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Hysteretic Nonlinear Response of Contacting Interface to Harmonic Acoustic Waves

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Abstract

A hysteretic nonlinear model for solid-solid contact interfaces is developed by means of Coulomb friction to investigate the possibility of nonlinear interface waves propagating along hysteretic nonlinear contact boundaries. Dispersion equation of the interface waves is derived by combining the wave equation with boundary conditions: hysteretic nonlinear discontinuity in displacements and tractions. In the model, the traction on the interface is related to the discontinuity using a hysteretic interfacial stiffness of contact interfaces. The existence of the interface waves is demonstrated by harmonic wave analysis along the contact interface. Analysis results show that there exists only anti-symmetric mode of interface waves along hysteretic contact interfaces and the phase velocity of the waves is highly dependent on contact conditions represented by linear and hysteretic nonlinear stiffness.

Résumé

Un modèle hystérétique non-linéaire pour des interfaces de contact solides-solides est développé au moyen d'une loi de friction de Coulomb pour examiner la possibilité d'ondes d'interface non-linéaires se propageant le long des frontières de contact hystérétiques non-linéaires. L'équation de dispersion des ondes d'interface est obtenue en combinant l'équation d'onde avec des conditions aux limites : discontinuité hystérétique non-linéaire dans les déplacements et tractions. Dans le modèle, la traction sur l'interface est associée à la discontinuité en utilisant une rigidité hystérétique interfaciale des interfaces de contact. L'existence des ondes d'interface est démontrée par l'analyse d'onde harmonique le long de l'interface de contact. Les résultats d'analyse montrent qu'il existe seulement un mode anti-symétrique d'ondes d'interface le long des interfaces hystérétiques de contact et la vitesse de phase des ondes est fortement dépendante des conditions de contact représentées par la rigidité linéaire et hystérétique non-linéaire.

Keywords

Interface wave, contact acoustic nonlinearity, dispersion, hysteresis

1 Introduction

Contact-type discontinuity such as closed cracks leads to an anomalously high level of nonlinearity. The variation of contact area within the interface due to deformation of asperities is known to cause nonlinear elasticity of the interface. The physical nature of the contact acoustic nonlinearity(CAN) has been explained by developing linear mathematical models of contact-type interface[1,2]. In their works, this interface is considered as a linear spring whose stiffness is proportional to the contact area within the interface under an assumption that ultrasonic waves produce only a small amount of deformation. The linear model of spring-type crack interface connects displacements with stresses on both sides of the interface by employing linear spring stiffness. These interfacial stiffnesses are known to offer useful information on the nature of the contact interface. They have carried out theoretical analysis of the steady-state wave propagation along a spring-type interface between two elastic half-spaces and have shown the existence of two distinct modes of propagation; namely, the symmetric mode and the anti-symmetric mode that are governed separately by the normal and the tangential interfacial stiffnesses. These waves are a special kind of guided waves propagating along the interface and can be quite useful to detect cracks which are closed so tightly that they do not produce linear scattering waves. However, it is also well known in experiment that contact interface has hysteresis during its compression/tension cycle while interacting with acoustic waves.

In this paper, a hysteretic nonlinear displacement discontinuity model for contact interface is sought and analyzed to investigate the possibility for interface waves to propagate along the nonlinear contact boundaries and to estimate contact state of the interface. Dispersion equation of the waves is derived by combining the wave equation with the boundary conditions: nonlinear discontinuous displacements. In the model, the traction applied to the interface is defined to be hysteretic to the discontinuity in displacement using the hysteretic nonlinear specific stiffness of the contact interface. The existence of the waves is verified in theory by plane wave analysis along contact interface.

2 Hysteretic Nonlinear displacement discontinuity model

At the micro-scale, contact interface appears as two surfaces of irregular topology which intersect to form micro-void spaces and asperities of contact. The presence of the asperities and voids within planar crack define a thin, compliant zone with effective normal and shear stiffnesses that can range from near zero for open crack to almost infinite values for completely closed crack which are bonded or subjected to high compressive stresses. Typically a crack loaded in shear or compression exhibits a highly nonlinear stress-displacement relationship resulting from deformation of the asperities. In addition, hysteresis appears during the loading and unloading[3,4], which indicates the presence of inelastic deformation of the asperities of contact and frictional sliding between contacts. In linear displacement discontinuity model or imperfect interface model, two contact surfaces of each boundary are assumed to be continuous in stress but not in displacement at which the specific stiffness is defined as a linear spring such that the stress is proportional to the displacement difference[5-7]. But the stress continuity is not guaranteed in a discontinuity that has a hysteresis caused by friction like closed or partly closed cracks.

Suppose that two identical elastic bodies put into contact by static pressure undergo a hysteretic nonlinear-oscillatory deformation cycle between two deformation states of the interface, u_1 and u_2 caused by acoustic wave. An irreversible deformation starts from u_1 to u_2 following a loading curve and returns to the original point u_1 via an unloading curve different from the loading curve. In these processes, the contact interface is assumed to go through Coulomb friction which imposes a force opposite to the direction of motion on the sliding surface. This frictional loss can be modeled by a hysteretic nonlinear spring, whose stiffness is always equal in amplitude but changes in its sign (positive or negative) according to either loading or unloading process. On these assumptions, the stress-displacement relationship across the interface can be expressed for each loading and unloading process by superposition of a linear spring κ and hysteretic nonlinear spring κ_n such as

$$\begin{aligned} \sigma_{loading} &= \kappa\ (u_2 - u_1) + \kappa_n (u_2 - u_1) \qquad \text{for } \textit{loading} \\ \sigma_{unloading} &= \kappa\ (u_2 - u_1) - \kappa_n (u_2 - u_1) \qquad \text{for } \textit{unloading} \end{aligned} \tag{1}$$

Now consider that the identical contacting solids undergo a microscopically imperfect elastic-plastic deformation between the upper (denoted by superscript *u*) and lower (denoted by superscript *l*) rough surfaces as shown in Fig. 1. The displacement of the upper surface is $\boldsymbol{u^u} = (u_x^u, u_z^u)$ and the lower surface $\boldsymbol{u^l} = (u_x^l, u_z^l)$. The sizes of asperities are assumed to be much smaller than the wavelength of acoustic wave so that the incoherent scattering from the interface is negligible. Since both displacement and stress have discontinuity across the interface, some boundary conditions are necessary to connect those discontinuities. These conditions are obtained from the constitutive properties of the interface formulated by the hysteretic nonlinear spring κ_n in Eq. 1 between tractions and displacements across and along the interface in normal and shear directions (*z* and *x* in Fig. 1). The stress-displacement relations given by Eq. 1 and applied to the interface of Fig. 1 yield the following equations

$$
\begin{aligned}
\boldsymbol{t}_x^u &= \kappa_x(\Delta u_x) + \kappa_{nx}(\Delta u_x) = \kappa_x(u_x^u - u_x^l) + \kappa_{nx}(u_x^u - u_x^l) \\
\boldsymbol{t}_x^l &= \kappa_x(\Delta u_x) - \kappa_{nx}(\Delta u_x) = \kappa_x(u_x^u - u_x^l) - \kappa_{nx}(u_x^u - u_x^l) \\
\boldsymbol{t}_z^u &= \kappa_z(\Delta u_z) + \kappa_{nz}(\Delta u_z) = \kappa_z(u_z^u - u_z^l) + \kappa_{nz}(u_z^u - u_z^l) \\
\boldsymbol{t}_z^l &= \kappa_z(\Delta u_z) - \kappa_{nz}(\Delta u_z) = \kappa_z(u_z^u - u_z^l) - \kappa_{nz}(u_z^u - u_z^l)
\end{aligned}
\qquad (2)
$$

where, $\boldsymbol{t}_x^u, \boldsymbol{t}_x^l, \boldsymbol{t}_z^u, \boldsymbol{t}_z^l$ are normal and shear tractions on the upper and lower surface, κ_z and κ_x are the linear normal and shear stiffness, κ_{nx} and κ_{nz} nonlinear normal and shear stiffness, and $u_x^u, u_x^l, u_z^u, u_z^l$ are the displacements of the upper and lower surface in x and z as shown in Fig.1. The signs of the hysteretic nonlinear stiffnesses, κ_{nx} and κ_{nz} in Eq. 2, are assigned positive if the displacement and traction increase with the coordinate x and z, and negative if the displacement and traction decrease with the coordinate x and z. It could be possible to use the sign rule conversely. It can be found from Eq. 2 that if the hysteretic nonlinear stiffnesses κ_{nx}, κ_{nz} are all zero, Eq. 2 leads to $\boldsymbol{t}_x^u = \boldsymbol{t}_x^l$, $\boldsymbol{t}_z^u = \boldsymbol{t}_z^l$, which means stress continuity across the interface. It also represents the limiting cases for a traction-free boundary condition as κ_x and κ_z go to zero and for a welded interface as κ_x and κ_z become infinity. Eq. 2 gives a simple but pertinent model for the hysteretic nonlinear properties of non-welded contact between two identical media.

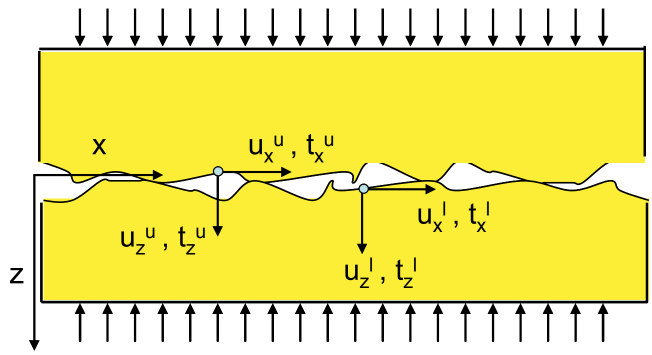

Figure 1. *Two surfaces in contact by static pressure*

3 Generation of guided waves along contact surface

Displacement vector for an inhomogeneous plane wave propagating along the contact interface in *x* direction with the amplitude that decays exponentially with distance *z* away from the interface can be expressed for the upper medium using the superscript *u* as [8]

$$u_x^u(x,z) = \omega[i\frac{A_1}{c}e^{-p\omega z} + qB_1 e^{-q\omega z}]e^{i(\kappa x-\omega t)}, \qquad u_z^u(x,z) = \omega[-pA_1 e^{-p\omega z} + i\frac{B_1}{c}e^{-q\omega z}]e^{i(\kappa x-\omega t)} \tag{3}$$

, for the lower medium using the superscript l

$$u_x^l(x,z) = \omega[i\frac{A_2}{c}e^{+p\omega z} - qB_2 e^{+q\omega z}]e^{i(\kappa x-\omega t)}, \quad u_z^l(x,z) = \omega[pA_2 e^{+p\omega z} + i\frac{B_2}{c}e^{+q\omega z}]e^{i(\kappa x-\omega t)} \tag{4}$$

In Eq. 3 and 4, $p^2 = 1/c^2 - 1/c_p{}^2$, $q^2 = 1/c^2 - 1/c_s{}^2$, ω is the angular frequency, t is time, A_1, A_2, B_1 and B_2 are unknown constants, c is the phase velocity of interface wave, and c_p and c_s are the longitudinal and shear wave velocity respectively. Tractions in the upper and lower medium obtained by substituting Eq. 3 and 4 into Hooke's law can be combined with the nonlinear displacement discontinuity condition in Eq. 2 to obtain a system of four homogeneous linear equations for four undetermined constants, A_1, A_2, B_1 and B_2.

$$\begin{vmatrix} \frac{i}{c}(2\kappa_x + 2\mu\omega p) & (2q\kappa_x + \mu\omega N) & -\frac{i}{c}(2\kappa_x + 2\mu\omega p) & (2q\kappa_x + \mu\omega N) \\ (2p\kappa_z - \omega Q) & -\frac{i}{c}(2\kappa_z + 2\mu\omega q) & (2p\kappa_z - \omega Q) & \frac{i}{c}(2\kappa_z + 2\mu\omega q) \\ \frac{i}{c}(2\kappa_{nx} + 2\mu\omega p) & (2q\kappa_{nx} + \mu\omega N) & -\frac{i}{c}(2\kappa_{nx} - 2\mu\omega p) & (2q\kappa_{nx} - \mu\omega N) \\ (2p\kappa_{nz} - \omega Q) & -\frac{i}{c}(2\kappa_{nz} + 2\mu\omega q) & (2p\kappa_{nz} + \omega Q) & \frac{i}{c}(2\kappa_{nz} - 2\mu\omega q) \end{vmatrix} = 0 \tag{5}$$

where, λ and μ are Lame's constants, $L = (\lambda + 2\mu)$ and $N = (c^{-2} + q^2)$, $Q = (\lambda c^{-2} - Lp^2)$. Examining the first two equations of Eq. 5 reveals that only two combinations of solutions are possible corresponding to two wave motions: one is symmetric about interface, the other is anti-symmetric. For anti-symmetric motion, the determinant in Eq. 5 reduces to

$$[4\alpha^2\sqrt{\alpha^2-1}\cdot\sqrt{\alpha^2-\beta^2} - (1-2\alpha^2)^2] + 2(\frac{\kappa_x}{\omega Z_s})\sqrt{\alpha^2-1} + 2(\frac{i\kappa_{nx}}{\omega Z_s})\alpha[1-2\alpha^2 + 2\sqrt{\alpha^2-1}\cdot\sqrt{\alpha^2-\beta^2}] = 0 \tag{6}$$

where, $\alpha = c_s / c$, $\beta = c_s / c_p$, $Z_s = \rho c_s$ is the shear acoustic impedance of the medium and $\kappa_x / \omega Z_s$, $\kappa_{nx} / \omega Z_s$ are the linear and nonlinear specific shear stiffnesses. In the same manner, the dispersion equation for symmetric wave motion of hysteretic contact interface is written by

$$[4\alpha^2\sqrt{\alpha^2-1}\sqrt{\alpha^2-\beta^2} - (1-2\alpha^2)^2] + 2(\frac{\kappa_z}{\omega Z_s})\sqrt{\alpha^2-\beta^2} + 2(\frac{i\kappa_{nz}}{\omega Z_s})\alpha[1-2\alpha^2 + 2\sqrt{\alpha^2-1}\sqrt{\alpha^2-\beta^2}] = 0 \tag{7}$$

where, $\kappa_z / \omega Z_s$, $\kappa_{nz} / \omega Z_s$ are linear and nonlinear specific normal stiffnesses. Eq. 6 and 7 are complete dispersion equations describing the interface waves propagating along the hysteretic nonlinear contact interface. The first terms of Eq. 6 and 7 represent the free Rayleigh equation for the free boundary condition. Second and third terms of Eq. 6 and 7 come from the linear and hysteretic nonlinear contact of interface surfaces, respectively. Eq. 6 and 7 include the angular frequency ω, which means that the waves are dispersive, so that the wave velocities vary with the frequency as well as material properties such as interfacial stiffness.

In order for the interface waves to exist and propagate, Eq. 6 and 7 should have real roots. However, the symmetric wave motion of Eq. 7 doesn't have real roots except that the hysteretic nonlinear stiffness is zero regardless of the linear stiffness. Therefore the symmetric wave does not exist all the time. From Eq. 6 it is obvious too that the ant-symmetric wave have real roots only if the nonlinear stiffness κ_{nx} is pure imaginary complex, i.e. $\kappa_{nx} = i\eta$ (η is real number). Otherwise the wave motion exists only as leaky waves with energy loss through acoustic radiation. If the interface is free of stress, i.e., $\kappa_x = \kappa_{nx} = \kappa_{nz} = \kappa_{nz} = 0$, Eq. 6 and 7 lead to the famous Rayleigh characteristic equation for free surface.

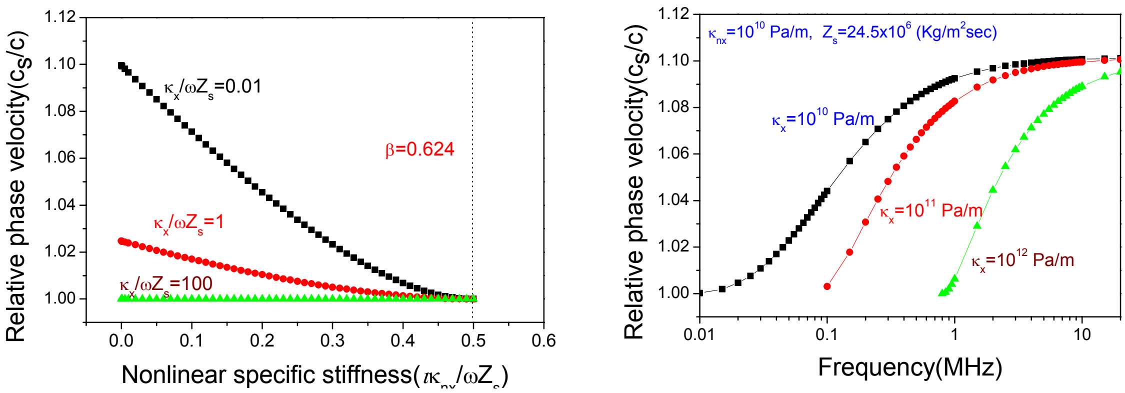

Figure 2. *Dispersion curves of anti-symmetric wave with respect to, (a) hysteretic nonlinear stiffness, (b) frequency*

4 Dispersive characteristics of anti-symmetric waves

Fig. 2 depicts the dispersion curves of interface wave (anti-symmetric mode, A-wave) for three different values of linear stiffness as a function of hysteretic nonlinear stiffness. Clearly observed in Fig. 2(a) is the monotonous decrease of the phase velocity ratio, $\alpha = c_s / c$ (increase of the phase velocity c), with the increase of hysteretic nonlinear stiffness κ_{nx}. Even when the nonlinear stiffness is less than $i\kappa_{nx} / \omega Z_s = 0.1$, the phase velocity ratio decreases significantly with the increase of the nonlinear stiffness. In the limiting case that the nonlinear stiffness $i\kappa_{nx} / \omega Z_s$ approaches 0.5 in Fig. 2(a), the phase velocity of interface wave asymptotes to the Rayleigh wave velocity regardless of the linear stiffness and material properties. No propagating waves exist beyond the value of $\kappa_{nx} / \omega Z_s = 0.5$ as shown in Fig. 2(a), where the phase velocity of anti-symmetric interface wave is bounded by both hysteretic nonlinear stiffness κ_{nx} and linear stiffness κ_x.

It is also found from Fig. 2(b) that the linear stiffness κ_x plays a key role in the dispersion characteristics in entire region. Phase velocity is increased up to 10% by the increase of linear stiffness κ_x. On the contrary to the hysteretic nonlinear stiffness, the higher linear stiffness makes A-wave faster. However, if the linear stiffness κ_x is very high in Fig. 2(b), the phase velocity is almost constant and insensitive to the nonlinear stiffness κ_{nx}. The dispersion curve in Fig. 2(b) represents the effect of wave frequency on the phase velocity of A-wave. The A-wave velocity is reduced with the use of higher frequency and it approaches the Rayleigh wave velocity as the frequency goes to GHz level. Conversely it increases and asymptotes to the shear wave velocity as the frequency decreases down to KHz order. It can be deduced that this dependency of phase velocity on the linear and hysteretic nonlinear stiffness can be used to analyze and estimate the contact state of non-welded interface such as closed cracks. If the contact interface is so tight and completely closed that the interface has a very large values of stiffness, the wave velocity of A-wave gets close to the shear wave velocity. If the contact interface is loose, the stiffness becomes small and the wave velocity is close to the Rayleigh wave velocity. Thus the dispersion curve gives a tool for estimation of the contact state of non-welded interface such as cracks or welded joints.

5 Conclusions

Interface waves propagating along a hysteretic nonlinear contact surfaces are demonstrated in theory based on discontinuous stress-displacements on both sides of the contact interface employing hysteretic nonlinear interfacial stiffness. Analytic solutions are derived to obtain dispersion curves for interface waves. Dispersion equations indicate that symmetric mode exists only as leaky wave and anti-symmetric mode can propagate along the interface as guided wave if the nonlinear stiffness is pure imaginary complex. Theoretical results show that the phase velocity of the anti-symmetric wave is sensitive to contact state and changed as much as 10% depending on both the linear and hysteretic nonlinear stiffnesses. It is also observed that the phase velocity of the anti-symmetric wave is bounded between the shear wave velocity and the Rayleigh wave velocity.

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