CT Reconstruction on Unstructured Mesh for Multi-material Object

Yukie NAGAI, Yutaka OHTAKE, Hiromasa SUZUKI
The University of Tokyo, Tokyo, Japan
Contact e-mail: yukie@den.t.u-tokyo.ac.jp

Abstract. Computed tomography (CT) is an inspection tool attracting increasing attention in industry for the last decade because of its ability to inspect an object with a non-destructive manner. In general, the boundary is defined as an isosurface of a CT image that is a surface consisting of points with the same CT value. However, a boundary between two materials with different densities tends to incorrectly be located inside of the object with the lower density. In this presentation we propose a novel algorithm extracting such a boundary accurately. In order to suppress a miss location of boundary, we give another definition of boundary based on the directional derivative of CT value. The input of our algorithm is a sinogram, a set of radiographic images projected from various angles. A CT image is computed from a sinogram, so it can be expected that artifacts on a sinogram are less than CT image. Therefore, by taking a sinogram as input, we can compute accurate derivatives and avoid artifacts arising on a CT image. Along with the computation of differentials, we compute a CT image from the sinogram. For this process, we adopt an unstructured mesh, for instance a triangular mesh or a tetrahedral mesh, as an underlying structure on which a CT image is reconstructed, since it has strong advantages over a structured grid (voxels) on a well representation of surface and less number of triangles on the resulting surface mesh. We will also show the effectiveness of the proposed algorithm with some experimental results.

Introduction

1.1 X-ray CT Scanning in the Manufacturing Field

X-ray CT scanning is nowadays getting introduced into the manufacturing aimed at non-destructive inspections, measurements, inspections of the assembled products, acquisition of the shape, and generation of geometry data for physical simulations. The reason of this movement is that the accuracy of measurement by an X-ray CT scanning was improved in the last decade.

Industrial applications mentioned above, however, require very high accuracy of measurement of objects, for instance, in the order of micrometer or nanometer. The technology realizing such a higher accuracy measurement is highly required. To achieve such an accurate measurement, not only the improvements of the hardware as scanning machines, but also improvements of the software including CT reconstruction algorithm and data structure for a CT volume are necessary. Imprecise expression of the surface of an object is particularly remarkable on the boundary between two materials of a multi-material object. Unfortunately this problem often happens for an industrial product where multi-material
objects are quite ordinary. In general the data amount obtained by an X-ray CT scanning tends to be so large as several gigabytes. For promoting the use of X-ray CT scanning in the manufacturing field, solving these problems is required.

The goal of this paper is to propose a method which can realize an accurate expression of the surface of an object with less amount of memory and deal with a multi-material objects.

1.2 CT scanning

Fig. 1 shows the most common setting of the industrial X-ray CT scanning. The object to be scanned is located on the top of a rotation table and rotates around the rotary axis while scanning. The X-ray emanates from the X-ray source and a part of its radiation is absorbed while penetrating the object and reach the detector panel on the other side of the object, and then the intensity of the attenuated X-ray is measured at each pixel of the detector panel. The amount of attenuation depends on the material of the object and the length of the intersection of the X-ray and the object. The values of attenuation ratio can be expressed as a grey scale image referred to as a projection image, as you can see in the right of Fig. 1.

Fig. 2 shows a general flow of X-ray CT scanned data processing. The first data acquired by an X-ray CT scanning is a set of projection images obtained in many different projection directions and called a sinogram.

The sonogram is converted to a tomogram which is called a CT volume to emphasise that we are treating 3D data in this paper, by a computation known as a CT reconstruction. CT volume is intuitively a 3D image consisting of a piled up 2D images of the slice of the object. The integral transform from a CT volume to a sinogram is called Radon transform.

Since a CT volume is not the best data structure for many applications as geometry processing and measurement, usually the boundary of the object is extracted from the CT volume as a surface mesh (e.g. STL data) by a surface extraction algorithms as the marching cubes [1] and dual contouring [2]. Surface mesh is easier than a CT volume to be processed and modified, so it is used in variety areas in manufacturing. The image in the right of Fig. 2 is an example of a surface mesh extracted from a CT volume and you can see many polygons approximating the surface of the object in its partially enlarged image.
CT reconstruction has been investigated mainly in the medical imagens field in its early days and many excellent algorithms with different advantages have been proposed. The algorithms solving the inverse problem of Radon transform are known as analytical algorithms. Filtered back projection [3], FDK [4] and algorithms using Fourier transformation [5] are analytical algorithms. Among them, FDK can fast compute a CT volume and the quality of the resulting CT volume is relatively high when the effects of possible disturbance as scattered radiation is small.

Other algorithms basically obtain a CT volume by solving a linear system derived from a discretized Radon transform with an iterative optimization. This is why this type of algorithms are called as iterative algorithms. OS-CONVEX [6] and OSEM [7] are iterative algorithms, just to name a few. The largest advantage of iterative algorithms is that the linear system can be developed taking into account the effects of the causes of artifacts arising in a CT volume. This consideration enables to generate a higher quality CT volume by reducing the artifacts took into account. The drawbacks of iterative algorithms is its high computational cost and long computational time which may be more than ten times of an analytical algorithm in some cases.

2. CT Reconstruction on an Unstructured Grid

In this section, firstly we are describing the problems of conventional CT reconstruction methods. Introduction of a previous algorithm overcoming these problems comes next. It reconstructs a CT volume on a tetrahedral mesh instead of a grid. And then, we are proposing a CT reconstruction algorithm which realises precise surface expression and quality FEM mesh generation with less amount of memory even for a multi-material object.

2.1 Problems of the CT Reconstruction on a Uniform Grid

In general, a CT volume is a 3D image composed of a set of regularly arranged uniform-sized voxels. A surface mesh extracted from such a conventional CT volume is usually suffered from artifacts caused by the regular pattern of the grid. Such artifacts can be reduced for some extent by using a CT volume with a finer grid, see Fig. 3. But it causes a severe increase
of data amount of the CT volume and ends up to a surface mesh with much more triangles, while the essential problem on accuracy is left unsolved.

Nowadays it is not surprising that just a single CT volume can be several gigabyte of memory. A data with large amount of memory like this is difficult to even render on a display or process for a successive application. The stream of increasing the resolution of CT volume along with the advance of technology will never stop, so another data structure for a CT volume which realizes high accuracy expression of an object with small amount of memory is needed. A series of data conversions including CT reconstruction and surface mesh extraction also decreases the accuracy of the surface mesh. The number of data conversion should be as small as possible.

Fig. 3. Surface meshes extracted from CT volumes with different resolutions. From left to right, the approximate numbers of triangles are 80,000, 330,000, and 1,350,000 and the size of voxels of CT volumes are 0.4mm, 0.2mm, and 0.1mm.

2.2 The Sinogram Polygonizer

To solve the above-mentioned problems, Yamanaka et al. [8] proposed and algorithm named the sinogram polygonizer which succeeds to generate a highly isotropic tetrahedral mesh (unstructured grid) whose faces are confirm to the surface of the scanned object and requires a smaller amount of memory, by reconstructing a CT volume on the tetrahedral mesh instead of a conventional uniform grid.

Fig. 4 and Table 1 are the comparison of the sinogram polygonizer with the conventional CT reconstruction on a uniform grid. Fig. 4 shows the effect of CT reconstruction on an unstructured grid, a tetrahedral mesh. Each tetrahedron works as a voxel of a conventional CT volume. The number of triangles is less than the one required for a conventional CT reconstruction with almost the same accuracy.

The sinogram polygonizer works well when the object is made of a single material. For a multi-material object, determining the boundary between two different materials is difficult due to the blur of CT value and fails to determine the boundary correctly. Fig. 5 is a result surface of the sinogram polygonizer applied for an object composed of two blocks with different attenuation coefficient values. The smaller block with a lower attenuation coefficient value allows some extent of penetration of another block due to the blur of CT value. It results to the appearance of an undesired bump on the top of the bottom object. So for the sinogram polygonizer, handling multi-material object has been left as a problem to be solved.
Fig. 4. Resulting surface obtained by conventional method for a CT volume with size of $64^3$ (left), $512^3$ (middle), and the sinogram polygonizer (right) for a single material case.

<table>
<thead>
<tr>
<th></th>
<th>Conventional ($64^3$)</th>
<th>Conventional ($512^3$)</th>
<th>The sinogram polygonizer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean square error</td>
<td>0.101</td>
<td>0.0089</td>
<td>0.0091</td>
</tr>
<tr>
<td>Maximum error</td>
<td>0.387</td>
<td>0.051</td>
<td>0.076</td>
</tr>
<tr>
<td>Number of triangles</td>
<td>10,329</td>
<td>700,037</td>
<td>12,964</td>
</tr>
</tbody>
</table>

Fig. 5. Resulting surface for the bottom cuboid generated by the sinogram polygonizer (left) for a multi-material object (right).

2.3 Extension for Multi-material Objects

The following is our proposal algorithm to reconstruct a CT volume of a multi-material object on an unstructured mesh. We developed it based on the original sinogram polygonizer.

The input of this algorithm is a sinogram, and the output is a CT volume on a tetrahedral mesh some of whose faces are conforming to the boundaries of the object. By taking a sinogram which contains less artifacts than a CT volume as input, we can compute accurate derivatives of CT value which are used to determine the surface of an object and avoid artifacts arising on a CT volume.

This algorithm basically iterates a combination of the computation of CT value and update of the grid. Fig. 7 shows how this algorithm works with a 2D example accompanied by intermediate 2D CT volumes. Fig. 8 is an illustrations of the concrete flow of the algorithm.

Algorithm
1. Generate a point cloud
2. Generate the Delaunay tetrahedrization for the point cloud
3. Compute the CT value at the centroids of tetrahedra
4. Extract boundary faces
5. Find surface points and compute their normals
6. Move boundary vertices to fit the boundary faces to the surface of the object
7. Move non-boundary vertices
8. If the amount of vertex movement is sufficiently small, then exit. Otherwise go back to step 2.

Fig. 6. CT reconstruction on an unstructured grid for a 2D example.

Fig. 7. The flow of the proposed algorithm.

In step 1, the generated points are randomly located covering the CT-reconstruction area (a vertical cylinder). Their positions will be optimized in the succeeding steps. The number of points should be sufficiently large so that the edge length of the tetrahedron which will be generated in step 2 is smaller than the finest feature of the scanned object.

The Delaunay tetrahedrization used in step 2 triangulates the convex hull of the given points with tetrahedra whose vertices are the given points. The circumsphere of each tetrahedron does not contain a point other than the vertices of the corresponding tetrahedron. It gives a highly isotropic tetrahedral mesh and is suitable for mesh generation aimed at FEM. We generated a Delaunay tetrahedral mesh for the points generated in the step 1 by TetGen [9].

In step 3, the CT value is computed with FDK algorithm. Although a quality CT value is a key of a conforming surface mesh generation, this analytical method is adopted instead of an
iteration CT reconstruction algorithm since an iterative CT reconstruction algorithm imposes too much computational cost to our iterative CT reconstruction, nevertheless the computation speed has been increased in these days thanks to the development of high-performance CPUs and GPUs. FDK can compute CT values in much shorter time than an iterative algorithm and give a relatively quality CT volume. FDK computes the CT value at a point \( \mathbf{p} \) with the following equation:

\[
    f(\mathbf{p}) = \frac{1}{2} \int_{0}^{2\pi} \alpha(\theta, \mathbf{p})^2 S(\theta, \mathbf{p}) \, d\theta. \tag{1}
\]

See Fig. 1 for the setting. The value \( \alpha(\theta, \mathbf{p}) = \frac{d}{d + p_z} \) where \( d \) is the distance from the X-ray source to the rotation axis and \( p_z \) is the \( z \) coordinate of \( \mathbf{p} \) after a rotation by an angle of \( \theta \).

The value \( S(\theta, \mathbf{p}) \) is the value of the projection image at the pixel corresponding to \( \mathbf{p} \) rotated by an angle of \( \theta \). An important thing is that CT value can be computed with this equation at any point in the reconstruction area. This fact enables CT reconstructions on an unstructured grid.

We define the surface of the object as the maximizers of the norm of the gradient of CT value:

\[
    \left\{ \mathbf{x} \mid \frac{\mathbf{g}}{\|\mathbf{g}\|} \cdot \nabla\|\mathbf{g}\| = \frac{\mathbf{g}^T H \mathbf{g}}{\|\mathbf{g}\|^2} = 0 \right\} \tag{2}
\]

where \( \mathbf{g} \) is the gradient and \( H \) is the Hessian matrix of CT value. In our observation this definition is less affected by the blur of CT value than the isosurface of CT volume as the original sinogram polygonizer does. Step 4 extracts the boundary faces each of that is a triangle shared by a tetrahedron expressing one material with a negative value of \( \frac{\mathbf{g}}{\|\mathbf{g}\|} \cdot \nabla\|\mathbf{g}\| \) and a tetrahedron expressing another material with a positive value of \( \frac{\mathbf{g}}{\|\mathbf{g}\|} \cdot \nabla\|\mathbf{g}\| \).

In step 5, a point on the surface is computed for each boundary face and called a surface point. A surface point can be found by the bisection method on the segment between the centroids of the tetrahedra sharing the boundary face. In this step, the normal \( \mathbf{n} \) of the surface at each surface point is also computed for the QEM method performed in the next step. The normal \( \mathbf{n} \) can be computed with the following equations derived by analytically differentiating equation (1):

\[
    \mathbf{n} = \nabla f(\mathbf{p}) / \|\nabla f(\mathbf{p})\|, \tag{3}
\]

\[
    \nabla f(\mathbf{p}) \approx \frac{1}{2} \int_{0}^{2\pi} \alpha(\theta, \mathbf{p})^2 R_{-\theta} \left( \nabla_{(x,y)} S(\theta, \mathbf{p}) \right) \, d\theta. \tag{4}
\]

where \( R_{-\theta} \) means the rotation matrix by the angle of \( -\theta \) and \( \nabla_{(x,y)} S(\theta, \mathbf{p}) \) is the gradient of the projection image.

In step 6, the boundary vertices that are the vertices of the boundary faces are moved following the QEM algorithm as in [10] and [11]. QEM algorithm minimises the sum of the distances from specified points to a target surface and in this case it makes the boundary vertices fit to the surface of the object.

After the fitting of the boundary faces to the surface of the object, other vertices of the tetrahedral mesh are relocated by the optimal Delaunay triangulation [12] for improving the level of isotropy in step 7.

The main difference between the proposal and the original sinogram polygonizer is the definition of the boundary of the object. The sinogram polygonizer uses an isosurface as the definition of the boundary of the object. As shown in Fig. 5, the CT value of the object with the lower attenuation coefficient becomes higher around the boundary of the two different materials. It makes the isosurface deformed for a multi-material object.
3. Results

In this section we are showing some experimental results obtained by the proposed algorithm. Fig. 8 is the resulting surface meshes of the cuboids for the same object as the one in Fig. 6. The input sinogram has been obtained by a simulation for an STL data. The square-shaped bump observed on the top of the surface mesh of the bottom cuboid obtained by the sinogram polygonizer was reduced although some tetrahedra appeared on the surfaces.

We also applied the proposal to a real sinogram of a product made of lubber and aluminum. Fig. 9 shows the cross-sections of the tetrahedral meshes of the whole object, lubber part, and aluminum part. The resolution of the sinogram is $500 \times 500$ and the number of projections is 500. Several fragments appeared in the lubber mesh, but even the part where the lubber, aluminium, and air met were well reconstructed. The computational time was 5,526 seconds.

4. Conclusion and Future Work

In this paper we proposed a CT reconstruction algorithm which can handle a multi-material object with less memory by computing a CT volume on an unstructured grid. Investigating for a better reconstruction of a smooth surface and sharp feature is one of the issues to be solved. Using adaptive grid is considered helpful for more reduction of the number of tetrahedra.
References