Non-Destructive Evaluation of Elastic Modulus in Metals
Using Lamb Wave Technique

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Abstract:
Estimation of Elastic constants of metals is of paramount importance for the evaluation of the structure's mechanical performance. Tensile testing is widely used for estimating elastic properties of metallic material. Such destructive tests may not aid in in-situ assessment of elastic properties of end products viz. pressure vessels, air-frame structures, reactor components etc. A simpler and accurate NDT method is required for such applications. This paper describes a novel NDT method implemented using Lamb waves for the determination of elastic modulus. Lamb waves are similar to longitudinal waves and occur as a result of compression and rarefaction of waves. The pulse-echo method was employed in the experimental procedure to calculate the phase velocity of the Lamb wave. Modes such as $S_0$ and $A_0$ were identified at a constant frequency of 300 kHz and for a fixed distance of 30mm between the probes. The same procedure was adopted for materials such as Maraging steel and 15CDV6 Steel. Three samples of each material were tested and the phase velocity thus arrived at, was used to calculate the modulus value. The same was verified using conventional testing, i.e., Tensile test. The results obtained by both methods agree well with a maximum error of 8.6%. A proven NDT technique for in-situ evaluation of elastic properties of metallic structures is implemented. Various sources of errors in the estimation of elastic constants and their contributing factors are also discussed in detail.

Keywords: Lamb Wave, Elastic Modulus, NDT, Phase Velocity

Introduction:
In recent years, non-destructive evaluation of material properties using Rayleigh-Lamb wave method has been receiving increased attention \cite{1, 2, 3}. The bulk waves used in traditional Ultrasonic methods, travel in the thickness direction of the material. The limitations in using the traditional non-destructive method are: 1) The wavelength must be smaller than the thickness of the specimen and hence higher frequencies are required for thin specimens; 2) the method is not applicable to dispersive waves; 3) material considered has to be isotropic as the properties are measured in the thickness direction of the specimen. The Lamb waves are guided waves that travel in the plane of the plate which are generally of lower frequency and can travel for longer distances with lesser attenuation. The wavelength of the Lamb waves are greater than the thickness of the plate and can be generated at various frequency ranges. The Lamb waves are used to measure the in-plane and out-of-plane material properties. Lamb wave method has also been implemented in anisotropic materials (composites) to measure the material properties.
Rayleigh – Lamb Wave Propagation:

Elastic displacements propagating in a solid plate with traction-free boundaries are referred to as Lamb waves \(^5, 6\). They are guided waves formed by interference of multiple reflections and mode conversion of longitudinal waves and shear waves at the free surfaces of the plate. These guided waves exists only at certain frequencies and phase velocities. These set of permissible frequencies and phase velocities are referred to as dispersion curves. There are three distinct types of guided waves in a plate with traction-free boundaries: horizontally polarized shear waves, longitudinal waves and flexural waves. In our study, only longitudinal (symmetric) and flexural (anti-symmetric) waves are of interest. The Rayleigh-Lamb dispersion equations for isotropic plate are given by \(^7\):

For symmetric modes:

\[
\frac{\tan \beta d}{\tan \alpha d} = \frac{4\alpha \beta k^2}{(k^2 - \beta^2)^2},
\]

(1)

And for anti-symmetric modes:

\[
\frac{\tan \beta d}{\tan \alpha d} = \frac{(k^2 - \beta^2)^2}{4\alpha \beta k^2}.
\]

(2)

Where,

\[
\alpha^2 = \frac{\omega^2}{C_L^2} - k^2.
\]

(3)

\[
\beta^2 = \frac{\omega^2}{C_T^2} - k^2.
\]

(4)

\(d\) is the thickness of the plate, \(k (= \frac{\omega}{C_p})\) is the wave number, \(\omega (= 2\pi f)\) is the circular frequency, \(C_p\) is the Lamb wave phase velocity, \(C_L\) is the longitudinal wave velocity, \(C_T\) is the shear wave velocity.

The longitudinal and shear velocities are related to the material constants by:

\[
C_L = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-\nu)}}.
\]

(5)

And,

\[
C_T = \sqrt{\frac{E}{2\rho(1+\nu)}}.
\]

(6)

The relationship between phase velocity and material properties are also governed by dispersion curves.
When $C_L$ and $C_T$ are known, one can find the elastic constants $E$ and $\nu$ by:

$$\nu = \frac{1 - 2\left(\frac{C_T}{C_L}\right)^2}{2 - 2\left(\frac{C_T}{C_L}\right)^2}.$$  \hspace{1cm} (7)

And,

$$E = 2\rho C_T^2 (1 + \nu).$$  \hspace{1cm} (8)

**Forward and Reverse Problems:**

Lamb waves are used to address two types of problems. The first one being the forward problem where the material properties are known and the dispersive characteristics of the wave are studied with respect to the plate geometry. The second case is the inverse problem where the phase velocities are measured experimentally and the elastic constants are derived by substituting and iterating the wave equations \(^4\). The primary objective of this work is to develop a simple and effective method to measure the elastic constants of different materials by measuring the phase velocity of Lamb waves. Secondary objective is to discuss the various possibilities of error and its effect on the result.

**The Forward Problem - Dispersion Curve:**

The primary aim of the forward problem as discussed earlier is to generate the dispersion curve and to study the effect of frequency and geometry of the plate on the existence of various wave modes. The theoretical dispersion curve (using eqns. 1 to 8 above) for maraging steel of 5.55mm thickness is shown in figure 2. It can be seen that when the wave is excited at values of $fd$ lower than 0.7 MHz mm, the zero modes i.e., $\alpha_0$ and $s_0$ only exist. Since $S_0$ is highly attenuating, it can be easily eliminated. As the frequency increases, it can be observed that the higher order modes also come into existence. For example, at $fd=1$ MHz mm, both $\alpha_0$ and $a_1$ exist. It may be difficult to differentiate between two nearer modes experimentally. Often this created confusion in
identification of the peaks. Such a condition also occurs when the amplitude of the received wave undergoes interference and hence varies its position on the time axis.

![Dispersion curve for maraging steel specimen 5.55mm thick](image)

**Fig. 2 Dispersion curve for maraging steel specimen 5.55mm thick**

**Inverse Problem:**

In the inverse problem, i.e., determining the material properties by measuring the phase velocity and frequency of the Rayleigh – Lamb wave is carried out. The phase velocity – material properties relationship is however governed by the wave equations Eqns (1) - (8). The unknowns in this case are the material properties. The equations here are supplied with initial guess values of the material properties i.e. E & ν and are continuously iterated using Matlab to obtain convergence. In other words, the error arising out of the difference between Matlab calculated $C_p$ value and experimental $C_p$ value is minimized. The input values at the convergence step yield the material properties i.e. elastic modulus.

**Experimental Method:**

Figure 1 is a schematic diagram of the experimental setup. The transmitting transducer is a piezo-electric sensor, was excited using a sine wave tone burst from the Function Generator at 300 kHz frequency. The function generator used has 40MHz frequency range. The oscilloscope used has a Frequency range of 600MHz.

![Schematic of Experimental Setup](image)

**Fig. 1 Schematic of Experimental Setup**
The receiving transducer placed at 30mm from the transmitting transducer was connected to an oscilloscope. Best results with least reflections and attenuation were observed when the transducers were placed 30mm apart. The phase velocity of the Lamb wave was measured using:

\[ C_p = \frac{D}{t}. \]  \hspace{1cm} (9)

Where \( D \) is the distance between the transducers and \( t \) is the time taken for the wave to propagate. Three samples of each specimen with 50mm gauge length of Maraging steel of 5.55mm gauge thickness and 15CDV6 Steel of 3.67mm gauge thickness were measured.

**Results and Discussion:**

The Elastic modulus for different specimens are calculated from the measured phase velocities using Matlab\(^8\), is given in table 1.

Table 1 Elastic modulus measurement by Inverse problem

<table>
<thead>
<tr>
<th>S. No</th>
<th>Computed E [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maraging Steel</td>
</tr>
<tr>
<td>1</td>
<td>204</td>
</tr>
<tr>
<td>2</td>
<td>186</td>
</tr>
<tr>
<td>3</td>
<td>186</td>
</tr>
</tbody>
</table>

The Elastic modulus for maraging steel specimens was found to be 186GPa and 204GPa and for 15CDV6 steel it was 185GPa and 199GPa. Further, tensile test was done to verify and find the deviation from the computed result. It is observed that, a similarity in the Elastic modulus was observed between the two specimens used. While the tensile test reports suggest that though there is a similarity in the modulus values, the elongation of the two materials were not the same. The maraging steel specimen elongates between 8-10% while the 15CDV6 steel specimen elongates between 10-14%. The data from the tensile test was plotted, the graph was linearized and the median of the modulus values were found using Matlab. A similar procedure was followed for all the specimens used and the error percentage was calculated based on the Lamb wave experimental values and the tensile test values. For all the specimens used, a similar percentage of error was observed and the uncertainties account for the variation in the material thickness, inaccurate measurement in the distance between the sensors, approximations involved in numerical calculation by Matlab program and due to assumption of \( v \) value. Keeping the causes of error in mind, one can improve the accuracy of the results by either minimizing the cause of error or by analyzing the effect of error on the result and apply suitable corrections.

As stated above, the first cause of error may be due to inaccurate measurement of the distance between the sensors; the effect of which is discussed below. We have kept the distance between the sensors as 30mm between the sensors and assuming a tolerance of ±1mm, the effect of this error is discussed below.

Table 2 Error percentage due to inaccurate distance measurement

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Distance[m]</th>
<th>time [s]</th>
<th>Velocity [m/s]</th>
<th>Error%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.030</td>
<td>0.00001314</td>
<td>2283.10502</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.029</td>
<td></td>
<td>2207.00152</td>
<td>3.3333333333</td>
</tr>
<tr>
<td></td>
<td>0.031</td>
<td></td>
<td>2359.20852</td>
<td>-3.3333333333</td>
</tr>
</tbody>
</table>
There is a 3.33% error in the value of phase velocity $C_p$ due to a 1mm difference in the measured distance.

The second source of error may be due to approximations involved in the numerical method used. This is verified by conducting conventional tensile test and observing the deviation from the computed result. The tables 3&4 show the difference in the values.

For thick maraging steel specimens:

Table 3 Error in $E$ values of maraging steel

<table>
<thead>
<tr>
<th>Computed $E$[GPa]</th>
<th>Calculated $E_{\text{exp}}$[GPa]</th>
<th>Error%</th>
</tr>
</thead>
<tbody>
<tr>
<td>204</td>
<td>190.98</td>
<td>6.382353</td>
</tr>
<tr>
<td>186</td>
<td>186.8701</td>
<td>-0.4678</td>
</tr>
<tr>
<td>186</td>
<td>196.8311</td>
<td>-5.82317</td>
</tr>
</tbody>
</table>

For 15CDV6 specimens:

Table 4 Error in $E$ values of 15CDV6

<table>
<thead>
<tr>
<th>Computed $E$[GPa]</th>
<th>Calculated $E_{\text{exp}}$[GPa]</th>
<th>Error%</th>
</tr>
</thead>
<tbody>
<tr>
<td>187.19</td>
<td>203.2969</td>
<td>-8.60457</td>
</tr>
<tr>
<td>199</td>
<td>213.327</td>
<td>-7.199</td>
</tr>
<tr>
<td>195</td>
<td>202.726</td>
<td>-3.962</td>
</tr>
</tbody>
</table>

The third cause of error is the effect of assumption of $\nu$ values. It can be observed that, $E$ has a direct impact on the value of $C_p$ and $\nu$ has an insignificant impact on the value of $C_p$ \[9\]. For example, a 10% change in $E$ results in 5% change in $C_p$, a 10% change in $\nu$ results in 1% change in $C_p$ and a 10% change in $E$ and $\nu$ together results in a 4% change in $C_p$. This is represented in Figures 3 &4.

![Effect of variation of E on C_p](image)

Fig 3. Effect of E variation on $C_p$
Summary and Conclusion:

An experimental procedure for the estimation of elastic constants of metallic materials by measuring Rayleigh – Lamb wave phase velocity has been discussed. Based on the experiment, it has been observed that by measuring the phase shift of the harmonic wave, phase velocity can be measured with good accuracy.

Elastic modulus was calculated using Matlab with the measured phase velocity and the known frequency value using the inverse problem technique. Traditional tensile test was carried out to estimate actual Elastic modulus and the error percentage was calculated. Other possibilities of error and its effect on the result have also been discussed.

This paper shows that estimation of elastic constants can be achieved using Lamb wave non-destructive evaluation method for any metallic materials. This method can be implemented in situations where tensile testing is not feasible. This method can widely be used for finished products irrespective of the size and shape to estimate the modulus values non-destructively.
References:


