared to reflection phenomena, the Kirchhoff field is equal to the Kirchhoff edge diffraction contribution and so cancels it so that the Kirchhoff & GTD model leads to similar results than the GTD model.

\[
U^{KA}(x) \approx D^{KA}_{\alpha\beta}(x) \frac{e^{i\lambda r}}{\sqrt{kr}} \quad \text{and} \quad U^{PTD}(x) \approx D^{GTD}_{\alpha\beta}(x) \frac{e^{i\lambda r}}{\sqrt{kr}} = U^{GTD}(x) \tag{4}
\]

Flaws which can be modelled thanks to Kirchhoff & GTD are the same than with the GTD model: planar flaws (rectangular, semi-elliptical or CAD contour planar flaws), multi-faceted flaw and branched flaw.
VALIDATIONS OF THE KIRCHHOFF & GTD MODEL

Both theoretical and experimental validations of the new developed Kirchhoff & GTD model have been performed on steel specimens.

a. Theoretical validations: direct modes in pulse echo configurations

Theoretical validations have consisted in comparing different models including the new developed one for the simulation of direct echoes in pulse echo configurations. In most configurations, the hybrid CIVA/ATHENA model using finite elements (FEM) for flaw scattering modeling is employed as reference model.

The first studied configuration is presented in Figure 2.a and deals with the pulse echo inspection with SV45° waves at 5MHz of a rectangular 5mm high defect of various tilt angle $\alpha$. In Figure 2.d is compared the amplitude (in dB) of the flaw simulated echoes versus the tilt angle $\alpha$ for different Civa11 models: GTD, Kirchhoff (KA) and Kirchhoff & GTD (PTD). This comparison illustrates the unified modeling provided by the GTD-KIRCHHOFF (PTD) model. Indeed, when the flaw is observed by the probe near specular reflection (for $\alpha=-45^\circ$, Figure 2.b and yellow area of Figure 2.d), GTD is invalid (the theoretical divergence is avoided by a treatment in the CIVA integrated GTD model without removing the invalidity) but KA is very effective and consequently, the PTD model leads to results equivalent to Kirchhoff ones. When diffraction effects are observed (in blue areas of Figure 2.d), KA leads to prediction errors: for instance for the classical case of a vertical flaw inspected with S45° waves (for $\alpha=0^\circ$, Figure 2.c), the KA error compared to FEM (see Figure 3) is of 6dB order. For diffraction modeling, GTD is much more effective than KA and PTD leads to equivalent simulation than GTD in the blue areas.

![Figure 2](image)

Figure 2: a) pulse echo inspection with SV45° waves at 5MHz of a rectangular 5mm high defect of various tilt angle $\alpha$; b) specular reflection configuration for $\alpha=-45^\circ$; c) for $\alpha=0^\circ$: classical case of a vertical flaw inspected with S45° waves and observed diffraction effects; d) comparison of the simulated echoes amplitude (in dB) versus the tilt angle $\alpha$ for different Civa11 models: GTD, Kirchhoff (KA) and Kirchhoff & GTD (PTD).

In Figure 3, the same configuration as in Figure 2 is still studied but the results from 2D FEM CIVA/ATHENA are also provided. The simulated amplitudes shown in Figure 3.b are this time absolute and still represented versus the tilt angle; Figure 3.c is a zoom devoted to small amplitudes. A perfect agreement between PTD and FEM is obtained near specular direction (reflected echoes) in Figure 3.b for $\alpha=-45^\circ$. On the other hand, the PTD model can lead to some prediction errors in diffraction compared to the FEM reference model but only for S incident waves. Indeed, the FEM curve presents beyond L critical angle oscillations due to interferences between head waves (detailed in Figure 4) and the S->S diffracted wave which are not accounted by the PTD model. This
The phenomenon appears for angles between observation and specular directions more than 33° (in steel). The PTD error is generally still acceptable except exactly at the critical angle (α=-78° or -12°). When increasing flaw height, the oscillations phenomenon fades and an improvement in the prediction for diffraction is observed.

Figure 3: a) pulse echo inspection with SV45° waves at 5MHz of a rectangular 5mm high defect of various tilt b) and c) zoom: comparison of the simulated absolute echoes amplitude versus the tilt angle α for different 2D Civa11 models: FEM, Kirchhoff (KA) and Kirchhoff & GTD (PTD).

Figure 4 details the paths of the waves that interfere together at critical incidence. These waves arise only from the bottom edge and consists in: a) classical diffraction of the incident SV bulk wave into a SV bulk wave; b) due to critical incidence of the S wave, creation of a P creeping wave propagating along the flaw which is diffracted by the bottom edge into a S wave notably in the backscattering direction; c) diffraction at the bottom edge of a P creeping wave propagating along the flaw which radiates a S head wave along the critical direction (equal to the backscattering one). The interferences between these three contributions lead to the PTD peaks at critical angles (Figure 3.c) and to the less effective prediction for SV waves near L critical angles (notably for small flaws).

Figure 4: a) Classical diffraction of the incident SV bulk wave into a SV bulk wave b) diffraction of a P creeping wave by the bottom edge c) radiation of a S head wave along the critical direction.

P waves are now considered. The last presented theoretical validation involves the pulse echo inspection of a rectangular 5mm high defect at 5MHz with P waves of various incidences. To impact the flaw with various α incidences, a cylindrical specimen is used and the probe is rotated along the specimen surface and positioned so as to be at normal incidence to this entry surface (Figure 5.a). Using such a procedure, the beam impacts the flaw with different P wave incidences during the probe scanning. In Figure 5.b is compared the amplitude (in dB) of the flaw simulated echoes versus the tilt.
angle $\alpha$ for different Civa 11 2D models: CIVA/ATHENA (FEM), Kirchhoff (KA) and Kirchhoff & GTD (PTD). Figure 5.c is a zoom devoted to small amplitudes. The Kirchhoff & GTD model leads to a very good agreement with FEM for all tilt angles. A logical equivalence with KA is observed near the specular reflection configuration ($\alpha=90^\circ$, see Figure 5.b) and with GTD (not represented here for simplification) for a diffraction configuration. In the latter case, the Kirchhoff model leads to prediction errors for tilt angles corresponding to observation directions far from the specular one.

Figure 5: a) inspection at 5MHz with P waves of various incidences of a rectangular 5mm high defect b) and c) zoom: comparison of the simulated echoes amplitude in dB versus the tilt angle $\alpha$ for different 2D Civa11 models: FEM, Kirchhoff (KA) and Kirchhoff & GTD (PTD).

Note that for both P and SV waves, a deterioration of the PTD prediction can be observed for small flaws (rarely encountered in NDT) in diffraction configurations since the diffraction at edges of the Rayleigh waves propagating along the defect are not currently predicted in the Kirchhoff & GTD model.

b. Experimental validations

Intensive experimental validations of the developed PTD model have been carried out both in pulse echo and TOFD configurations by studying both diffraction echoes and corners echoes resulting from specular reflections.

In order to validate simulation of corners echoes in pulse echo configuration, measurements were performed on a mock-up with vertical backwall breaking notches of various heights (Figure 6a). A 64 elements contact matrix phased-array probe with a wedge dedicated to generate 45° longitudinal waves was used (Figure 6.c and d). A 2D scanning of the probe was performed over each reflector. At each probe position, different focusing laws were applied: several depths focusing (from 10 to 40mm depth) with L45° deviation. The reference amplitude corresponds to the maximum amplitude of the specular L45° direct echo obtained on a 2mm diameter SDH at 72mm depth. To check the experimental reproducibility, the experimental measurements were carried out several times (minimum 2 times for all the flaws); a maximum difference of less than 1dB was obtained. Figure 6.c and d present experimental and CIVA 11 simulated reconstructed B-scans obtained over the 15mm height backwall breaking notch obtained for a L45° deviation with focusing at 30mm depth. A very good agreement between experimental and simulated B-scan is obtained. Note that in this case, the corner echo is due to a specular reflection on both the backwall and the flaw and consequently the CIVA 11 Kirchhoff & GTD simulation is equivalent to the Kirchhoff one with Civa 10.
Applications to TOFD inspection are then considered. First a detailed comparison with Ravenscroft’s results [14] is discussed hereafter. Ravenscroft notably employed a symmetrical TOFD arrangement (Figure 7.a) of both transmitting and receiving probes over a cylindrical mock-up containing a fatigue crack (mimicking a real one) and investigated the theoretical and experimental behaviour of the amplitude of the flaw edge diffraction echo versus the transmitting (and receiving) orientation of the beam (Figure 7.b).

On the simulated curve one clearly observes a minimum amplitude of the flaw response about 38°, which accurately corresponds to the minimum of the longitudinal-longitudinal GTD coefficient [14]. CIVA 11 simulation results show a pretty good agreement with Ravenscroft’s results. The Civa 11 PTD model leads to an improvement for small incident angles on bottom edges compared to 2D GTD Civa 10.

Another experimental validation was performed to evaluate the effect of skew flaw angle on edge diffraction amplitude. Tests were carried out on a planar rectangular backwall breaking flaw using a pair of transducers (2.25 MHz, 45° longitudinal wave, 6.35 mm diameter). The probes were
positioned in a TOFD configuration with a 60mm Probe Space Center and with flaw skew angle varying from 0° to 70° (Figure 8.a).

Figure 8: a) TOFD configuration for measuring skew angle effect on edge diffraction b) top edge diffraction echoes amplitudes versus skew angle for measure, 2D GTD Civa 10 and 3D PTD Civa11 simulations.

The experimental and simulated results are presented in Figure 8.b. Experimental results show that there is negligible effect of the skew angle on the diffraction echoes amplitudes. Moreover there is a good agreement between experimental and simulated results with a maximum difference of about 2 dB for 3D PTD CIVA 11 model whereas the 2D GTD CIVA 10 model breaks down for important skew (>40°).

CONCLUSIONS

An elastodynamic GTD-Kirchhoff model based on physical theory of diffraction (PTD) has been developed in Civa 11. This model provides a unified modeling of both specular reflection and diffraction phenomena. Whereas corner echoes were previously (CIVA 10) simulated by the Kirchhoff model and direct diffraction ones by the GTD (geometrical theory of diffraction) model, all phenomena are accounted by the CIVA 11 GTD-Kirchhoff model. Theoretical comparisons carried out on direct echoes with a FEM model has shown a very good overall prediction for P waves and also for S waves except in the latter case for incidences near the longitudinal critical angle (existence of head waves). A deterioration of the GTD-Kirchhoff prediction can be observed for small flaws (less significant case in NDT) in diffraction configurations. Successful experimental validations have been obtained in numerous NDT configurations: 2D and 3D, pulse echo and TOFD, on diffraction or corner echoes.

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