Image Reconstruction of Corrosion under Coating Film by Dynamic Shear Strain Analysis of Lamb Waves

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Abstract. Most of artificial structures are made by carbon steel plates, and are facing the possibility of corrosion. Paint coatings are applied to the surface of the steel plate structures in order to resist rust and corrosion. However, because of long-term usage, rust progresses from small scratches of the coating film and is leading to debonding of structure. In last decades, Lamb waves have been studied intensively as sensitive tools to detect subsurface damages in large areas. However, these media are often characterized by multimodal and frequency dispersive behavior. This implies that the received signals can be expressed as a sum of wave modes that travel at different frequency dependent velocities. Consequently, these characteristics make it difficult to reconstruct images of damages by Synthetic Aperture Focusing Technique (SAFT) or Time of Flight Diffraction Technique (TOFD). Thus for non-destructive testing using a guided wave, it is an important issue to construct an imaging technology for extracting the information of the flaw from the observed signals. This paper proposes a novel method for reconstructing image of the corroded region under coating films. The proposed method has an ability to characterize the corroded region through evaluating the linearity between the orthogonal pair of out-of-surface shear strains. Acoustical experiment has been carried out to validate the outcome of the method. The squared shape specimen is made of S45C steel. Employing laser interferometer, single A0 mode Lamb wave has generated to excite the model structure having a single hollow shaped corroded region under aluminum film.

1. Introduction

Carbon steel plate is one of the more versatile options for application when it comes to a number of different steel components. Products from carbon steel are common in many settings, including manufacture as well as construction. Detection of damage of these structure has been an important area of concern from the early the of the product life cycle. Carbon steel can suffer from corrosion in a wide range of environments. After occurring corrosion, detection of such area is difficult to inspect because of limited access, and such an approach may also incur significant time and associated penalties because of downtime. Lamb waves are guided dispersive waves propagating in an elastic isotropic plate with traction free boundaries [1]. In this type of elastic wave, particle motion lies in the plane that contains the direction of wave propagation and the plate normal (the direction perpendicular to the plate) [2]. Typical dispersion curve of Lamb wave always exhibits two modes of
propagation, i.e. Symmetric (S) and Anti-symmetric (A) [3], whereas each mode contains several order depending on the propagating frequency.

Lamb wave based non-destructive testing (NDT) technique always deals with shorter wavelength than the dimension of the part being inspected. The advantages of lamb waves over bulk waves for damage detection are variable mode selection by using various combinations of different striking incident angles and frequencies. Because this type of wave can travel long distance with little attenuation, they have been studies intensively by different researchers worldwide for Structural Health Monitoring (SHM). The $S_0$ mode was used for corrosion detection in aircraft structures [4] and, the longitudinal modes were employed for plate inspection [5]. This type of waves allows for the inspection of structures over reasonably long distances, and can be used even if local access to that inspected part is not possible [6]. $A_0$ and $S_0$ modes were used in a synthetic aperture array system based on full matrix capture methods for detection of corrosion defects [7]. Several experiment have been carried out to detect the corrosion in welded structure by Sargent [8-10]. Kundu et. al. [11] has identified the most efficient lamb wave modes for detecting defects. Rose [12], described that, the particle motion of wave distribution through the thickness of the plate can be varied to increase the sensitivity to different defects. Limitation of damage investigation in large plate using lamb wave have overcome by Ghosh et. al. [13]. The development of computational tools, along with a more widespread understanding of the nature of lamb waves, made it possible to devise techniques for NDT.

Corrosion under coating is not visible until it has caused paint to blister, crack, or chip. Identifying this type of damage before it becomes visible would minimize repairs and costs and potential structural problem. A number of investigations have carried out to detect corrosion under paint in the recent years. Conventional NDT technologies (infrared, eddy current, ultrasonic, and radiography) have been used to inspect for under coating corrosion with the most promising being thermographic testing [14]. A demonstration has given thoroughly for detecting rust under common paint and composite laminate coatings by using an open-ended rectangular waveguide [15]. Near-field microwave techniques have also been used to detect for the presence of corrosion under paint and primer [14]. Terahertz NDE imaging under paint has been examined as a method to inspect corrosion by examining the terahertz response to paint thickness and to surface roughness [16]. Investigation has also been carried out for the detection of corrosion through layers of paint, using millimetre wavelength signals at 20 GHz [17]. The CUPID (Corrosion Under Paint Integrated Detection) system has offered an extremely flexible and highly customized corrosion management system, which can detect corrosion under paint from remote place [18].

In this contribution the potential of high frequency $A_0$ mode Lamb wave for the detection of corrosion under coating film in a thin carbon steel plate has been investigated. The algorithm of the proposed image reconstruction is summarized as following steps: [a] obtaining a time series signals of the orthogonal pair of out-of-surface shear strains, [b] calculating the determinant of covariance matrix composed of above vector, and [c] reconstructing the wave frontal image of the scattered wave field from the boundary of the corroded region. The outcome of the mathematical concept has been compared to laboratory acoustical experiments using a laser interferometer.

2. Problem Formulation

2.1 Dispersive Curves of Lamb waves

Considering single sound source at the origin on the plate of thickness $d$, density $\rho$, longitudinal velocity $C_L$, and shear velocity $C_T$, it can be found that the general
displacements normal to the plate are given by the combination of the following symmetric
and anti-symmetric waves:

\[
u_{sz}(x,y,t) = \left( a_{szL} \left( \exp(\im \beta_L) - \exp(-\im \beta_L) \right) + a_{szT} \left( \exp(\im \beta_T) - \exp(-\im \beta_T) \right) \right) \\
\cdot \exp \left( \im \left( \omega t - k \sqrt{x^2 + y^2} \right) \right),
\]

(1)

\[
u_{az}(x,y,t) = \left( a_{azL} \left( \exp(\im \beta_L) + \exp(-\im \beta_L) \right) + a_{azT} \left( \exp(\im \beta_T) + \exp(-\im \beta_T) \right) \right) \\
\cdot \exp \left( \im \left( \omega t - k \sqrt{x^2 + y^2} \right) \right),
\]

(2)

where \( \omega \) is the angular frequency, \( k \) is the wave number of propagating Lamb wave, and
\((x,y)\) denotes the location on the surface of the plate. The wave numbers for the normal
particle displacement satisfy the following relations:

\[
\beta_L = \sqrt{\left( \frac{\omega}{C_L} \right)^2 - k^2}, \quad \beta_T = \sqrt{\left( \frac{\omega}{C_T} \right)^2 - k^2}.
\]

(3)

where \( C_L \) and \( C_T \) denote the longitudinal and translational phase velocity respectively. Under
the stress free surface boundary conditions, the symmetric and anti-symmetric of Lamb
waves satisfy the following Rayleigh-Lamb's dispersion equations respectively:

\[
\frac{\tan(\beta_T d/2)}{\tan(\beta_L d/2)} = -\frac{4 \beta_L \beta_T k^2}{(k^2 - \beta_T^2)^2}, \quad \frac{\tan(\beta_T d/2)}{\tan(\beta_L d/2)} = -\frac{4 \beta_L \beta_T k^2}{(k^2 - \beta_T^2)^2}.
\]

(4)

Therefore, the dispersive curves of Lamb waves in a steel plate are plotted as Fig.1.

\[\text{Fig. 1 Dispersion curves of (phase velocity) vs (thickness x frequency) for Lamb waves following modes, } A_0: 0^\text{th} \text{ anti-symmetric, } S_0: 0^\text{th} \text{ symmetric, } A_1: \text{ first anti-symmetric, and } S_1: \text{ first symmetric modes.}\]

The phase velocity curves indicate the speed of propagation of a wave front and are therefore
the curves of particular interest for long range propagation for NDT applications. The
low-frequency \( A_0 \) mode Lamb wave is attractive for NDT because it has a slower phase
velocity. Consequently, it has a short wave length that is equally sensitive to defects at any
depth in the plate. In \( A_0 \) mode, particle displacements and stresses are dominated by the
out-of-surface components. Ideally for long-range NDT, this mode is best exploited at the
lowest possible frequency, in order to realize higher spatial resolution. However, higher
dispersion causes difficulties to localize defects by the conventional Time-of-Flight (TOF)
technique. Avoiding these problems from dispersion, this study introduces an analysis of
linearity between the orthogonal pair of out-of-surface shear strains for localizing corrosion under coating.

2.2 Mathematical model

Assuming an $A_0$ mode Lamb wave field, the normal displacement $u_z(x, y, t)$ is governed by the 2-dimensional wave equation. Analyzing the near field of the scattering object, let us consider the scattered wave field from a subsurface cylindrical cavity, which is covered by a coating film. Figure 2 shows the geometrical relations between the hollow cylinder and the incident plane wave.

![Fig. 2 Geometrical model of the subsurface defect in the plate, S1: rim of the cylindrical cavity, a: radius of the cylindrical cavity, L1: surface layer over the cavity, L2: bottom layer, k: wave-number vector of the incident plane wave, k1: wave-number vector of the incoming wave, r: the vector of an observation point, P, a: the vector of a scattering point, Q.](image)

The radius of the cylindrical cavity, $a$, satisfies the condition as: $2\pi a \ll \lambda$, where $\lambda$ denotes the wavelength of the $A_0$-mode Lamb wave. The observation point $\mathbf{r}$, the scattering point on the rim of the cylindrical cavity $\mathbf{a}$, the normal vector $\mathbf{n}$ (at Q), and the wave-number vector $\mathbf{k}$ are denoted respectively as:

$$
\mathbf{r} = \begin{pmatrix} \rho \cos \phi \\ \rho \sin \phi \end{pmatrix}, \quad \mathbf{a} = \begin{pmatrix} \rho \cos \psi \\ \rho \sin \psi \end{pmatrix}, \quad \mathbf{n} = \begin{pmatrix} \cos \psi \\ \sin \psi \end{pmatrix}, \quad \mathbf{k} = \begin{pmatrix} k_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\pi / \lambda \\ 0 \end{pmatrix} = \begin{pmatrix} \omega / c \end{pmatrix}
$$

In the lowest frequency band, $A_0$-mode is the most dominant Lamb wave and its phase velocity is decreasing proportionally with the square root of the product of frequency and plate-thickness. The incident and outgoing wave-number, $k$ and incoming wave number $k_1$ satisfy the following relation:

$$
\frac{k}{k_1} = \frac{d_1}{d}
$$

(6)

Assuming the following incident plane wave:

$$
u_{i2}(\mathbf{r}, t) = \exp(i\mathbf{k}^T\mathbf{r})\exp(i\omega t)
$$

(7)

The Neuman series expansion is obtained as follows:
\( u_{iz}(r, t) = \left( J_0(kr) + 2 \sum_{n=1}^{\infty} i^n J_n(kr) \cos(n\phi) \right) \exp(-i\omega t), \) (8)

where, \( i, k^T, J_n(\cdot) \) denote the imaginary unit, the transpose of the wave number vector, and \( n \)th order Bessel function respectively. Focusing on the normal displacement of the top surface, the reflected wave can be expressed as following multiple expansion form:

\( u_{rz}(r, t) = \left( A_0 H_0^{(1)}(kr) + 2 \sum_{n=1}^{\infty} A_n H_n^{(1)}(kr) \cos(n\phi) \right) \exp(-i\omega t), \) (9)

where \( H_n^{(1)}(\cdot) \) denotes the first kind \( n \)th order Hankel function and the above function is defined at the exterior of the cylindrical cavity. The incoming wave over the cylindrical cavity can be denoted as:

\( u_{cz}(r, t) = \left( B_0 f_0(k_1 r) + 2 \sum_{n=1}^{\infty} B_n f_n(k_1 r) \cos(n\phi) \right) \exp(-i\omega t). \) (10)

At the boundary of the cylindrical cavity, \( r = a \), three Lamb wave fronts satisfy the continuity of the wave functions and derivatives across the boundary as follows:

\[
\begin{align*}
  u_{iz}(a, t) + u_{rz}(a, t) &= u_{cz}(a, t), \\
  \nabla(u_{iz}(a, t) + u_{rz}(a, t))^T \mathbf{n} &= \nabla(u_{cz}(a, t))^T \mathbf{n}
\end{align*}
\] (11)

Consequently, the coefficients, \( A_n \) and \( B_n \) (\( n = 0, 1, \ldots \)) are obtained as follows:

\[
\begin{align*}
  A_n &= -\frac{k_1}{k_{k+1}} \frac{1}{i^n} \frac{d}{da} f_n(ka), \\
  B_n &= \frac{2}{k_{k+1}} \frac{1}{i^n} f_n(ka).
\end{align*}
\] (12)

Being that the circumference of the delamination is much smaller than the wavelength, the outgoing scattered wave field and the incoming wave field can be expressed respectively as [19, 20]:

\[
\begin{align*}
  u_{rz}(r, t) &\approx \frac{k_1 - k}{k_1 + k} \left( -i\pi \left( \frac{ka}{2} \right)^2 H_0^{(1)}(kr) - 2\pi \left( \frac{ka}{2} \right)^2 H_1^{(1)}(kr) \cos(\phi) \right) \exp(-i\omega t), \\
  u_{cz}(r, t) &\approx \frac{2k}{k_1} \left( f_0(k_1 r) + 2i \frac{k}{k_1} f_1(k_1 r) \cos(\phi) \right) \exp(-i\omega t).
\end{align*}
\] (13, 14)

Figure 3 shows the distribution of RMS value of the outgoing and incoming wave filed. Hence, the total wave field outside the cylindrical cavity, \( ||r|| > a \), is approximated as:

\[
\begin{align*}
  u_z(r, t) &\approx \left( \exp(ikr\cos(\phi)) + \frac{k_1 - k}{k_1 + k} \left( -i\pi \left( \frac{ka}{2} \right)^2 H_0^{(1)}(kr) \right) \\
  &\quad - 2\pi \left( \frac{ka}{2} \right)^2 H_1^{(1)}(kr) \cos(\phi) \right) \exp(-i\omega t).
\end{align*}
\] (15)

In the same way, the wave field inside the cylindrical cavity can be expressed as,
\[
\begin{align*}
    u_z(r, t) \approx \frac{2k}{k_1 + k} \left( I_0(k_1 r) + 2i \frac{k}{k_1} J_1(k_1 r) \cos(\phi) \right) \exp(-i\omega t).
\end{align*}
\] (16)

Fig. 3. 2-dimensional distributions of RMS values of (a) scattered wave \( u_{zs}(r, t) \) and (b) incoming wave field \( u_{zz}(r, t) \), the wavenumber \( k \) and the radius of the cylindrical cavity \( a \) satisfies \( ka = 1/8 \).

2.3 Dynamic shear strain analysis

Analysing the linearity between the orthogonal pair of shear strains, a covariance matrix is adopted as:

\[
C \equiv \begin{pmatrix} c_{rr} & c_{r\phi} \\ c_{\phi r} & c_{\phi\phi} \end{pmatrix} = \lim_{T \to \infty} \int_{-T/2}^{T/2} \nabla u_z(r, t)(\nabla u_z(r, t))^T dt.
\] (17)

Therefore, when \( ||r|| > a \), the determinant of the above correlation matrix can be given as:

\[
|C| \approx \frac{1}{4} \left( \frac{k_1 - k}{k_1 + k} \right)^2 \left( \frac{2\pi}{r} \right)^2 \left( \frac{k a}{2} \right)^4 \sin^2 \phi \cdot \left( 16 k^2 \cos^2 \phi \left| H_1^{(1)}(kr) \right|^2 + k^4 r^2 \left| H_1^{(1)}(kr) \right|^2 + 4k^2 \left| H_2^{(1)}(kr) \right|^2 \right). \] (18)

On the basis of above the discussion, the determinant of the covariance matrix, \(|C|\), possesses following characteristics:

- \(|C| > 0\), when the scattered wave fronts propagate along the any different direction of the incident wave front.
- \(|C|\) concentrates its own energy inside the near-field of the cavity. Therefore, the distribution of \(|C|\) is utilized for the near-field imaging of defects

On the contrary, inside the cylindrical cavity region, the determinant of the covariance matrix, \(|C|\), becomes zero always.

3. Acoustical experiment

In order to assess the effectiveness of eigenvalue imaging algorithms we have carried out acoustical experiment. The structure model investigated in this contribution is made of S45C carbon steel having 2.3 mm thickness with a length and width of 140 mm. The specimen is excited by single A_0 mode Lamb wave and has single corroded region beneath the paint coating. Figure 4(a) shows the block diagram of the proof of concept model. Normal displacements of the plate are observed by the Michelson interferometer. The foot print of the laser beam is shown in Fig.4(b). The inspected area is covered by an aluminum film with 12\( \mu \)m in thickness. This film is used in stead of paint coating.
Figure 5(a) shows the geometrical sketch of the circular cavity, which is covered with aluminum film. The aluminum film is adhered to the surface of the specimen except the circular cavity. In order to simulate the detection of corrosion under paint coating film, the circular region is engraved about 0.5mm in depth, Figure 5(b) shows the distributions of the normalized vertical displacement of particle. The corresponding snapshot, Fig.5(c), shows the reconstructed image of the subsurface defect obtained by the proposed dynamic shear strain analysis. As a result, the proposed method can detect and localize the corroded area smaller than the incident wavelength ($\lambda=24\text{mm}$).

4. Concluding remarks

This study proposes dynamic shear strain analysis for reconstructing a silhouette of corroded-region in the paint-coated carbon steel plate. The experimental results through the proof of concept model are summarized with following conclusions and remarks:

- A corrosion in the plate reflects the wave front of the incident $A_0$ mode Lamb wave. This scattered wave front can be observed on the surface of the plate.
- A novel inspection method based on the analysis of the covariance matrix of dynamic shear strains is proposed.
By analysing the covariance matrix composed the out-of-surface shear strain vector, it is found that the intensity of its determinant becomes increased at the boundary of the corroded region.

References