Fractal Dimension of Waveforms as a Useful Feature in Ultrasonic Imaging

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Abstract
In this paper an alternative way of generating C-scan images using Fractal Dimension (FD) of any digitized waveforms as an ultrasonic feature is proposed. The work is carried out on the composite specimen having flaw in the form of inclusion and damage created by the drop weight impact. The technique to estimate FD of a 2D image is modified in this work and a procedure to compute FD of a digitized waveform is evolved. FD is seen to be a representative or a defining parameter of the waveforms received from different regions of the scanned domain for being used in generation of C-scan image. A feature selection procedure based on the ID3 algorithm is also performed on the dataset. It substantiates FD as a potential ultrasonic feature.

Keywords: Fractal Dimension, C-scan, Data clustering and ID3 algorithm.

1. Introduction

The concept of fractals has been used extensively for graphical simulation of natural phenomena, study of image textures and analysis of material surfaces. The defining characteristic of a fractal is that it has a fractional dimension from which the word fractal generates. Historically, the word fractal and fractal dimension have been interpreted in many ways. Mandelbrot [1] described fractal as a “shape made up of parts similar to the whole in some way.” This property of self similarity or scaling is one of the central concepts of fractal geometry. It has closely connected with the intuitive notion of dimension. Voss [2] showed the generation of a series of random fractal shapes to provide a visual introduction to the concepts of fractal geometry. One such example is the Von Koch curve where the property of self similarity and dimension is well illustrated. He discussed generation of the curve from a single straight line and subsequently the fractal dimension of a D-dimensional self-similar object was deduced. Pentland [3] extended the shape-form-shading and shape-form-texture methods (applicable to smooth surfaces) to rough surfaces. He used the model of natural surface shapes to derive a technique for 3-D shape estimation for treating the shaded and textured surfaces in a unified manner. Lundahl et al. [4] demonstrated the use of fractal theory in analyzing X-ray medical images. Chen et al. [5] also applied the fractal concept to the classification and analysis of images obtained in X-ray medical imaging. Bhatt et al. [6] used fractal concept to analyze the quality of the reconstructed images in non medical areas. The book by Falconer [7] provides a general framework for study of irregular sets by means of Hausdorff measure and Hausdorff dimension, which play a central role in defining the fractal sets and their corresponding fractal dimensions. Munshi et al. [8, 9] investigated the applicability of the fractal concept in the field of non destructive evaluation of real tomographic images. They applied fractal concept to analyze tomograms obtained by an X-ray CT scanner.
In the present investigation a methodology for automated imaging of inclusion and impact inflicted flaw in glass-epoxy composite specimens are discussed. The methodology is essentially based on automated grouping of dataset pertaining to fractal dimensions computed from the waveforms for each location of the scanned domain. Dataset are subjected to systematic grouping as per algorithm developed by Wong et al. [10] and the results are used for imaging the scanned zone.

2. Experimental Setup

Ultrasonic C-scan of the composite laminate is done for making precise measurement of the variation of strength of the ultrasonic signal when the transducer is moved over a selected region around the zone of flaw. This is accomplished through controlled and automated movement of the transducer in a plane parallel to the surface of the laminate. The facility comprises an immersion tank made of acrylic glass and a housing frame furnished with two lead screws in mutually perpendicular directions. Two stepper motors drive the lead screws while a common nut, holding the probe holding device, moves linearly due to their rotation. The transducer, fitted in the probe holding device, can move along two mutually perpendicular directions in precise steps and is capable of scanning any predefined two-dimension region. The transducer is connected to an ultrasonic board (PCUS11 [11]) that acts as the pulsar, receiver and digitizer of the ultrasonic waveform. The board seamlessly interacts with the controlling software [12] that has the capability to condition, gate and zoom the digitized signal. At each location, relevant portion of the waveform is digitized and is stored as an ASCII file for future post-processing.

3. Algorithm to Find Fractal Dimension (FD) of an Ultrasonic Waveform

The algorithm of Bhatt et al. [6] to calculate the fractal dimension of a 2-D digitized image is modified to suit calculation of the fractal dimension of a digitized waveform. The ultrasonic signal is digitized over a gate length L along the time axis with a pre-selected sampling rate. Depending on the chosen sampling rate the waveform is digitized at a number of points n, i.e., amplitude values are obtained at these points either in dB, mV or in percentage of screen height (%SH). A typical signal, in RF form, looks like having several peaks and valleys over the base (zero) line as shown in Fig. 1. These deviations take place over a certain portion of the gated length and thus could be treated as a roughness (dents) in a 2-D domain. Had the amplitude been same at all digitized locations, it would result in a 2-D domain with no dents. The algorithm primarily identifies this difference in the waveforms and quantifies it in the form of a fractal dimension.

In this way the 1-D digitized waveform can be represented as a fractal having a fractal dimension between 1 and 2 and is computed from the fractal graph. A waveform containing n digitized points is shown in Fig. 1. Let $I_i$ and $I_j$ be the intensities of two digitized points, i and j of the waveform respectively. The fractal graph of the waveform is visualized as the plot of log (NMSID) vs log (NSR). Here, NSR corresponds to the normalized scale range vector. It consists of reference scale and generally corresponds to the possible normalized distances between any pair of points in the concerned waveform. Thus,

$$NSR = \{ndr(1), ndr(2), \ldots, ndr(k), \ldots, ndr(n-1)\}$$

(1)
where,
\[ n_{dr}(k) \leq (j-i) < n_{dr}(k+1) \]  
(2)

The NMSID vector corresponds to the Normalized Multi-Scale Intensity Difference vector. It consists of different absolute-intensity difference averages around each normalized reference scale (NSR) i.e,
\[
NMSID = \{ndi(1), ndi(2), \ldots, ndi(k), \ldots, ndi(n-1)\} \]
(3)

where,
\[
ndi(k) = \frac{\sum_{i=1}^{n} \sum_{j=i+1}^{n} |i-j|}{npn(k)}
\]
(4)

Figure 1. A waveform containing \(n\) digitized points

In Eq. 4, \(npn\) is the Normalized Points-pair Number vector, which consists of elements that represent the number of point pairs with scale (distance) values similar to reference scale. Plotting \(\log_e\) (NMSID) vs. \(\log_e\) (NSR) for \(k=1, 2, \ldots, (n-1)\), results in a curve consisting of \((n-1)\) pairs of points. This curve is the fractal graph which, for a practical waveform, exhibits an initial rise up to some NSR value and then starts to dip. The initial linear rise with a slope of less than 1 corresponds to the fractal nature of the curve and gives an idea of the limit of the NSR for such nature. A linear fractal graph represents a perfect fractal; otherwise a least square linear regression on the relevant part of the graph gives the required slope \(b\). FD of the waveform is then calculated by the relation, \(FD=2-b\).
4. Grouping of Data Set

An effort has been made for automatic generation of C-scan images for a set of given data using clustering technique. It is a way to create group of objects in such a way that the profiles of objects in the same group are very similar and those in different groups are quite distinct. Grouping of data set is performed by implementation of algorithm developed by Wong et al. [10]. This method is based on regulating a similarity measure and replacing movable vectors so that the appropriate number of clusters is determined by the best performance index. At initial stage a data point is chosen as a reference vector and those vectors that have high similarity with the reference vector has to be found. Then reference vector will be replaced with the average of those vectors with high similarity with the reference vector. In this way one time will reach when all the replaced vectors will tend toward their cluster centres. In the iterative process of the algorithm, the width of the similarity measure function can be changed. The value of this width plays an important role in determining how large or small range of data can be grouped in the same cluster. Each different width value may result in different number of clusters and their centres. In the iterative process, the width is increased by some increment, calculated on the basis of similarity measure, and for each width the classification process is evaluated through calculation of a performance index. In this way the optimum numbers of clusters along with the classification results are obtained that correspond to the maximum performance index.

5. Generation of C-scan Image based on Fractal Dimension

In this section steps of generating C-scan image, based on fractal dimension of the digitized waveforms are discussed and results are presented. As per the algorithm presented in section 3, computer code has been developed for composite specimens having flaw in the form of inclusion and impact. Initially the scanned domain is composed of two regions, (i) the non-inclusion region and (ii) the inclusion region. One representative waveform, each from Zone (i) and (ii) and shown in Fig. 2 and Fig. 3 respectively, are taken to illustrate the FD computation procedure.

![Representative waveform for a non-inclusion zone](image)

**Figure 2. A-scan trace from a point in the non-inclusion zone**
As per the algorithm outlined in section 3, the fractal graphs are obtained through calculation of NSR and NMSID vectors. As there are 986 digitized points (gate-length of 12.3079 µs, digitized at 80 MHz) in the waveform, both NSR and NMSID are 985 element vectors. The corresponding fractal graphs are shown in Figs. 4 and 5 respectively.
It is evident from both the figures that all normalized distance scales in the NSR do not exhibit the fractal behavior. In order to extract the differences between the waveforms in terms of fractal characteristics the initial near linear part of the fractal graphs is used to compute the fractal dimension for both the cases. The truncated fractal graphs with the best fit line in the least square sense are shown in Figs. 6 and 7 respectively.

Figure 5. Fractal Graph of the waveform shown in Figure 3

Figure 6. Relevant part of the Fractal Graph shown in Figure 4 with the best fit line
Corresponding slopes (b) for the non-inclusion and inclusion cases are found as 0.9269 and 0.9434 respectively based on a normalized distance cut off at 25. The higher value of the slope indicates a lower FD (FD=2-b) signifying a higher roughness in the waveform. Thus the slope itself could be a representative feature of the waveform.

The representative slopes of the fractal graph are obtained in a similar fashion for all the scanned points in the composite domain and are subjected to automated classification using Wong’s algorithm discussed in section 4. The classification yields 3 and 5 natural clusters for the composite having inclusion and impact inflicted flaw respectively. This classification results is utilized to generate C-scan images that are shown in Figs.8 and 9 respectively.

Figure 7. Relevant part of the Fractal Graph shown in Figure 5 with the best fit line

Figure 8. Image of Inclusion zone by Fractal analysis
The image, shown in Fig. 8, identifies the inclusion as the dark region. The colour of the inclusion zone is dark in this case. A few black points are scattered in the non inclusion zone but that may be due to the statistical variation in the waveform. In Fig. 9, the image clearly visualizes the core region of the damage defined by the white shade. A careful observation will reveal that it is also able to extract other regions of damage surrounding the core defect region. Pixels belonging to the level defined by the black shade seem to constitute the unflawed region, unaffected by the impact.

6. Sensitivity Analysis by ID3 Algorithm

It is a mathematical procedure developed by J.R. Quinlan [13] to calculate the homogeneity of a data sample. Training set is used to create rules for predicting and information gain is used to select the most useful attribute for classification. It is based on the information theory invented by Shannon et al. [14].

\[ \text{Entropy}(S) = - \sum_{i=1}^{N} p_i \log_2 p_i \]  

(5)

Here \( N \) is the number of distinct symbols appearing in the digitized signals and \( p_i \) is the number of occurrence of the \( i^{th} \) symbol in it. Gain \((S, A)\) is information gain of example set \( S \) on attribute \( A \) is defined by Eq. (6)

\[ \text{Gain}(S, A) = \text{Entropy}(S) - \sum \left( \frac{|S_v|}{|S|} \right) \times \text{Entropy}(S_v) \]  

(6)

where, \( \sum \) is each value \( v \) of all possible values of attribute \( A \). \( S_v = \) subset of \( S \) for which attribute \( A \) has value \( v \), \(|S_v|\) = number of elements in \( S_v \), \(|S|\) = number of elements in \( S \).

In the initial stage of the algorithm the entropy of the total dataset is calculated using Eq. (5). The data set is then split on the different attributes. The entropy for each branch is calculated. Then it is added proportionally, to get total entropy for the split. The resulting entropy is subtracted from the entropy before split to get the information gain or decrease in entropy. The attribute that yields the largest information gain is chosen for the decision node. The algorithm, in this way, determines the sensitivity of different features in respect of a target flaw type.
present case, it has been utilized to check the sensitivity of fractal dimension of the waveform pertaining to composite specimens having inclusion and drop weight impact. Training data set is provided to the algorithm for determining the sensitivity of classification. For the glass-epoxy composite specimen with inclusion type flaw, the test has been conducted for 60 data, out of which 40 are selected from good region and the remaining is from the inclusion region. For impacted specimen, a total of 45 data are used from non-impacted and impacted region. The values of the information gain (i.e., ability to classify classes) are calculated for the fractal dimension and slope of the fractal graph for the stated samples. The calculated values of information gain for FD (these values for slopes are same) are listed in Table 1. For comparison, the information gains of other features for the same scanned zones are also included.

**Table 1. Sensitivity (Information gain) value of different features on flawed specimen**

<table>
<thead>
<tr>
<th>Composite Specimen</th>
<th>Feature Name</th>
<th>Sensitivity (information gain) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass Epoxy with inclusion flaw</td>
<td>Shannon Entropy</td>
<td>0.7299</td>
</tr>
<tr>
<td></td>
<td>Signal Energy</td>
<td>0.1827</td>
</tr>
<tr>
<td></td>
<td>Peak Amplitude</td>
<td>0.0737</td>
</tr>
<tr>
<td></td>
<td>Signal Amplitude</td>
<td>0.0852</td>
</tr>
<tr>
<td></td>
<td>Fractal Dimension</td>
<td>0.3520</td>
</tr>
<tr>
<td>Glass epoxy with drop weight impact</td>
<td>Shannon Entropy</td>
<td>0.9183</td>
</tr>
<tr>
<td></td>
<td>Signal Energy</td>
<td>0.9183</td>
</tr>
<tr>
<td></td>
<td>Peak Amplitude</td>
<td>0.9183</td>
</tr>
<tr>
<td></td>
<td>Signal Amplitude</td>
<td>0.9183</td>
</tr>
<tr>
<td></td>
<td>Fractal Dimension</td>
<td>0.9183</td>
</tr>
</tbody>
</table>

From the above table, it is evident that the sensitivity values of fractal dimension are comparable to other ultrasonic features such as peak amplitude, signal amplitude, signal energy, Shannon entropy etc. This indicates that FD can also be treated as a resourceful ultrasonic feature in analyzing structural integrity of composite panels. In particular, the performance of FD in detecting inclusion type flaw is better in comparison to conventional features. However it is not as good as Shannon entropy. Performance of FD as a feature in identifying drop weight impact zone is found to be at par with the amplitude based features.

### 7. Conclusions

In this work the Fractal Dimension (FD) of digitized ultrasonic waveforms is proposed as a potential feature in generating C-scan images. Its usefulness is judged by generating FD based images and the feature sensitivity study. From the generated images it has been observed that the FD is a potential ultrasonic feature to identify the inclusion as well as the impact inflicted flaw. Features sensitivity checking by the ID3 algorithm reveals that the performance of this feature in identifying drop weight impact zone is at par with the amplitude based features. In case of inclusion flaw it is better than the conventional features such as peak amplitude, signal amplitude and signal energy but not as good as Shannon entropy.
References