Reconstruction of Limited View Tomography Data by DIRECTT

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Abstract
Incomplete tomographic data sets such as limited view (missing wedge) data represent a well-known challenge for reconstruction algorithms, since they unavoidably lead to substantial image artefacts. Such data sets may occur in industrial computed tomography of limited access (e.g. extended components, fixed objects), directional opacity, limited sample life time or laminographic set-up. We present strategies to effectively suppress the typical elongation artefacts (e.g. lemon-like deformed pores) by our iterative algorithm DIRECTT which offers the opportunity to vary the versatile reconstruction parameters within each cycle. Those strategies are applied to experimental data obtained from metallic foams as well as model simulations. Comparison is drawn to state-of-the-art techniques (filtered backprojection and algebraic techniques). Further reference is made to reconstructions of complete data sets serving as gold standards. For quantitative assessment of the reconstruction quality adapted techniques based on spatial statistics are introduced.

Keywords: Computed tomography, Limited tomographic data, Iterative reconstruction, DIRECTT, Limited view, Missing wedge, Spatial statistics

1. Introduction

In industrial computed tomography a special type of incomplete data sets refers to the so-called Limited View problem, also known as Limited Angle or Missing Wedge. In case of limited access to extended components and/or large aspect ratio shapes, accompanied by strong attenuation, the resulting measurements are limited to an angular projection sector of less than half a circle. Therefore the reconstructions severely suffer from improper deposition of material density elements.

The mentioned restrictions occur especially in the established techniques of electron tomography and laminography. In the first case the Limited View is mostly caused by restrictions of the manipulation stage or by the sample life time (potential degradation), respectively, and is always accompanied by several other deviations from ideal conditions [1], which hamper studies of the isolated restriction. Concerning laminographic set-ups the Limited View is inherent to the principle.

Since all types of filters which are applied for tomographic reconstruction tasks rely on the completeness of measurements (regarding projection angles and regions of interest) their application unavoidably must lead to typical image artefacts such as elongations along the mean direction of projections.

In our study we aim to plumb the potentials of suppressing those artefacts by application of appropriate strategies to the DIRECTT (Direct Iterative Reconstruction of Computed Tomography Trajectories) algorithm. The algorithm has been demonstrated to cope with non-standard reconstructions tasks very well [2,3]. The applied strategy was found empirically and is described in detail in section 3.
In order to study the isolated restriction we consider simulated data as well as purposely reduced experimental X-ray (synchrotron) data with structural details in common. While the real data images pores in metallic foam, the model data is built up from ideal circular pores in a homogeneous matrix.

2. DIRECTT Algorithm

The DIRECTT algorithm has been described in detail elsewhere [3-5]. Here, we restrict ourselves to emphasize its key features. As any iterative algorithm DIRECTT is based on a sequence of alternating reconstructions and projections. The following items list the distinctive features:

**Initialization**
- compute line integrals of sinogram entries along all possible (sinusoidal) trajectories
- optional filtering of those integrals
- selection of a pre-defined percentage of the maximal amplitude
- weighted addition of the selected integrals to the (empty) reconstruction array

**Iteration**
- precise projection of the intermediate reconstruction
- subtract those projections from the measured sinogram
- reconstruction of the obtained residual sinogram as before
- weighted addition of actual elements to the intermediate reconstruction array
- iterative repetition

**Termination**
- terminate the algorithm at a pre-defined number of cycles or when the residual sinogram converges

It should be emphasized that the selection implies an alteration of only a subset of reconstruction elements in each cycle. The mentioned parameters can be varied in each cycle either by the operator after assessing the intermediate result (interactive reconstruction) or automatically depending on the iteration progress. The choice of filters depends on the reconstruction task, i.e. either to overcome certain restrictions of the measurement (regarding completeness, geometric or intensity distortions) or to emphasize special reconstruction features of interest. The selected fraction \( \alpha \) ranges from 0 (accept all reconstructed values; no selection) to 1 (accept only the maximal absolute value).

The precise projection results from treating pixels actually as volume elements instead of points. Given very precise projection input data, DIRECTT is even capable of computing subpixel reconstructions. Although hard to believe, model calculations prove that DIRECTT yields superior results without any filtering.

Concerning the task to reconstruct limited view sinograms, two features make DIRECTT a promising candidate to solve it: (i) the correct deposition of the line integral values within the reconstruction array, (ii) the tolerance of the reconstruction quality for the chosen type of filters if the iteration steps are small enough, as a valid filtering for all angles is generally impossible in the limited view case.

In model calculations, DIRECTT has been shown to overcome the Limited View problem even under severe restrictions. Figure 2 displays the direct comparison of results computed by DIRECTT and FBP for identical angular sectors of 30, 60, 120 and 180 degree. While the FBP exhibits dominant elongation artefacts along the mean direction of projection, the iterative procedure proves to suppress those artefacts effectively. On the expense of numerous
iterations the only remaining deviations from the model refer to details of the imaged elements.

Figure 1. Reconstruction principle of the iterative procedure at the example of a 14-dots model object: (a) the measured (or simulated) sinogram, (b) an intermediate (incomplete) reconstruction, (c) the residual sinogram and (d) the final reconstruction.

Figure 2. Limited view model calculations: comparison of reconstruction results obtained from DIRECTT (top row) and Filterd Backprojection (bottom row) for identical angular sectors of 30, 60, 120 and 180 degree (from left to right).

3. Reconstruction strategies

Some of the above mentioned parameters are altered in the course of iteration. Under the basic assumption of an essentially binary mass assembly we casually obtain a rough
estimation of the density $\rho_0$ from the ratio of the integral mass and the number of non-zero elements in the initial reconstruction.

The course of iteration can be subdivided into three stages. The first two of them rigidly impose binary reconstructions while the last stage is free of restrictions. It is our experience that reconstructions work best when the iteration is performed very cautiously. In terms of the above introduced parameters, the selection $\alpha$ is initially chosen close to unity and the very few accepted elements are added to the intermediate reconstruction with a small weighting factor $F$. Thus the iteration progress, which is obtained from the residual sinogram’s integral weight, is purposely slow. The advantage of this strategy is to avoid the formation of strong artefacts (amplitudes) which cannot be purged in the further course of iteration.

In the first stage it proves useful to purposely assume a substantially smaller binary value than estimated from the initial reconstruction. This eases to heal out erroneous mass locations implied by the missing wedge of projections. From cycle to cycle the selection $\alpha$ is reduced until the residual sinogram’s integral weight converges to a non-zero constant. At the end of stage one we obtain a map of locations nearly free of artefacts.

In the second stage the obtained map is set to the initially estimated value $\rho_0$ (by simple multiplication). Iterations are strictly binary ($0$ and $\rho_0$) again. Starting from high values $\alpha$ they are reduced until convergence to nearly zero is reached. At the end of stage two we obtain an approximate solution of the reconstruction.

Stage three which usually comprises up to five cycles is free of all restrictions regarding selection and discretization.

4. Quality Assessment

In order to get a measure of the reconstruction quality beyond pure eye inspection we pursue two approaches of assessment. The first is to assess the remnants of the missing wedge in the reconstruction as an integral criterion. This is done by evaluating the spectral amplitudes of the respective Fourier transforms inside and outside the wedge. Local criteria refer e.g. to edge contrast and mass overshoots.

The second approach employs techniques from spatial statistics. Missing wedge tomography causes an erroneous elongation of objects and typically also results in streak artefacts, which occur at random locations in the reconstructions. Since local object configurations have a dominant impact on both the formation of streak artefacts and the local contrast at phase boundary pixels, we simulate random samples resembling foam structures with non-overlapping but randomly positioned pores. Due to the random nature of the phantom data and the resulting artefacts, the assessment of reconstruction quality is naturally a statistical issue. Our approach is based on statistical orientation analysis of high grayscale gradients in the reconstruction. These primarily correspond to pore boundaries or result from streaks. Hence typical elongation and streak artefacts are reflected by the orientation of these gradients. The pixelpaths of the gradients are converted to polygonal tracks by a fully automatic algorithm. The directional distribution of the resulting fiber system is investigated by a length-weighted kernel density estimation of its rose of directions (RDR). The RDR is the probability distribution of the local orientation in a randomly chosen boundary point.

A general benefit of using simulated phantoms is that in contrast to the reconstruction of experimental projection data, the input data can be exactly controlled and reconstruction conditions such as signal to noise ratio (SNR) may be systematically varied.
5. Examples

5.1 Experimental Data

A metallic foam sample (alloy: Al$_{50}$Ni$_{25}$Fe$_{25}$) has been subjected to a tomography experiment using monochromatic ($E = 50$ keV) synchrotron radiation (storage ring BESSY II in Berlin). The sample (cross-section app. $1.1 \times 0.65$ mm$^2$) was projected under 1200 angles of rotation in a 180 degree sector (corresponding to an angular increment of 0.15 deg). The attenuated radiation intensity was detected with a $4008 \times 2672$ pixel detector (pixel size: 1.1 $\mu$m). In order to study missing wedge effects, a 120 deg sector of projections was used for the reconstruction. In contrast to electron tomography data, the experimental imponderabilities are reduced to a minimal extend, thus the missing wedge problem is isolated.

Due to the lack of an ideal solution, the reconstruction of the full 180 degree data serves as the gold standard.

We discuss reconstruction details at the example of a single slice as displayed in Figure 3. The (arbitrarily chosen) mean direction of the projection sector corresponds to the horizontal axis. Closer inspection of the FBP reveals some peculiarities:

(i) Elongation artefacts, which distort the roughly circular pores to lemon-like objects, point along the mean direction of the projection sector. This involves that adjacent but separated pores (as seen in the gold standard) seem to connected.

(ii) In the perpendicular direction walls of increased mass are formed at the pore edges (see Fig. 4, right).

(iii) Roughly 20 % of the total mass are located outside the sample area (see the cross-hatching structure to the left and right of the sample).

(iv) A smaller pitch of the edges.

![Figure 3. Reconstruction of one slice of the metallic foam. Left: reconstruction of the full (180 deg) sinogram serving as the gold standard, middle: FBP of the 120 deg sinogram, right: DIRECTT reconstruction of the 120 deg sinogram (11 iterations).](image)

Argumentation is straightforwardly that all these observations are due to the missing wedge. The missing spectral information along the mean direction of the projection sector hampers
the correct deposition. The most direct way to visualize the anisotropy are the respective 2D Fourier transform where the missing wedge spectral amplitudes is obvious (Fig. 4 left). Application of the DIRECTT algorithm according the strategy described in section 3 removes almost the listed artefacts. However, two aspects do not match the gold standard reconstruction: while the histogram of the gold standard is a bimodal density distribution the DIRECTT reconstruction reveals a spiky histogram with dominant peak introduced at the second stage of iteration. Furthermore, some curved streak-like objects are introduced perpendicular to the mean direction of the projector sector. They can be seen e.g. in the lower right part of the reconstructed slice in Fig. 3. They are preferably formed in regions where long connected patches of mass are found.

![Figure 4. Enlargements of identical details of the gold standard, 120 deg FBP and 120 deg DIRECTT reconstructions (top row, from left) and the respective 2D Fourier transforms (middle). The bottom row depicts a polar plot of the Fourier transforms (left) and a comparison of profiles along an arbitrarily chosen way within the reconstructions.](image-url)
5.2 Simulated Data

The simulated model data emulates the foam structure to a certain degree: the pores are distributed randomly and do not overlap. In contrast to the real foam the density is strictly binary except of the pore edge pixels whose values correspond to the partial pixel coverage. Furthermore the pore size is kept constant (monodisperse) in the single models. These phantoms are generated by random sequential adsorption (RSA) models [6]. The models are realized in a 500×500 pixel window at a pore size of 20 pixels in diameter. Results for other pore sizes, are qualitatively similar [7].

100 phantoms are generated, each of which is superimposed with different signal-to-noise ratios (ranging from 20 to 100) after projection and converting to intensity.

Figure 5. An projected example model (20 pixels pore size), 120 deg FBP and the 120 deg DIRECTT result (16 iterations) for an simulated SNR of 100 (top, from left) and the respective 2D Fourier transforms.

Fig. 6 shows that even under a rather large missing wedge of 60°, DIRECTT reconstructions preserve the phase boundary orientations in a much better way than FBP. As in particular indicated by the integrated mean-squared error of the density estimations for the RDR, DIRECTT substantially reduces object elongations over the entire range of SNRs between 20 and 100 compared to FBP. Nevertheless, the improvements of DIRECTT become less pronounced at low SNRs corresponding to high noise levels in the projection data. DIRECTT reconstructions under high noise levels suffer in particular from a more frequent occurrence of streak artefacts with a strong directional preference. These are extracted by the gradient filters for the segmentation of phase boundaries and thus the estimated densities for the RDR are a sensitive tool for the assessment of streak artefacts.
6. Conclusion

Application of the DIRECTT algorithm reduces the typical missing wedge artefacts substantially. The presented strategy to suppress the artefacts by the iterative algorithm is realized by variation of the reconstruction parameters in the course of iterations. Those strategies perform with experimental data obtained as well as with model simulations. Comparison to filtered backprojection reveals the advantages. For quantitative assessment of the reconstruction quality adapted techniques based on spatial statistics are used. These are necessary since missing wedge artefacts are in principle dependent on local spatial object configurations, which thus need to be randomly varied in order to obtain a more objective assessment of reconstruction quality. In the current study we only briefly present some descriptive statistical results clearly demonstrating the superior quality of DIRECTT reconstructions. Novel statistical tests assessing deviations in the RDR of reconstructions from the phantom data are discussed in [7], where the statistical comparison also includes reconstructions computed by the frequently used SIRT algorithm. The presented findings do not finally characterize all the potentials of novel algorithm and further efforts are required for full exploitation of its properties.

Figure 6. Estimated densities for the rose of directions of phase boundaries in FBP (a) and DIRECTT (b) reconstructions of the simulated models under a missing wedge of 60°. The rose of directions of the object boundaries in the original phantoms is depicted in black. (c) shows the integrated mean squared deviations ($L^2$-distance) of the densities in the reconstructions from the original phantoms.
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References