Modeling the Radiation Field of Normal-Beam Ultrasonic Transducers by Multi-Gaussian Beam Technique

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Abstract
The modeling of ultrasonic tests can help in gaining a better understanding of the test process. The model can also predict the test results and provide a basis for choosing the optimum parameter settings. One of the first steps in modeling an ultrasonic test is to understand the characteristics of the ultrasonic wave field. In this paper, the multi-Gaussian beam (MGB) technique is used for modeling the radiation field of a normal-beam ultrasonic transducer. By using this approach, one can avoid the computational difficulties encountered in other modeling techniques such as Rayleigh-Sommerfeld and Green function integrals. By modeling the radiation field of a 10 MHz transducer, the beam profile of the reflected signal have been calculated. Experiments confirm the computational results.

Keywords: Ultrasonic Modeling, Multi-Gaussian Beam, Wave Field.

Introduction
Modeling of wave field generated by a transducer can help in nondestructive evaluation of materials by prediction of echoes, simulation of defects, and also in signal processing. Moreover, a good model can be very helpful in the design and optimization of a testing procedure and in the interpretation of experimental results. It is also much simpler and cheaper to perform parametric studies with by using a good model compared to conducting experiments. By using the model, the probability of detection of various types of defects can be assessed.

In the earliest works of wave field modeling the transducer surface was considered to act as a source of point forces (Cherry 1962). This method had computation difficulties. By using Johnson's Green function, Tang et. al. (1990) derived integral expressions of the elastic displacements for both compression and shear piston sources acting on the surface of an elastic half-space. The shortcoming of this method is its inability to model the near-field of the wave field as well as oblique incidence modeling. Harris (1981) reviewed three approximations including Rayleigh surface integral, King integral, and Schoch solution to find the pressure field of a planar probe on an infinite half-space. Other numerical methods were also developed to show the displacement field in a solid media or the pressure field in a fluid due to wave radiated from normal or angular transducers (Alia et. al. 2004). Based on an elastodynamics solution, Bostrom and Wirdelius (1995) modeled the wave field for elliptical and rectangular transducers in a normal or angular position, emitting wave into a solid half space.

Although the exact on-axis field and an approximation to the far field can be expressed in a closed form, describing the near field, the transient zone, and off-axis beams are difficult and time consuming in numerical methods by integration over points of the probe surface.
Wen and Breazeale (1988) proposed an alternative approach. They computed the total field by superimposing a number of Gaussian beam solutions. They have shown that by superimposition of only 10 Gaussian solutions, the field radiated by a circular piston transducer can be accurately modeled.

In this paper, the multi-Gaussian beam technique is used for modeling the radiation field of a normal-beam ultrasonic transducer. By using this approach, one can avoid the computation difficulties encountered in other techniques including the Rayleigh-Sommerfeld and Green function integrals. In some ultrasonic tests, it is required to use ultrasonic beams with large spread angles. Based on the location of defects in the material, the wave can be reflected from any point on the beam cross section. By modeling the radiation field of a 10 MHz transducer, the beam profile and frequency spectrum of the reflected signal have been calculated.

**Multi-Gaussian Beam Modeling**

Modeling the fields radiated by ultrasonic transducers is a challenging task because of the large number of possible transducer types, sizes, and configurations that are used in practice. The multi-Gaussian beam model can describe the wave field of a circular piston transducer by superposition of a number of individual Gaussian beams with a proper set of weighting factors. Although ultrasonic transducers do not generate Gaussian beams but this modeling works well in considering the transducer to act as a piston source. One of the advantages of multi-Gaussian beam modeling is significant reduction in computation time as well as ability in modeling the wave field in anisotropic materials. In Gaussian beam modeling of circular transducers, it is first assumed that the transducer is a time harmonic Gaussian source with time dependency and waves are emitted perpendicular to the transducer surface ($x_3$ direction) as a quasi-plane wave. The displacement field in Gaussian beam is obtained from the following formulation (Kim et. al. 2004),

$$u^p (x_1, x_2, x_3, \omega) = \frac{iv_0}{\rho c_p \omega} \exp(i k_p x_3) \frac{\sqrt{\det M^p(x_3)}}{\sqrt{\det M^p(0)}} \exp \left( \frac{i k_p x^T [M^p(x_3)] x}{2} \right)$$  \hspace{1cm} (1)

where $v_0$ is the velocity on the transducer surface, $k_p = \omega / c_p$ is the wave number, $\omega$ is frequency in radians per second, $c_p$ is longitudinal wave speed in specimen, and $\rho$ is the density of the medium in which the wave radiates. Moreover, $x = [x_1, x_2]^T$ and $M^p$ is a 2x2 complex-valued symmetric matrix. The multi-Gaussian beam modeling can finally be written as (Schmerr and Song 2007),

$$u^p (x_1, x_2, x_3, \omega) = \sum_{n=1}^{N} \frac{i A_n}{1 + i B_n x_3 / D_r} \exp(i k_p x_3) \exp \left( i \omega x^T [M^p_n(x_3)] x \right) \hspace{1cm} (2)$$

where $N$ is the number of Gaussian beams, $D_r = k_p a^2 / 2$ is the Rayleigh distance, $a$ is the radius of the transducer and...
$\left[ M'_i(x_3) \right]_n = \begin{bmatrix} \frac{iB_n}{c_p D_R} & 0 \\ 1 + iB_n x_3 / D_R & \frac{iB_n}{c_p D_R} \end{bmatrix}$ \hspace{1cm} (3)

$A_n$ and $B_n$ are complex-valued expansion coefficients that need to be determined to match the velocity field on the face of the transducer. Wen and Breazeale (1988) found ten coefficients by an optimization method for circular planar piston transducers.

**Experiment**

The results of multi-Gaussian beam model have to be verified by comparison with other models or with experiments. An experiment was carried on a 69 mm thick aluminum block. The compressional and shear velocity in aluminum are 6200 m/s and 3150 m/s, respectively. The wave was generated by a 6.35 mm diameter transducer having a center frequency of 10 MHz. The test was performed by ultrasonic pulse-echo technique in which the single transducer acted as both the transmitter and receiver.

**Results and Discussion**

In the first step, to show the ability of multi-Gaussian beam for modeling both the near field and far field of the transducer, the computed pressure field of a circular transducer with ?? mm diameter and 5 MHz central frequency is illustrated in Fig. 1.

![Fig. 1- Modeling of the transducer wave field](image)

The backwall echo obtained from the aluminum test block is shown in Fig 2a. The frequency spectrum of this echo is also shown in Fig. 2b. Both the multi-Gaussian model and the Green function model were used for modeling the backwall echo. The Green function model was developed following the approach used by Tang et. al. (1994). The modeled echo obtained from this approach is shown in Fig. 2c along with its frequency spectrum shown in Fig. 2d. The echo obtained from the multi-Gaussian model is shown in Fig. 2e and its relative spectrum frequency is shown in Fig. 2f. Both modeled echoes are in good agreement with the experimental result.
Fig. 2- Comparison of experimental signal (a), Green function model (c) and multi-Gaussian beam modeling (e). Frequency spectra of three echo is shown in (b), (d) and (f) respectively.

Next, we compare the capability of the two models in modeling the radiated ultrasonic wave at various angles. We consider a 5 MHz transducer with 6.35 mm diameter to be transmitting the ultrasonic wave into a semi-circular specimen having a radius of 75 mm. The wave is then received by an identical transducer on the cured surface of the specimen as shown in Fig. 3.
As shown in Fig. 4, by increasing in angle with respect to the central axis of the transmitting transducer, the frequency spectrum of the received echoes change. This change of frequency spectra is due to the destructive interference of waves originating from various points on the transducer surface. Again the two models are in good agreement. Experimental verification of the two models is currently underway.

**Conclusion**

In this paper, the multi-Gaussian beam (MGB) approach was used to model the wave field of a normal beam transducer. The advantage of using this method is modeling the near-field as well as the far-field of the transducer in both fluids and solids. It also avoids the computational difficulties encountered with other modeling techniques. To examine the capability of this model, a backwall echo was modeled by the multi-Gaussian beam approach and compared with the experimental as well as Green function modeling results. The result obtained from the MGB model was in good agreement with the experimental result. Moreover, the frequency spectra of beam radiated from a normal transducer was modeled by the MGB approach and compared with Green's function results. Again, the two models were in good agreement. The MGB technique can be used in modeling the refraction and reflection from interfaces without any computation difficulties.

**Fig. 4** Frequency spectrum comparison of MGB (solid line) and Green's function (dashed line)
Reference


