Investigation of Suspended Sediment Properties based on Scholte wave

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Abstract. The propagation of the Scholte wave at liquid/solid interface is studied to measure the suspended sediment properties. Based on ultrasonic suspension models, i.e. Uttrick, Uttrick-Ament, HT and Mcelements, the Scholte wave propagation characteristics are analyzed at the interface between Sediment concentration two-phase fluid and porous medium solid. The relationship between Scholte velocity and two-phase fluid’s properties, e.g., dispersion, volume content, particle size are discussed. The Scholte wave propagation and signal are obtained by simulation, and the obtained Scholte wave velocity are in accord with Uttrick-Ament and HT mode. The proposed work is practically feasible and of great scientific and technological importance in sediment research.

Introduction

The characteristic of ultrasound propagation in the two-phase has always been the focus of research scholars. Some scholars have proposed a series of classic ultrasonic propagation models, i.e. Uttrick[1], Uttrick-Ament[2], HT[3], ECAH[4,5], Mcelements[6]. The propagation and attenuation characteristics can be used for analyzing the volume content, compressibility or density of either the dispersed or continuous phase, which provide a non-invasive and non-destructive measurement be widely used in detection. The Scholte wave has large amplitude, long propagation distance, and low attenuation loss. When propagating along the interfaces, these waves carry information of the medium on both sides, which can be applied for inversion analysis of the medium properties. The dependence of the fluid/solid interface wave on the sediment property is a subject of profound theoretical and practical significance. In this paper, the time-domain waveform of interface wave between sediment containing two phase fluid and porous solid is simulated, whilst the solid interface wave and two-phase fluid ultrasonic propagation model are combined. By analyzing the dispersion effect and influence of volume fraction, particle size of sediment impact on the Scholte wave, we provide some theoretical guidance for the detection of sediment character of two-phase.

1. Establishment of characteristic equation

A fluid/solid interface is depicted in Fig. 1, where y=0 represents the interface and x indicates the propagation direction of the interface waves. y< 0 represents the semi-infinite space, y >0 denotes the semi-infinite porous solid space.
The potential function of medium (3)

where \( \eta_1, \eta_2, \) and \( \eta_3 \) are coefficients of \( P_1, P_2, \) and \( SV \) acting on the phase.

\[
\eta_1 = [\rho_1 R - \rho_{12} Q - A(\frac{1}{c_{pf}^2})]/(\rho_{22} Q - \rho_{12} R) \quad \eta_2 = [\rho_1 R - \rho_{12} Q - A(\frac{1}{c_{ps}^2})]/(\rho_{22} Q - \rho_{12} R) \quad \eta_3 = -\rho_{12}/\rho_{22}
\]

\( A = PR - Q^2 \), \( \rho_{11} = (1 - \beta)\rho_s - \rho_f \) is the effective density of the solid phase, \( \rho_{22} = \beta\rho_f - \rho_{12} \) is the effective density of the phase, and \( \rho_{12} = -\beta\rho_f(\alpha - 1) \) is the coupling coefficient of solid and; \( \rho_s \) is the density of solid skeleton, \( \rho_f \) is the density of pore, \( \beta \) is the porosity of the porous solid, and \( \alpha \) is the coefficient of pore twists and turns. Displacement, stress can be expressed by potential function as

\[
\begin{align*}
\mu_{yy} = \frac{\partial \phi_s}{\partial y} - \frac{\partial \phi_f}{\partial x}, & \quad \mu_{xx} = \frac{\partial \phi_s}{\partial x} + \frac{\partial \phi_f}{\partial y} \\
\mu_{yz} = \frac{\partial \phi_s}{\partial y} - \frac{\partial \phi_f}{\partial x}, & \quad \mu_{xz} = \frac{\partial \phi_s}{\partial x} + \frac{\partial \phi_f}{\partial y}
\end{align*}
\]

Fig. 1. Schematic of Semi-infinite porous solid-interface

For the porous solid, the potential function can be constituted by solid phase and phase potential function. For solid phase, the potential function can be expressed as \( \phi_s, \phi_s \). For phase, the potential function can be expressed as \( \phi_f, \phi_f \). The potential function of medium can be showed as \( \phi_{sh} \). The displacement, stress can be expressed by potential function as

\[
\begin{align*}
\nabla^2 \phi &= \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \\
\nabla^2 \phi &= \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}
\end{align*}
\]

Applying the Fourier transform to time \( t \) and space \( x \), which are presented with * and _, the potential function solutions in transform field are obtained as

Solid phase:

fast p-wave \( P_1: \phi_{pf}^* = Be^{-k_{pf}y} \) slow p-wave \( P_2: \phi_{ps}^* = c_{ps}e^{-k_{ps}y} \) shear wave \( SV: \phi_{sh}^* = De^{-k_{sh}y} \)

where

\[
\begin{align*}
\phi_s &= \phi_{pf}^* + \phi_{ps}^* = Be^{-k_{pf}y} + Ce^{-k_{ps}y} \\
\phi_h &= \phi_{sh}^* = De^{-k_{sh}y}
\end{align*}
\]

(2)

where \( \eta_1, \eta_2, \) and \( \eta_3 \) are coefficients of \( P_1, P_2, \) and \( SV \) acting on the phase.

Displacement, stress can be expressed by potential function as

\[
\begin{align*}
\mu_{yy} = \frac{\partial \phi_s}{\partial y} - \frac{\partial \phi_f}{\partial x}, & \quad \mu_{xx} = \frac{\partial \phi_s}{\partial x} + \frac{\partial \phi_f}{\partial y} \\
\mu_{yz} = \frac{\partial \phi_s}{\partial y} - \frac{\partial \phi_f}{\partial x}, & \quad \mu_{xz} = \frac{\partial \phi_s}{\partial x} + \frac{\partial \phi_f}{\partial y}
\end{align*}
\]

(4)
\[
\begin{align*}
\sigma_{11} &= (P-2\mu)(\frac{\partial \mu_{s11}}{\partial x} + \frac{\partial \mu_{s12}}{\partial y}) + 2\mu \frac{\partial \mu_{s21}}{\partial x} + Q(\frac{\partial \mu_{s22}}{\partial x} + \frac{\partial \mu_{s23}}{\partial y}) \\
\sigma_{12} &= Q(\frac{\partial \mu_{s21}}{\partial x} + \frac{\partial \mu_{s22}}{\partial y}) + R(\frac{\partial \mu_{s23}}{\partial x} + \frac{\partial \mu_{s24}}{\partial y}) \\
\sigma_{s2} &= \mu(\frac{\partial \mu_{s11}}{\partial y} + \frac{\partial \mu_{s12}}{\partial x})
\end{align*}
\]  

(5)

The potential function of the semi-infinite medium, for p-wave:
\[
\varphi_x = Ae^{ikx}
\]

(6)

where, \( k_i = \sqrt{k^2 - \omega^2/c_i^2} \).

Displacement, stress can be expressed by potential function as
\[
\mu_{s1} = \frac{\partial \varphi}{\partial y}, \mu_{s1} = \frac{\partial \varphi}{\partial x}, \sigma_{y1} = K \left( \frac{\partial \mu_{s1}}{\partial x} + \frac{\partial \mu_{s1}}{\partial y} \right)
\]

(7)

The boundary conditions at the interface \((y=0)\)
Continuity of total stress at the interface:
\[
\sigma_{y1} + \sigma_{y2} = \sigma_{y1} + F(\omega)G(k) \quad \text{And} \quad \sigma_{y2} = 0
\]

(8)

Where \( F(\omega) \) and \( G(k) \) are transform domain expression of the excitation source.

Conservation of volume:
\[
(1 - \beta)\mu_{s2} + \beta \mu_{s12} = \mu_{s11}
\]

(9)

Proportionality between discontinuity in pressure and relative velocity to solid in porous medium
\[
p_{L1} - p_{L2} = T\beta(\mu_{21} - \mu_{s2})
\]

(10)

Equation (12) assumes that the rate at which flows relative to the solid is governed by the pressure drop across the surface. The \( T \) is a kind of surface flow impedance. Two special limiting cases we will be considering are: \( T=0 \) corresponds to an open pore situation with free flow of in and out of pores; \( T=\infty \) corresponds to a sealed pore situation, and there is no relative flow.

Substituting equations (6)–(9) into (10)-(12), the characteristic equation can be obtained.

Open pore \((T=0)\):
\[
\begin{vmatrix}
m_{11} & m_{12} & m_{13} & m_{14} \\
m_{21} & m_{22} & m_{23} & m_{24} \\
m_{31} & m_{32} & m_{33} & m_{34} \\
m_{41} & m_{42} & m_{43} & m_{44}
\end{vmatrix} = 0
\]

(11)

Sealed pore \((T=\infty)\)
\[
\begin{vmatrix}
m_{11} & m_{12} & m_{13} & m_{14} \\
m_{21} & m_{22} & m_{23} & m_{24} \\
m_{31} & m_{32} & m_{33} & m_{34} \\
m_{41} & m_{42} & m_{43} & m_{44}
\end{vmatrix} = 0
\]

(12)

where
\[
m_{11} = -K_j(k_j^2 - k^2) \quad m_{12} = [P + Q_1(Q + R)](k_j^2 - k^2) + 2\mu k^2
\]

\[
m_{13} = [P + Q + \eta_3(Q + R)](k_j^2 - k^2) + 2\mu k^2 \quad m_{14} = 2\mu k_{sh}
\]
\[ \begin{align*}
m_{21} &= 0, \\
m_{22} &= -2\imath kk_{pf}, \\
m_{23} &= -2\imath kk_{ps}, \\
m_{24} &= k_{sh}^2 + k^2, \\
m_{31} &= k_i, \\
m_{32} &= (1 - \beta)k_{pf} + \eta_1\beta k_{pf}, \\
m_{33} &= (1 - \beta)k_{ps} + \eta_2\beta k_{ps}, \\
m_{34} &= 0, \\
m_{41} &= K_i\beta(k_i^2 - k^2), \\
m_{42} &= -(Q + \eta_1R)(k_{pf}^2 - k^2), \\
m_{43} &= -(Q + \eta_2R)(k_{ps}^2 - k^2), \\
m_{44} &= 0. \\
m_{51} &= 0, \\
m_{52} &= (1 - \eta_1)k_{pf}, \\
m_{53} &= (1 - \eta_2)k_{ps}, \\
m_{54} &= (1 - \eta_3)k_i.
\end{align*} \]

The transient displacement can be obtained by inverse Fourier transform

\[ \mu_{ij\lambda} = \int (\int \mu_{ij\lambda} e^{-\imath \omega t} dk) e^{\imath \omega t} d\omega \]  
(13)

2. Time-domain waveform

Let us simulate the transient respond firstly, the Parameters of porous solid and fluid are listed in Table 1.

<table>
<thead>
<tr>
<th>Medium</th>
<th>Density (kg/m³)</th>
<th>Bulk modulus (Gpa)</th>
<th>Shear modulus (Gpa)</th>
<th>Porosity (100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porous solid</td>
<td>Solid skeleton</td>
<td>2.48</td>
<td>499</td>
<td>61</td>
</tr>
<tr>
<td>Porous medium</td>
<td>Solid skeleton</td>
<td>1.00</td>
<td>2.2</td>
<td>--</td>
</tr>
</tbody>
</table>

Fig. 2. The transient respond of interface wave for open pores the collect point is 6.25mm from excitation source.

In the simulation \( G(k) = 1, \quad F(\omega) = e^{-\imath \omega f_0^2/k^2} \quad f_0 = 70 KHz \). Fig. 2. shows the transient respond of interface wave for open pores, the collect point is 6.25mm from excitation source. It can be found there are many wave exist at the interface between and porous solid, however, only Scholte wave is caused by pole (point), while other three wave are leded by branch (point) whose amplitude are relatively small. In the case of the sealed pores, the result is relatively complex, more waves exist and, two larger waves leded by pole i.e. Scholte and true surface propagating with difference velocity. All the results are in agreement with the reference [7].

Fig. 3. The transient respond of interface wave for open pores the collect point is 6.25mm from excitation source.
3. Theoretical models of the suspension

Four typical suspension models are introduced to describe two-phase containing sediment.

3.1 Urick model

Urick model is the earliest and simplest research about ultrasonic wave propagation in the suspension of particle [1,2]. In 1984, Urick proposed two separate equations for the acoustic velocity and attenuation within the two-phase. The velocity expression used an effective density and compressibility approach, was given by $c_{\text{eff}} = (\rho_{\text{eff}} \beta_{\text{eff}})^{-1/2}$, these effective properties were additive functions of the individual phases that were dependent on relative amount of each phase, were given by $\rho_{\text{eff}} = \rho(1-\phi) + \rho'\phi$ and $\beta_{\text{eff}} = \beta(1-\phi) + \beta'\phi$. Where $\phi$ is the volume fraction of particle phase, $\rho$, $\beta$ are the density and compressibility of water, and $\rho'$, $\beta'$ are those of suspended particle. In this model, the assumptions of Urick models includes: the two-phase is ideal suspension in the conditions of no velocity dispersion, no viscous scattering, no thermal process, and the particle is smaller than the wavelength. The equation for the attenuation was derived in the consideration of the velocity potentials of waves scattered from the particle.

$$\alpha = \frac{\phi}{2} \left( \frac{1}{3} k^4 R^3 + k(\sigma - 1)^2 \right) \frac{36b^2 R^2 (bR + 1)}{[9(bR + 1)^2 + [4\sigma b^2 R^2 + 2b^2 R^2 + 9bR]^2]}$$

Where $b = \left( \frac{\omega \rho}{2\eta} \right)^{1/2}$, $\sigma = \frac{\rho'}{\rho}$, $\eta$ was the viscosity, $R$ was the particle radius.

3.2 Urick-Ament model

Urick and Ament [2] derived a complex propagation equation for a composite medium and gave a wholly real wave number expression, by considering the relationship between the incident, transmitted and reflected waves when a compression wave impinged on a slab of scatters that was suspended in the continuous phase. The complex propagation equation was given by:

$$k_i^2 = k^2 \left[ \frac{\beta_{\text{eff}}}{\beta_i} - i \frac{3bR(2bR + 3) + 3i(bR + 1))}{bR[4\sigma^2 R^2 + 6bR + 9] + 9i(bR + 1)} \right]$$

where: $b = \left( \frac{\omega \rho}{2\eta} \right)^{1/2}$, $\xi = \frac{\rho'}{\rho}$; $k$ was the lossless compression wave number for continuous phase. According to the equation $k_i = \omega/c_i + i\alpha_i$, in which the real part refers to velocity and the imaginary part refers to attenuation.

The assumptions made are:
(a) Velocity and attenuation are dependent on the frequency and $bR$;
(b) The particles are spherical, rigid, compressible and mobile;
(c) There is negligible absorption in the continuous phase;
(d) The particles does not rescatter acoustic energy and occupied negligible volume.

3.3 Urick-Ament model HT model

Harker and Temple considered acoustic propagation through suspensions from a hydrodynamical perspective in 1988. They derived equations for the viscous drag of one phase acting on the other, and came up with the concept of coupling phase model, as known as HT model [3]. The complex wavenumber equation was given
by: \( k_i^2 = \beta_{\phi} \omega^2 \left[ \rho (\phi \rho' + (1 - \phi) \rho) \right] \) \( S(\omega) \) \( \frac{1}{(\phi \rho + (1 - \phi) \rho) + \rho S(\omega)} \]

Where, the coupling coefficient \( S(\omega) \), is a function of the density, viscosity, radius and the concentration of particle, which was given by:

\[
S(\omega) = \frac{1 + 2\phi}{2(1 - \phi)} + \frac{1}{1 - \phi} \left[ 1 + \left( 1 + \frac{\varepsilon_v}{R} \right) \right]
\]

And \( \varepsilon_v = \sqrt{2 \eta / \omega \rho} \) was the viscous skin depth.

The assumptions made were: there was no gravitational field, no thermal conduction, no mass transfer between phases, and no viscous damping losses in the pure continuous phase, no attenuation in the pure particle phase, and the was an infinite continuum.

### 3.4 Mcclements model

Mcclements model[6] is an improved model of classical ultrasonic suspension ECAH model. The wave number of ECAH model can be expresses as:

\[
\left( \frac{k_i}{k} \right)^2 = 1 + \frac{3\phi}{i k \tau} \sum_{n=0}^{\infty} (2n+1)A_n
\]

Where \( k \) is the plural wave number, \( \varphi \) is the solid volume fraction, \( r \) is the radius of suspended sediment particles, \( A_n \) is a n order matrix of the scattering coefficient. In order to simplify the calculation, Mcclements proposed to only consider the first two coefficient of the scattering coefficient matrix, which play a dominant effect.

\[
A_0 = i (kr)^2/3 \delta (\rho k^2 - (\rho k^2 - 1) - k^2 rc \tau H(\beta / \rho C_b - \beta / \rho C \delta v)^2
\]

\[
A_1 = -i(kr)^2/3 [2(\rho - \rho')(1 + 1)\delta /2 r + 3i \delta v^2/2 r^2] + 3\rho
\]

Where \( H = \left[ 1/(1 - iz) - 1/z' \cdot \tan(z')/\tan(z') \right] - 1 = (1 + i) \cdot r / \delta v, \delta v = \sqrt{(2\pi / \omega \rho)} \)

\( \delta v \) is absolute temperature, \( \beta \) is thermal expansion coefficient, \( C_b \) is specific heat at constant pressure, \( \tau \) is coefficient of thermal conductivity, \( \eta \) is viscosity, \( \delta v \) is hot skin depth, \( \delta v \) is sticky skin depth, parameters with \( \delta \) are particle parameters, others are continuous phase parameters.

### 4. Influence of sediment content on Scholte wave

Let us study the effect of sediment content on the propagation of the Scholte wave first, the sample parameters used for simulating the real river situation are listed in Table1 and Table2 and the open pore model is adopted in the simulation.

#### Table 2. The physical parameters of sediment suspension

<table>
<thead>
<tr>
<th>Media</th>
<th>Density / (kg·m⁻³)</th>
<th>Longitudinal wave velocity / (m·s⁻¹)</th>
<th>Shear wave velocity / (m·s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>1000</td>
<td>1500</td>
<td>--</td>
</tr>
<tr>
<td>Sediment</td>
<td>2640</td>
<td>6600</td>
<td>2750</td>
</tr>
</tbody>
</table>

#### Table 3. Parameters of solid skeleton of porous solid

<table>
<thead>
<tr>
<th>Media</th>
<th>Density / (kg·m⁻³)</th>
<th>Bulk modulus (GPa)</th>
<th>Porosity (100%)</th>
<th>Pore size (um)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid skeleton</td>
<td>2650</td>
<td>38</td>
<td>0.15</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Based on Eq.13, the phase velocity dispersions are plotted in Fig.3. From the dispersion curves we find that almost no dispersion and decay when Scholte wave propagation along the semi-infinite space between the ideal fluid and solid, which demonstrates that the porous medium impact a little on dispersion, while sediment bring out relative more effect on dispersion. The velocity of all four models increase with the frequency although the wave velocity are different; the dispersion is more obvious in low frequency range 0.1 to 0.4 MHz and approximately trends to be stable in high frequency. The dispersion of Mcclements model is most obvious, and the Utrick model demonstrates no dispersion because the frequency effect is not considered in model.

Now, we investigate influence of sediment characteristics on Scholte wave, the frequency is fixed on 1MHz, in which the dispersion effect can be neglected. Fig.5 shows the relation between and sediment volume content based on four models, in the calculation; it can be found that Scholte wave velocity show different tendency for four models. For HT model and Utrick-Ament model, Scholte wave velocity increase with sediment volume content.
monotonously, while it decrease monotonously for Mcclements model, but for Urtick model, Scholte wave velocity reduces before it increases. It can be explained from expression $\rho_{\text{eff}} = \rho(1-\phi) + \rho_\phi \phi$ and $\beta_{\text{eff}} = \beta_\phi(1-\phi) + \beta_\phi \phi$, when $\phi$ is smaller, the front of expression play a leading role, although $\rho_\phi$ is greater than $\rho$, otherwise, the latter leads to velocity increasing. It is also showed, the velocity of Scholte for four models are different at the same frequency, travelling fastest for Utrick-Ament and slowest for Mcclements model. Figure 6 shows the relationship between particle radius in suspension and the Scholte velocity, in this paper the frequency is 1MHz and the sediment volume content is 10%. We see that change tendency of Scholte velocity for three models are in consistent except Utrick model. Scholte wave velocity shows an obvious increase with the particle size when particle size less than 10 $\mu m$, and relative stable in range of 10-100 $\mu m$, the velocity is different at the same particle size, and fastest for Utrick-Ament model. The proposed conclusion is practically feasible and of great scientific and technological importance for sediment measurement.

![Fig. 6. The effect of particle size on the velocity of Scholte wave for four models](image)

5. Conclusion

In this paper, we have explored the propagation characteristics of the Scholte wave at sediment fluid/porous solid interface. The transient responds of interface wave are simulated and Scholte wave feature is found; Based on four ultrasonic suspension models, i.e. Utrick, Utrick-Ament, HT and Mcclements, the Scholte wave propagation characteristics are analyzed at the interface between sediment containing two phase fluid and porous medium solid. The dispersion of Scholte wave is found to be more obvious in low frequency range and becomes stable in high frequency; the dispersion of Mcclements model is most obvious, while the Utrick model illustrates no dispersion. The relationship between Scholte velocity and two-phase fluid properties, e.g., volume content, particle size are discussed, it shows the variation tendency of the Scholte wave velocity of the four models are different, for HT model and Utrick-Ament model, Scholte wave velocity increase with sediment volume...
content monotonously, while it decrease monotonously for Mcclements model, but for Urtick model, Scholte wave velocity reduces before it increase; Scholte wave velocity shows an obvious increase with the particle size when particle size is smaller than 10 μm, the velocity is different at the same particle size, and fastest for Utrick-Ament model. The research lay down the theoretical base for the detection of sediment characteristics of sediment in two-phase fluid.

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