APPLICATION OF RADON TRANSFORM IN CT IMAGE MATCHING

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Abstract: When Industrial Computerized Tomography (CT) techniques are adopted to inspect quality of industrial products, we lay emphases on their installation validity or inner defects. In order to do so, we always try to perform matching operation between reference object and test objects. However, conventional image matching methods can't meet on-line detection demand because of their slow recognition speed and low throughput rate. In this paper, we attempt to directly use CT projection data or object's Radon transform to solve matching parameters. Firstly we discuss some properties of Radon transform invariant to geometric distortion such as translation, rotation and uniform scaling, and then discuss recognition methods of matching parameters. Experimental results confirm that this scheme is robust to geometric distortions and greatly improve matching speed and precision.

Introduction: X-ray Computed Tomography enjoys rapidly growing interest in industrial quality inspection. However, to take advantage of this technique in the automated manufacturing process, one has to employ robust image processing algorithms that perform inspection tasks without human interaction. Image matching method is one of basic image processing algorithms and is a technique of estimating the similarity of different images. It is fundamental in image analysis and pattern recognition, and has a wide variety of application, such as detecting changes in a scene, estimating object motion, locating targets, identifying objects, integrating information from different types of image, etc. Among those applications considered most frequently are geometrical transformation such as translation, uniform scaling and rotations. The early interest in this problem comes from computer vision, which requires transformation from one coordinate space to another. Target recognition in remote sensing is another field of practical application. Recently with the advent of multimodality images in medical diagnosis, this problem becomes more and more important in studies and implementations.

The fundamental operation for image matching is to compare various correlations under different names and guises depending on specific implementations. Basically, there are three categories of algorithm in the existing literatures. They are (i) techniques based on image intensities, such as moments [1-3], correlations [4-5], integral kernel technique, and Fourier methods. (ii) Featured-based methods and (iii) elastic model-based methods [6]. A general review was given in [7,8]. In the recent applications of medical diagnosis and therapy treatment planning, many combined methods appeared.

However, conventional image matching methods can't meet the on-line detection need because of their slow recognition speed and low throughput rate. We attempt to directly use CT projection data or object's Radon transform (RT), to solve matching parameters. Generally geometric distortion, only referring to rigid distortion, such as translation, rotation and uniform scaling is the key problem of image matching, therefore, we focus on how to solve geometric deformation parameters by RT. In Section 2 we'll introduce some properties of RT invariant to geometric distortion and describe methods for determining the transform parameters in section 3. Many simulative experiments and results are shown in section 4.

2 Radon Transform and Its Prosperities: In recent years, Hough transform, Trace transform and the related Radon transform have received much attention. In the view of mathematics, Hough transform is a derivative of RT and RT is a special case of Trace transform [9]. These three transforms are able to transform two dimensional images with lines into a domain of possible line parameters, where each line in the image will give a peak positioned at the corresponding line parameters. These have lead to many line, circle and curves detection applications within image processing, computer vision, etc.

The following will only describe RT and its properties. Though several definitions of the RT are existed, they are related. A very popular form express is as what discussed in [10]. In computed tomography (CT) when
a bundle of x-ray goes through an object, its attenuation depends on content of object, distance and direction or
angle of this projection. This set of projections is called RT. In two dimensions, let f (x, y) be a 2D image, its RT denoted as $\Re f$ is the 2D function of the real variable t and angle θ, defined by:

$$\Re \mathrm{f}(\mathrm{t},\theta) = \int_{-\infty}^{\infty} f(t\cos\theta - \mathrm{s}\sin\theta, t\sin\theta + \mathrm{s}\cos\theta)ds$$

(1)

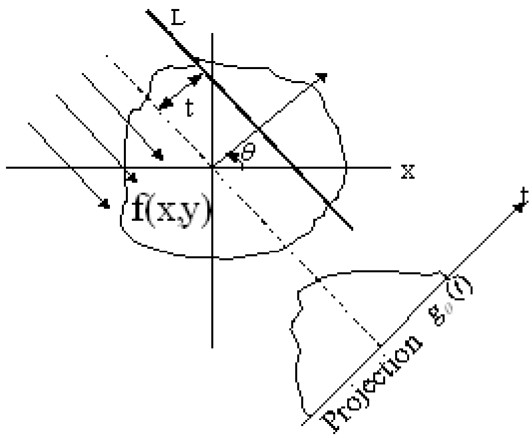


Fig1 Radon Transform

Geometrically, as show in Fig.1, $\Re f(t,\theta)$ is equal to the integral of the function on the straight line L passing through t and of direction orthogonal to θ. Where L is given by:

$$\mathrm{t} = x\cos\theta + y\sin\theta$$

(2)

For a given θ, the set of values of the RT which corresponds to integrals of the function on parallel lines is know as a projection in tomography. The projection of direction of θ is:

$$g_\theta(t) = \Re \mathrm{f}(\mathrm{t},\theta)$$

(3)

The mathematical expressions of RT lead to some very important properties. We show the invariance properties of RT against rotation, scaling, translation in the following.

2.1Fourier slice theory and Energy Conservation: The basic property for topographic applications is the Fourier slice theorem .It establishes that the Fourier Transform (FT) of a projection in direction θ is equal to a slice of the 2D FT of the image along the line of direction θ. According to [11], It can be written as:

$$\int_{-\infty}^{\infty} g_\theta(t)e^{-j2\pi wt}dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi w(x\cos\theta + y\sin\theta)}dxdy$$

(4)

This theorem shows the equivalence of the information contained within the image and its RT. Based on Fourier slice theorem, we similarly can consider that there is energy conservation in the Radon transform and in the space domain as following expressed:

$$\int_{-\infty}^{\infty} g_\theta(t)dt \leftrightarrow \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy$$

(5)

2.2 Translation: The effect of a translation on the image is a distortion on the RT, which can be described as follows: The spatial variable of the RT is translated by a quantity depending on the image translation vector
and the angular variable. However for a given θ, the RT of translated image is simply equal to the
translation of the image RT with respect to the special variable.
If image $f(x,y)$ is shifted by $(\Delta x, \Delta y)$, the RT is

$$f(x-\Delta x, y-\Delta y) \leftrightarrow \Re f(t-\Delta x\cos\theta - \Delta y\sin\theta, \theta)$$

(6)

2.3 Rotation Transform: If image $f(x,y)$ is rotated by ϕ, $f(x,y)$ becomes:

$$f(x,y) \leftrightarrow f(x\cos\phi - y\sin\phi, x\sin\phi + y\cos\phi)$$

(7)

Since the inner integral of equation (1) can be expressed by a rotation matrix, the RT of rotated image simply becomes:

$$f(x\cos\phi - y\sin\phi, x\sin\phi + y\cos\phi) \leftrightarrow \Re f(t, \theta+\phi)$$

(8)

2.4 Uniform Scaling: If image $f(x,y)$ is scaled by a factor k ($k > 0$), the RT is

$$f(ax,ay) \leftrightarrow \frac{1}{|k|}\Re f(kt,\theta)$$

(9)

The sinograms (all projections taken along the different θ direction) of Fig.2(c), Fig.2 (d), Fig.2 (e) and Fig.2 (f) show us respectively the rotation, translation and scaling property of RT.

3 recognition of parameters: When CT projection data are acquired, we directly turn to solve relations between original image (or reference image) and test image, in other words, to recognize the transform parameters k, ϕ, $\Delta x, \Delta y$. Due to matching operation manipulated in 1D projection instead of 2D grey level image, our matching method benefits for improving matching efficiency in essence.

From equation (5) and (9), the uniform scaling is easily derived. For a given spatial position t, as Fig.3 (b) illuminated, the rotation angle θ can be uniquely determined by local minima of two profile of two objects.

Now, we focus on the determination of the translation vector ($\Delta x, \Delta y$), the recognition procedures can be summarized as follows:

(1) Compute the Fourier transform of $f(x,y)$ and $f(x-\Delta x, y-\Delta y)$ respectively.

(2) For a fixed angle θ_1, let $\theta_2 = \theta_1 + \pi/2$, by phase-only technique [12] and the equation (6) we get:

$$\begin{pmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix}\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

(10)

Where a and b are two constants that depend on the translation vector ($\Delta x, \Delta y$).So the translation can be determined by

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

(11)

Thus, the rigid geometrical distortion is completely obtained.

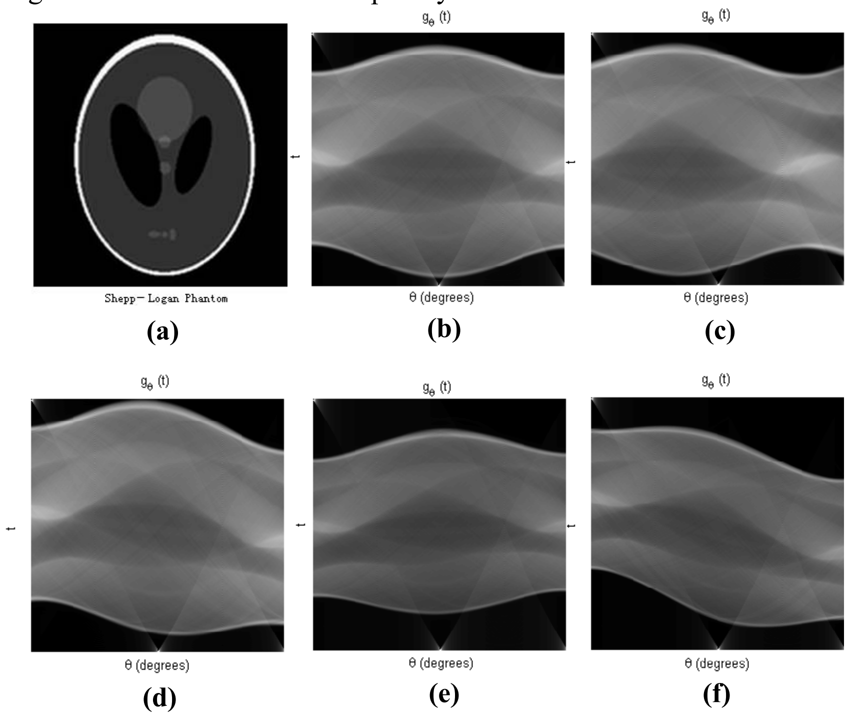


Fig.2: (a) Shepp-Logan phantom; (b) Sinogram of (a); (c) sinogram of (a) rotated by ϕ£½30° ; (d) sonogram of (a) translated ($\Delta x = 10, \Delta y = 10$); (e) sinogram of (a) scaled by four-fifths; (f) sinogram of (a) scaled by four-fifths and translated($\Delta x = 10, \Delta y = 30$)

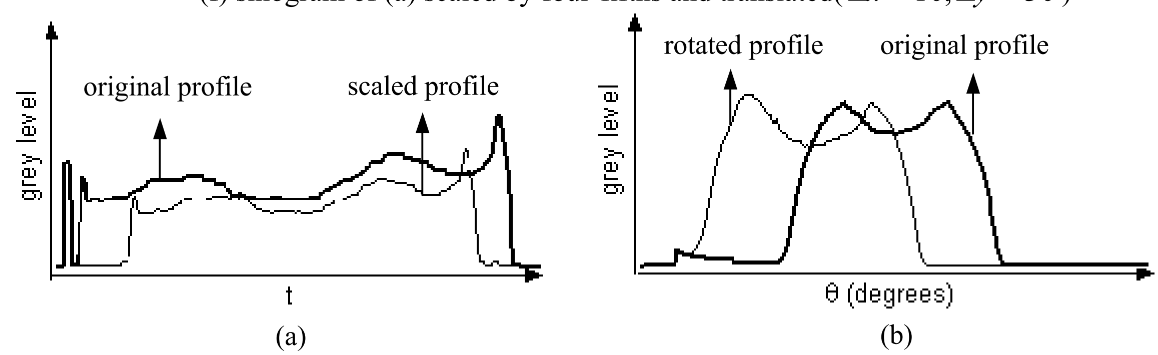
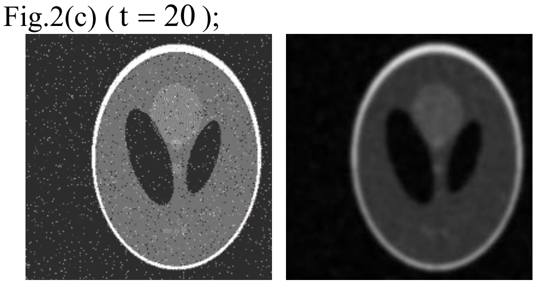


Fig.3:(a) profile of Fig.2 (b) and Fig.2 (e) (projection direction $\theta = 60°$); (b) profile of Fig.2(b) and Fig.2(c) (t = 20);

Fig.4 test images

(a) Shepp-Logan Rotated by 30¡ ãTranslated by (10,30) and noise-disturbed

(b) Shepp-Logan Rotated by 30¡ ãTranslated by (10,30) and blurred



4 Experiments and Results: After recognizing the transform parameters k, ϕ, $\Delta x, \Delta y$, the test image reconstructed from projections with filter back-projection algorithm [11] is transformed with parameters and then compared with original image (here, original image refer to image reconstructed from standard cross-section projections). We use root mean square error (RMSE) instead of the traditional normalized correlation as our similarity measure between original image and test image. Lets (X_R, Y_R) and (X_T, Y_T) refer to original image pixel and test image pixel then the RMSE between test image and original image is as following

$$\mathrm{RMSE} = \frac{1}{2}\sum\sum\left[(X_R - X_T)^2 + (Y_R - Y_T)^2\right] \quad (12)$$

If the RMSE is smaller than the threshold, the matching operation is over. The method has been applied to 256×256 grey level images represented in Fig.2 (a) and different images disturbed by noise or blurred as show in Fig.4 (a), Fig.4 (b), the RT computed on 256 projection directions and each direction with 256 projections. Different scaled, rotated and translated versions of Shepp-Logan Phantom were computed, the RMSE obtained for different values of scaling, rotation and translation are given in table1. Table1 not only show us our methods' feasibility and robust to noise.

Table1: RSME for different transformed images

Test Images	**RMSE**
Scaled by four-fifths	0.0091
Rotated by 30°	0.0086
Translated by (10,10)	0.0082
Rotated by 30°and Translated by (10,30)	0.0077
Rotated by 30°, Translated by (10,30) and nose-disturbed	0.0071
Rotated by 30°, Translated by (10,30) and blurred	0.0069

5 Conclusions: In this paper, we have discussed how to realize rapid image matching by RT and make enough preparation for further defects recognition. Our matching method benefits for improving matching efficiency in essence because matching operations are manipulated in 1D projection instead of 2D CT image. The method is well adapted to the recognition of objects in grey level image, even if they are blurred or noise-disturbed. Further improvements on the method are possible. For instance it is quite possible too recognize geometrical distortions parameters base on local RT.

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