Multiple Defect Detection by Applying the Time Reversal Principle on Dispersive Waves in Beams

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Abstract
A method for the localization and characterization of defects in waveguide-like structures is presented in this paper. The method eliminates the time-consuming scanning of a whole structure. Instead, a measurement of the displacement in one direction at one point together with a numerical simulation suffices to detect, localize and estimate the size of several defects. The method is based on the time reversal principle and the strong dispersion of the first bending mode at low frequencies In this paper, the method is described in detail and the feasibility is experimentally demonstrated. The mechanisms determining the capability and accuracy of the method are discussed and suggestions for improvements are given.

Keywords: Detection of defects, time reversal numerical simulation, structural wave propagation, multiple crack detection, defect characterization

1 Introduction
Many studies have been carried out on the topic of defect detection in beams using guided waves. Dimarogonas [1] gave a review on the dynamics of cracked structures and found over 500 papers published from 1971 to 1992. Using guided waves instead of ultra-sonic testing for NDT purposes is very attractive since it avoids the time consuming scanning of whole structures and is therefore especially suitable for large or remote structures that are difficult to access.

Dual [2] and Staudenmann [3] studied the detection of one and more notches in a beam by exciting narrow band waves to minimize the effect of distortion. They used a ‘generalized spring’ model, developed by Gudmundson [4], that uses stress intensity factors to analytically calculate the response at a notch in terms of an incident wave. Krawczuk et. al. [5] presented a finite spectral Timoshenko beam element with a transversal open crack. Numerical experiments showed that this element could be used for the localization of cracks. Rucka [6] applied the spectral element method to determine the location of structural discontinuities. Here also narrow band excitation was used. Ishak et. al. [7] used transverse impacts for broad band excitation. A multilayer perception network was used for the determination of cracks. Doyle [8] introduced stress intensity factors to model the dynamics of cracks with spectral elements. A generic algorithm was used to determine the notch parameters so that the spectral model fits the experimentally measured response.

The objective of the presented method is to locate several notches in a beam with only a single, one point measurement of one displacement component. The method makes use of the time reversal principle that is extensively studied by Fink et al. [9] and the strong dispersion of the first bending mode at low frequencies.
2 Method

A broad band longitudinal displacement pulse is excited at one end of a beam and propagates through the structure. As it hits an asymmetric defect, partial mode conversion occurs and a flexural wave is generated. The longitudinal wave pulse remains compact since this mode is almost non-dispersive. The generated flexural wave pulse distorts and diverges as it travels through the beam because of its dispersive behavior in the chosen frequency range. At the end of the beam, the lateral deflection due to the flexural wave and the lateral contraction of the longitudinal wave is measured.

The measured displacement data is reversed in time and set as the boundary condition for a numerical simulation. The beam is simulated without any notch. In the simulation, the distorted flexural wave form recompresses and reaches maximal amplitude at the location of its origin. This allows the detection of the notch by simply finding the local maxima in an x-t diagram, also known as Lagrange diagram. This procedure is illustrated in Figure 1.

![Figure 1. Lagrange diagrams of the two steps in the time reversed numerical simulation method. The vertical axis denotes the time coordinate, the horizontal axis denotes the axial coordinate. The left hand illustrates the wave propagation process in the experiment. On the right hand side, the simulated, time reversed propagation of the flexural wave is shown. The flexural wave “converges” at the position of its origin. The angle $\alpha$ is termed ‘focal angle’.](image)

After having converged at the notch, the different frequency components of the flexural wave will diverge again. The amplitudes of the flexural wave decrease again. This allows detecting multiple defects, provided that the distance between the notches is big enough so that the flexural wave of one notch will flatten out before reaching another notch.

This method is an extension of the time reversal numerical simulation method developed by Leutenegger and Dual [10]. The main conceptual difference is that Leuteneggers method detects defects mainly by interference of different wave modes. In the case of a beam with a notch, that would imply the determination of the notch location due to interference of the longitudinal and flexural wave mode. This procedure was successfully applied by Veres [11] however, it requires detecting and rejecting both wave modes and therefore makes it necessary to measure more than one displacement component. In contrast, the method described here requires only a single point, one direction measurement, since only dispersion of one wave mode is used. A further advantage is that multiple defects can be detected because the waves flatten out after having converged at the defect position.

A difference to other guided wave NDT methods is the use of broadband excitation as opposed to narrow band excitation. Most time of flight methods try to avoid the effect of phase dispersion by exciting only a narrow frequency band. However, since phase dispersion is well compensated in a time reversal process, the time reversal simulation acts as an adaptive spatio-temporal filter [12] that decodes the location of the defect. In the following sections, the three major steps of the method are described.
2.1 Experiment
First, the material parameters, that determine the wave propagation speed for the first flexural and longitudinal wave mode, are evaluated. This step is crucial for the reliability of the subsequent numerical simulation. All experiments are done on an aluminum beam with length \( L = 2 \text{m} \) and square cross section with side length \( b = 0.006 \text{m} \). The material properties were evaluated by measuring phase speeds of longitudinal and flexural waves and comparing them to analytically derived dispersion curves, see Figure 2.

The second objective is to measure the response of the defective beam due to an excited longitudinal wave. The excited pulse should be sharp and cover frequencies for which the flexural wave is most dispersive. With a single point measurement at a location within the beam, the propagation direction of a recorded wave remains unknown. Therefore, the measurement is done at the end of the beam. However, at a free end of a structure, one measures not only the incident wave but also the reflected wave and local vibrations. For harmonic waves, one can write the measured displacement as a sum of three waves (1):

\[
 w_m(x, t) = A e^{i(-\gamma x + \omega t)} + B e^{i(\gamma x + \omega t)} + C e^{i\omega x} e^{i\omega t}
\]  

where \( A \) is the amplitude of the incident wave, \( B \) the amplitude of the reflected wave and \( C \) the amplitude of the local oscillation. The amplitude ratios at a free boundary between incident and reflected- or incident and oscillating-wave according to Bernoulli theory are (2):

\[
 \frac{B}{A} = -i \\
 \frac{C}{A} = 1 - i
\]

Plugging these ratios into equation (1) and choosing \( x=L \), one gets:

\[
 w_m(t) = A(e^{i(\omega t - \gamma L)} + e^{i(\omega t + \gamma L - \frac{\pi}{2})} + \sqrt{2} e^{i(\omega t - \frac{\pi}{4})})
\]  

As can be seen in equation 3, the measured response differs from the incident wave both in amplitude and phase. However, the difference in amplitude and phase is frequency independent.

In contrast, by using Timoshenko theory, Mei [13] showed, that for the free boundary case, the reflected modes do have frequency dependent phase shift. In an upcoming study, we will discuss the effect of the frequency dependent phase shift. Though in this feasibility study, we chose not to modify the measured displacement vector. The time reversed displacement vector is set as the boundary condition for the time reversal simulation.

2.2 Time Reversal Numerical Simulation
The measured displacement vector contains the lateral contraction of the longitudinal wave and all generated flexural wave components. The beam geometry does not involve any notches. The lateral displacement at one end of the beam is prescribed by the time reversed displacement vector. The rotary degree of freedom is not prescribed. In terms of simulation methodology, we have successfully used finite element code and spectral elements. The finite element simulation was performed in ABAQUS Explicit. The simulation with spectral elements was done in MATLAB on the basis of [8]. Both techniques led to comparable results. However, since the geometry is simple, the spectral elements are much more efficient. A further advantage of the spectral elements in this method is the possibility to simulate semi-infinite structures. This eliminates disturbing reflections at the end of the beam.
2.3 Analysis

The analysis is based on the simulated lateral displacement along the beam. Since the problem is one dimensional, we can draw a Lagrange diagram, with the displacement amplitude given by the color tone. An example of such a diagram is shown in Figure 3. It shows the situation of the experiment, where the longitudinal wave, indicated by a dashed line, excites flexural waves at the notches. The Lagrange diagram gives an overview of the wave propagation process and focal spots can easily be identified. However, some local maxima are artifacts, resulting from the longitudinal mode or reflections of flexural waves and the analyst is challenged, to decide, which of the maxima in the Lagrange diagram are relevant. Nevertheless, there is a link to facilitate the task. Since the longitudinal wave has the highest propagation speed, one can identify the first displacement at the end of the beam as the lateral contraction of the arriving longitudinal wave. The propagation speed of the longitudinal wave is known from the experiments and based on this, one can draw a line into the Lagrange diagram that marks the characteristic of the longitudinal wave similar to the dashed line in Figure 3. All local maxima encountered along this line indicate notches or other discontinuities.

3 Experimental Setup

The experimental setup is depicted in Figure 4. The displacement is measured with a laser interferometer. For the dispersion relation measurements, the measurement angle $\alpha_m$ was 45° in order to have increased sensitivity for the axial displacement. The measurement of the beam containing the notches was done under an angle of $\alpha_m = 90°$. Two elastic nylon wires were used to support the beam. The piezoelectric plates used to excite the waves into the beam were additionally backed with a brass plate. This significantly enhances the energy transfer from the piezoelectric plate to the beam. To excite the beam with the desired pulse shape, the voltage signal was calculated according to an analytical model derived by Dual [14].

<table>
<thead>
<tr>
<th>Table 1. Material parameters for the beam</th>
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<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td>Young’s modulus</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
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<tr>
<td>Density</td>
</tr>
<tr>
<td>1st Timoshenko coefficient</td>
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Figure 2. Measured phase velocities for different frequencies and fitted dispersion curves.
Figure 3. Simulated Lagrange diagram of the lateral displacement in the center of a beam with four notches that is excited with a broad band longitudinal displacement pulse at \( x = 0 \) m. This is the wave propagation process that is expected to take place in the experiment. The dashed line indicates the longitudinal wave. At the notches at \( x_1=0.7 \) m, \( x_2=1.0 \) m, \( x_3=1.15 \) m and \( x_4=1.9 \) m, bending waves are generated.

Figure 4. Experimental setup. Parameters for the notch detection measurement: \( x_m = 2 \) m, \( \alpha_m = 90^\circ \); for the dispersion relation measurement : \( x_m = [1 \text{m}, 1.5 \text{m}] \), \( \alpha_m = 45^\circ \).

The geometry of the specimen is sketched in Figure 5. The dispersion relation was measured before the notches were milled into the beams. To determine the material parameters, the theories of Love and Timoshenko were compared to the measured dispersion curves, with Young’s modulus \( E \) and the 1st Timoshenko coefficient chosen as fitting parameters. The other parameters were taken from literature. The material parameters are listed in Table 1, the measurement of the dispersion curves is plotted in Figure 2.
Figure 5. Geometry of the beam with 4 notches. The beam has a square cross section with side length of 6mm.

4 Results

The simulation led to the Lagrange diagram depicted in Figure 6. It shows the lateral displacement along the beam’s axial coordinate in a time range from 0.8ms - 1.6ms. The geometry of the beam is sketched in Figure 5. The beam was simulated longer than its original length to avoid reflections from the free end. However, only the true length is plotted. The time reversed displacement data was set as the boundary condition at \( x_m = 2 \) m.

Figure 6. Simulated Lagrange diagram of a beam with several notches produced by a time reversal numerical simulation from experimental data. The dashed line marks the path of the longitudinal wave that generated the bending waves. Rectangles mark the windows, where the maxima were searched. White circles are plotted at the locations, where these maxima were found. Three notches were correctly identified, the smallest notch was not found.

The dashed line marks the path of the longitudinal wave that generated the flexural waves at the notches in the experiment. The rectangles indicate the search window used to find the local maxima of the displacement. The white circles mark the position, where the maxima were found. The results of the detected locations are listed in Table 2. Capital variables indicate simulated results, the other variables are meant for reference. The first column lists the positions where the notches were detected. The second column lists the true positions of the notches. \( W_i \) is the amplitude at the local maxima. The fourth column displays the depth of
the notch. The last column lists the orientation of the notch, whether it is open in positive or negative z-direction.

<table>
<thead>
<tr>
<th>X_i [m]</th>
<th>x_i [m]</th>
<th>W_i [mm]</th>
<th>e_i [mm]</th>
<th>z_i</th>
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<tr>
<td>1.925</td>
<td>1.90</td>
<td>-0.090</td>
<td>2</td>
<td>+</td>
</tr>
<tr>
<td>%</td>
<td>1.15</td>
<td>%</td>
<td>1</td>
<td>+</td>
</tr>
<tr>
<td>0.970</td>
<td>1.00</td>
<td>-0.087</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>0.705</td>
<td>0.70</td>
<td>0.087</td>
<td>2</td>
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</tbody>
</table>

Table 2. Results from the time reversal numerical simulation of a beam with several notches

Figure 7. Relative error with respect to the beam length L=2m.

5 Discussions

The dispersive behavior is clearly visible in the Lagrange diagram in Figure 6 and is apparent as ripples going out from the main pulse train. The wave amplitude strongly oscillates while propagating through the structure due to constructive and destructive interference of the phase components that travel with different velocity. At the measurement position x_m = 2m, the individual waves that were generated at the notches can’t be distinguished from each other. In the simulated back propagation process, these overlapping waves separate again.

5.1 Focal angle

The ‘focal angle’ is sketched in Figure 1 and is denoted with \( \alpha \). The predicted focusing and defocusing of the back propagating flexural wave is observable, although not as pronounced as expected. The dispersion curve of the flexural wave in Figure 2 shows that the higher the frequency, the smaller the increase in phase speed. The same is true for the group speed behavior. On the other hand, the system piezoelectric element and beam has a strong high pass behavior. In this experimental setup, it is difficult to excite a wave pulse that bears enough energy in the low frequency range around 20 kHz. It is hence difficult, to obtain a better focal angle with the current experimental setup. In order to increase the ‘focal angle’, the excitation needs more low frequency content and thus another excitation method is necessary. Ishak et. al [7] excited transient pulses by impacting the structure with a steel ball. The frequency spectrum he obtained covers the whole lower band up to 15 kHz. This would enhance the focal angle heavily and improve the sensitivity and resolution for small defects.

5.2 Accuracy and sensitivity

Referring to Table 2, all three larger notches were detected with an accuracy of about 2%. The relative error does not increase with the propagation distance as one would probably expect, but is smallest for the notch lying farthest away from the measurement location, see Figure 7. Roux et al. [15] performed time reversal experiments with ultrasonic waves in an unbound medium and in a wave guide. He reported, that the spatial refocusing was higher in the wave guide than in the unbound medium due to the fact, that not only the direct path between source and time reversal mirror contributed to the refocusing but also the several reflections that were recorded by the time reversal mirror. The effective aperture of the time reversal...
mirror was therefore increased by receiving the information on the source not only through the direct ray path but also through several reflected waves that arrived later in time. For the case of reflections occurring in a time reversal experiment, the effective aperture of a time reversal mirror is directly related to the number of reflections recorded and as a result to the duration of the time reversal window.

Here, dispersion replaces the effect of reflections to the effective aperture. Due to dispersion, the information on the location of a notch spreads on a larger time scale with increasing propagation path. A notch farther away from the detector is better resolved than a notch that is closer to the measurement position and hence can be located with higher accuracy. Effects of damping and noise will of course limit this effect.

The smaller notch could not be detected. The corresponding wave that should have converged at 1.150m, is slightly visible but not strong enough for the detection algorithm. The amplitudes generated by the small notch are buried under the dispersing amplitudes from the larger notches. Better sensitivity for defects that are surrounded by other scatterers is achieved with a higher ‘focal angle’. The focusing and defocusing would then occur faster and the focal spot would be smaller.

5.3 Amplitudes

The amplitudes at a notch position should correspond to the size of the notch and the sign of that amplitude should indicate whether the notch faces in positive or negative z-direction. Looking at the amplitude values in Table 2, this is the case for the two notches at 0.7m and 1.0m. The amplitudes for the corresponding notches are exactly equal but with different sign. This is according to our expectations and motivates to characterize the notch based on the simulated amplitude found at the focal spot.

However, the notch at 1.9m showed different behavior. Its corresponding amplitude is larger and its sign different compared to the amplitude that corresponds to the notch at 0.7m, although size and orientation of these notches are the same. This discrepancy could probably be explained with free end effects. However, after 0.1m distance from the free end, the exponential decreasing transient mode should not be noticeable anymore. Therefore, another explanation has yet to be found and the characterization of a notch due to the generated amplitudes by a broad band flexural wave is subject to further studies.

6 Conclusions

Three out of four notches were detected and localized with the proposed method. The missed notch was smaller in size and too close to another scatterer and was not detected.

Notches farther away from the measurement position were localized with higher accuracy. This interesting result is attributed to the effect of an increase in the effective aperture of a time reversal mirror due to dispersion.

The feasibility of the presented method is shown and impresses with its conceptual simplicity. The method can be improved with the use of an impact like excitation. This would enhance the low frequency range of the excitation and hence improve the sensitivity and resolution for small defects.

A simple beam is far away from structures in operation. The presented method does not easily translate into an applicable NDT method for real structures. However, one important result is, that the dispersion of a wave contains information about its propagated path and that the time reversal numerical simulation method is a handy tool to access this information. Where other methods struggle with the distortion of a wave due to dispersion and try to find a go around by using narrow band pulses, the TRNS method makes use of it. The information about the location of a defect is buried in the time domain wherefore a single point measurement
suffices. The only requirement is that a dispersive wave is generated at a defect. This property might also find interest in the field of acoustic emission and source detection. If acoustic emissions could be measured by a TR mirror and rejected in a numerical simulation, one would have a powerful method for structural health monitoring.

References