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Inverse parameters estimation of the functionally graded materials using surface waves measured with a laser interferometer

Laiyu LU[1], Mathieu CHEKROUN[1], Odile ABRAHAM[1], Géraldine VILLAIN[1] and Valerie MAUPIN[2]

[1] Laboratoire Central des Ponts et Chaussées, Route de Bouaye BP4129 44341 Bouguenais Cedex, Nantes, France

[2] Department of geosciences, University of Oslo, P.O.Box 1047, Blindern, 0316 Oslo, Norway

Abstract

In this paper, an inverse scheme of the surface waves is developed to invert for the mechanical parameters of functionally graded visco-elastic materials, in which the properties, that is the velocity and the quality factor of body waves, are supposed to be as continuous functions of depth. This scheme is validated by investigating a numerical model with continuous variation that can be expressed by a known smooth function. Then, in laboratory the surface waves are measured on mortar samples with a laser interferometer. The phase velocity and attenuation are then extracted from the seismograms. Based on the phase velocity and the attenuation, the profiles of body wave velocity, as well as the quality factor, are obtained as a continuous variation with depth by back-calculation.

Résumé

Dans cet article, un problème inverse est développé pour l'inversion des courbes de dispersion des ondes de surface dans des milieux continument variables avec la profondeur aux propriétés visco élastiques représentées par une vitesse de propagation et un facteur de qualité pour les ondes de volume. L'algorithme est validé sur des données numériques avec des variations des propriétés mécaniques qui suivent une fonction analytique. Il est ensuite appliqué sur des données obtenues en laboratoire avec un interféromètre laser sur des dalles de mortier. Les profils des vitesses de propagation des ondes de volumes et du facteur de qualité avec la profondeur sont obtenus avec les courbes de dispersion des vitesses de phase et de l'atténuation mesurées.

Keywords

Inversion, FGM, Surface wave, mortar

Introduction

Concrete is characterized by its extremely heterogeneous nature since, as a common construction material, it is a mixture composed of cement or asphalt as well as other materials such as aggregates and water. Non destructive evaluation of concrete is a major issue for monitoring the durability of civil engineering structures, especially for the cover concrete, which is directly subjected to aggressive attacks from the outside. Ultrasonic waves are often used to characterize such material properties. Some experiments show that dispersion and damping of waves are influenced by both variation of grain size and water/cement ratio[1,2]. However, it is difficult to give a quantitative estimation for these properties based on the measured ultrasonic signals, especially for the attenuation. The main reason is the multiple-scattering of the waves in such a heterogeneous media due to the random distribution of the pores, air bubbles and aggregates, when the wavelength has the same order of magnitude as the dimension of the heterogeneities. The conventional overall method, for example homogenization, has been widely employed to estimate equivalent or effective material properties. Even though it provides reasonable overall prediction of the mechanical behavior, such overall estimations, however, are insufficient to accurately predict the local behavior, as also shown by the experiment on damaged concrete [3,4]. In fact, the modulus of elasticity of the pavement, mainly the concrete materials, is not constant but changes with depth, due to different factors such as aging, moisture content, and temperature. It will be therefore more reasonable to consider thus material when modelling them as a non-homogeneous material with gradually varying properties, which is often referred to as Functionally Graded Materials (FGMs).

As a non-invasive method, surface waves are widely used to interpret the material properties in different fields and at different scales (geophysics, non destructive evaluation of composites, metals or concrete, etc.). This method has been successfully used to infer the properties of homogeneous or multi-layered media, and has been proved to be an efficient way to extract information of such medium. In this paper, we apply surface wave method to invert for the wave velocity and attenuation of FGMs, in which mechanical parameters are supposed to present a continuous variation with depth. As an application, surface waves are picked on the surface of mortar samples with a laser interferometer and spatially averaged to obtain the coherent field. The velocity, correspondingly the Poisson ratio, and the attenuation of the waves in the slabs are inverted based on the experimental data.

2 Formulation of the theory and method

In this paper, linear viscoelasticity and weakly dissipation are assumed. Under this assumption, the eigenvalue for a viscoelastic model can be obtained from that of the pure elastic case by perturbation theory. And as a result, the software developed for elastic media can be easily modified to visco-elastic case. We divide the medium into many thin constant layers to approximate the continuously varying properties. The equations of the Rayleigh wave eigenvalue problem for elastic case are omitted here, which can be found in many references. The software developed by Saito is used in this paper to calculate the Rayleigh wave eigenvalues and partial derivatives of phase velocity for isotropic elastic model [5].

For viscoelastic material, Liu et.al. [6] constructed a mathematical model in which the quality factor Q would be nearly constant, and according to the Kramers-Kronig relationship this yields the following dispersion equation

$$\frac{V(\omega)}{V(\omega_{ref})} \approx 1 + \frac{1}{\pi Q} \ln \frac{\omega}{\omega_{ref}} \tag{1}$$

where *ref* denotes a reference circular frequency. The dispersion relation is applicable only for weakly dissipative media, for example $Q > 10$, in which the dispersion caused by the intrinsic dissipation is low. Applying the variational principle to weakly dissipative media, one can obtain the dissipation factor of the Rayleigh wave, the phase velocity and the partial derivatives can then be expressed by that of the pure elastic case.

Concerning the inverse problem, an optimized method is required to minimize the difference between the predicted and measured dispersion and/or attenuation. For our problem, the relation between the data and parameters can be written as

$$\left[Q_R^{-1}(f), c(f)\right] = g\left(V_{pi}, V_{si}, \rho_i, h_i, Q_{si}^{-1}, Q_{pi}^{-1}\right) \tag{2}$$

Where the $Q_R^{-1}(f)$ and $c(f)$ are the dissipation factor and phase velocity of Rayleigh wave for each frequency f, and they are the nonlinear function of the parameters such as the velocity and dissipation factor of P-wave and S-wave, density, thickness of the layer. We write Eq.(2) as the general form

$$\mathbf{d} = \mathbf{g}(\mathbf{p}) \tag{3}$$

where **d** and **p** are the data and parameter set respectively. For the least square solution to discrete nonlinear inverse problem proposed by Tarantola and Valette (1982)[7], the model at $k+1$ iteration is given by

$$\mathbf{p}_{k+1} = \mathbf{p}_0 + C_{p0} \cdot G_k^T \cdot (C_{d0} + G_k \cdot C_{p0} \cdot G_k^T)^{-1} \cdot \{\mathbf{d}_0 - \mathbf{g}(\mathbf{p}_k) + G_k \cdot (\mathbf{p}_k - \mathbf{p}_0)\} \tag{4}$$

Where, G is the matrix of partial derivatives with respect to the model parameters. Superscript T means the transpose of matrix. p_o is the a priori model. d_0 the data vector. $g(p_k)$ the data predicted from the model p_k. C_{p0} and C_{d0} are the a priori covariance matrix of parameter and data, respectively. We introduce the Gaussian-shaped function as the a priori covariance function of the a priori model p_o.

$$C_{p0}(z,z') = \sigma(z)\sigma(z')\exp\left(\frac{-(z-z')^2}{2L^2}\right) \tag{5}$$

where z and z_0 are two depth points. L is the correlation length. This acts as a spatial filter to smooth the model, imposing correlation between points separated by distance of order L, and $\sigma(z)$ controls the amplitude of the model perturbation allowed at z. This algorithm is designed for a continuous problem. The smoothness in poorly sampled regions is mostly constrained by the correlation length. Therefore, the inversion is dependent on the a-priori information and correlation length, not dependent on the discretization with depth of the model[8.9].

3 Numerical results

In this section, a numerical model is considered. We first calculate analytically the velocity and attenuation of the Rayleigh wave propagated in this model. Then the velocity and attenuation are taken as the 'measured' data and used to invert for the model properties. Since we know everything of the model, it is helpful to validate the algorithm.

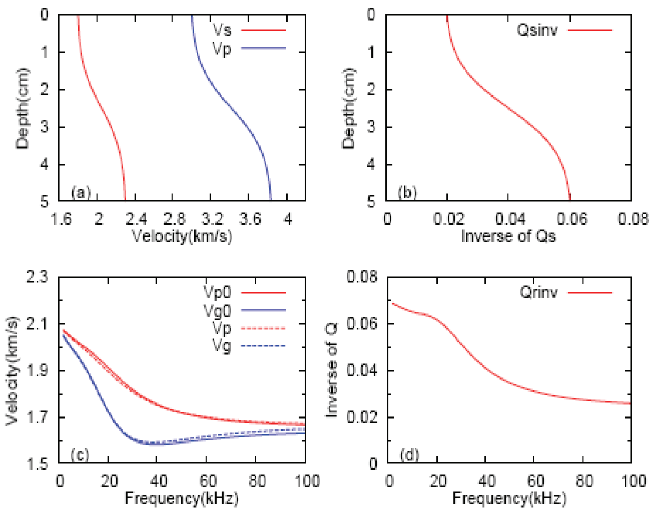

Figure 1 *The model with a smooth varying properties. (a). The velocity profiles. (b) The profile of the inverse of the quality factor Q_S. (c). The phase and group velocity of the Rayleigh wave propagated in this model. The velocity for pure elastic case is also shown, which is denoted by subscript 0. (d). The inverse of the Rayleigh wave Q-value.*

The model has a smooth variation of which V_s and Q_s^{-1} profiles is defined by

$$\begin{cases} \phi(z) = \phi(d)\left\{1 + \frac{1}{2}\frac{\phi(0)-\phi(d)}{\phi(d)}\left[\frac{\tanh[a(1-2z/d)]}{\tanh a}+1\right]\right\} & 0 \le z \le d \\ \phi(z) = \phi(d) & z > d \end{cases} \quad (6)$$

where z is the depth. V_p is related to V_s by assuming Poisson ratio is equal to 0.22. We chose a = 2 and d = 5 in Eq.(6), and the medium is assumed homogeneous below 5*cm*. As shown in Fig.1, the model has a smooth variation above 5*cm*. Baron et.al.[10] have adopted the function Eq.(6) to model a transition layer in which the material properties vary continuously without abrupt jump at the edge points. They have discussed the forward problem for elastic case by means of the Peano's series and gave an analytical expression of the dispersion relation. Here we extend the model to the visco-elastic case and investigate the inverse problem. By experience, we take $V_s(0)$ = 1.8*km/s*, $V_s(d)$ = 2.3*km/s*. It is assumed Q_s^{-1} has the same variation as V_s and $Q_s^{-1}(0) = 0.02$, $Q_s^{-1}(d) = 0.06$. Q_p^{-1} is related to Q_s^{-1} by approximation [11]

$$Q_p^{-1}(z) = \frac{4}{3}\frac{V_s^2(z)}{V_p^2(z)}Q_s^{-1}(z) \quad (7)$$

Fig.2 shows the inverted results for this model with known Poisson ratio (0.22) as the *a priori* information. The initial model, inverted profiles and true model are displayed. The initial value of V_s is the profile $1.1Vr$- 0.5λ . Initial V_p is obtained from V_s by known Poisson ratio. Initial Q_s^{-1} is estimated from that of the Rayleigh wave, and it is chosen as a moderate value. It can be found that from Fig.2 a good inverted results are obtained for Q_s^{-1} , V_s and hence V_p. On the other hand, if we do not have any *a priori* information on the Poisson ratio and V_p, we need to invert for V_s and V_p simultaneously. In this case, however, the inverted V_p is highly dependent on the initial model. Even if the inverted phase velocity and Q_s^{-1} of Rayleigh wave have a good agreement with the true model, the inverted V_p may not be reliable since the same phase velocity and Q_s^{-1} may allow multiple solutions of V_p because Rayleigh wave are less sensitive to V_p . Therefore, in practical applications we should try to get the *a priori* information on the Poisson ratio or P-wave velocity of the materials by other methods, such as reflection and refraction method. Or we estimate the Poisson ratio by experience. It's not a perfect scheme to recover the P-wave velocity using the Rayleigh wave velocity and attenuation. For Q_s^{-1} and V_s, we always get the reliable inverted result.

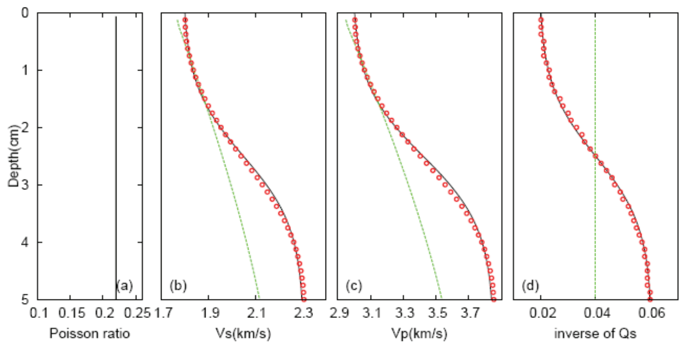

Figure 2 *The inverted results for the model with Poisson ratio as a known. (a),(b),(c) and (d) are respectively the profiles of Poisson ratio, S-wave velocity, P-wave velocity and the inverse of* Q_s*. Solid line-true model; dash line-initial model; circles-inverted model.*

4 Determination of the concrete properties from laser experiment

Experimental measurements are carried out on mortar samples. 2 series of mortar slabs are considered, the first one (M1) with a low water/cement ratio (*w/c* = 0.35) and the second one (M2) with a high ratio (*w/c* = 0.65), inducing a higher porosity. The water to cement ratio is the only parameter that changes between the samples. The maximum size of the aggregate in these mortars is 4*mm*. Each series is composed of 5 slabs with dimensions 600*mm*× 600*mm*×120*mm*. The 120*mm* thickness is selected so that the sample can be thought as half-space and the signals received at the surface are Rayleigh waves instead of Lamb wave. A piezoelectric transducer is used as a source. This transducer is equipped with a wedge in order to generate predominantly Rayleigh waves in the mortar slabs. The source function is a Ricker wavelet with a central frequency equal to 120kHz. Reception is performed with a laser interferometer (*Tempo from Bossa Nova Tech*) which acquires in a non-contact way the normal displacement of the surface of the slabs at different positions. The position of the laser beam is controlled by a robot so that we take an acquisition every 1 *cm* at a distance from the source varying from 10 *cm* to 45 *cm*. We obtain the equivalent of a common-gather shot in seismology. To evaluate the properties of the homogeneous effective medium, we take 36 equivalent common-gather shots at different positions of the 5 slabs for each series. Then a space average is carried out to evaluate the coherent field, representative of the effective medium[12].

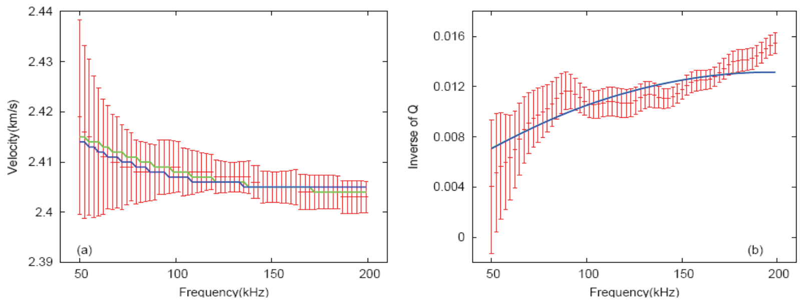

Figure 3 *The experimental (with error bars) and inverted phase velocity (a),Q^{-1}(b).*

By processing the multi-station signals, we can get the velocity and attenuation of the Rayleigh wave. As an example, Fig.3 gives experimental phase velocity and error bars for M1. Due to the limitation of the frequency bandwidth of the transducer, the ratio of the signal to the noise is not so good at the frequency lower than 60kHz and higher than 180kHz. Therefore, only the data between 60 *kHz* -180*kHz* are used in the inversion.

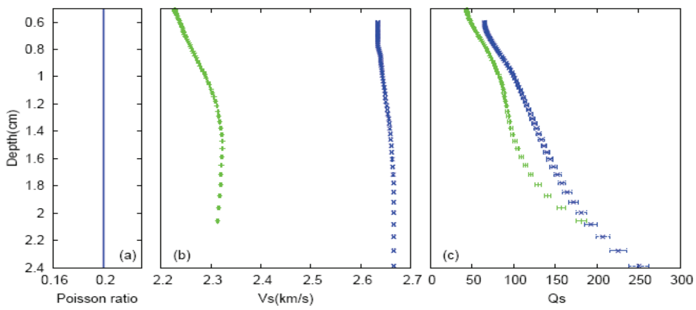

Figure 4 *The inverted Vs (b) and Qs (c) profiles for mortar M1 (blue) and M2 (green).*

Fig.4 shows the inverted results for M1 and M2. It can be found, as expected, that the phase velocity of mortar M1 is higher than that of mortar M2 since M2 has a higher porosity. Similarly the quality factor Qs of M1 is higher than that of M2 for all depth. For both mortars a slight dispersion in the phase velocity dispersion curve is visible meaning that the material properties varies with depth. At low frequency the phase velocity increases so that an increase of body waves shear velocity with depth is expected. This is in accordance with common knowledge of concrete mixes properties. The first few millimeters content less aggregates that beneath due to a wall effect, the proportion of large aggregate becomes constant after of depth equal to the radius of the largest ones. Furthermore those first millimeters which contain less large aggregates are also more porous than deeper due to the interaction with the wood box when the mortar is hardening. Both phenomena have been confirmed by gamma-densimetry [13]. For the inversion a constant Poison ratio, here equal to 0.2 a classical value for mortar, has been chosen. The densities for mortar M1 and M2 are also constant and have been measured independently. On the shear wave velocity profile we can see that below 1,5cm the mortar becomes homogeneous with a shear wave velocity equal to 2,66km/s for M1 and 2,33km/s for M2. The quality factor is not stable with depth.

5 Summary

In this paper, an approach is presented to invert for the parameters of visco-elastic FGMs, which is supposed to have a continuously varying properties. This method is not straightforward and an optimized method is needed to minimize the difference between the predicted and measured data on the dispersion and attenuation of the surface waves. Therefore, the forward and inverse problems are included in this method. For the forward problem, under the assumption of linear visco-elasticity and weakly dissipation, we use a model, that comprised of a series of piecewise homogeneous layers, to simulate the continuously variation, and calculate the predicted phase velocity and attenuation of surface waves. For the inverse problem, the solution for a continuous inverse problem developed by Tarantola and Valette [7] is used to minimize the difference between the predicted and measured data, in which a Gauss function is applied to smooth the model to suite for the continuously varying properties. This method is investigated based on a numerical model. The parameters of some concrete samples are inverted using the surface wave data by laser measurements.

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