The application of X-ray beam hardening correction based on polynomial fitting in local scan

Hao CHEN¹,², Yunbin CHEN¹,², Shoutao LI¹,²

¹Institute of Applied Electronics, Chinese Academy of Engineering Physics; Mianyang, China; Phone:+86 816 2493340; email: ultrablue@263.net, cloudbin@163.com, ssk_2000@163.com
²national X ray digital imaging center; Mianyang, China

Abstract
Beam Hardening effect would be induced due to broad energy spectrum of X-ray. Beam Hardening would lead to artifacts in CT reconstructed images, such as cupping artifact, streak artifact and metal artifact, so that the quality of CT image degrades. One efficient beam hardening correction method is building a mathematical law between polychromatic projection and intersection length, which can be calculated by forward projection in reconstructed images domain. However, to local scan, the intersection length obtained by forward projection is no longer equal to the real distance which X-ray passes through the object. The data used to mathematical fit is not valid. A cylindrical phantom was used in this paper. According to geometrical character of cylinder, the true distance X-ray passing through the object could be calculated analytically. Experimental result showed the beam hardening artifacts were suppressed by our approach.

Keywords: beam hardening correction, local scan, polynomial fit, forward projection

1. Introduction
Computed Tomography (CT) is one of the common equipments in the medicine and industry field. Reconstruction images can be obtained by specified CT reconstruction algorithm. It plays an important role in defect or disease detecting, diagnosing or assessment.

To traditional filtered backprojection reconstruction algorithm, it is assumed that X-ray tube output exhibits a single energy spectrum. However, the assumption deviates from actual physical property that the energy spectrum of X-ray is broad. When X-ray interacts with the materials, the photons with different energies show different attenuation effects, so that the spectrum of X-ray changes. Spectrum peak of X-ray shifts toward high energy band. The proportion of the photons with higher energy rises. This phenomenon is known as the beam hardening.

Beam hardening is one of the physical reasons that induce artifacts in reconstruction images, such as cup artifact, metal artifact. Linear correction is the most frequently method used for beam hardening correction, which establishes a relationship between polychromatic projection and the distance passing through object by polynomial fitting. Then monochromatic projection is acquired. The key point of this method is how to acquire the distance passing through object. It’s a feasible way that is calculating the distance by forward projection in reconstruction domain. One limitation about this method in the actual engineering is that the field of view (FOV) of CT must cover the object completely. In other words, this method is not valid in local scan. For the cone beam CT with small FOV, maybe local scan is only way. An illustration of local scan is shown in Fig 1. As local scan only reconstruct a part of the object, the distance calculated by forward projection in reconstruction domain is no longer equal to the real distance which X-ray passes through the object. The data
used for fitting is not valid.

In some case, we need a phantom to build a mathematical model indicating the projection variation with the intersection distance X-ray passing through the object. Let us define $\Phi$ as the diameter of FOV, $D$ is the diameter of circumcircle around phantom, $L$ is the intersection distance. An accurate attenuation model can be established by forward projection if $D \leq \phi$. But this model can only be used under the condition of $L \leq D$. In the case of $L > D$, one phantom with larger size is required to establish attenuation model. If $D > \phi$, local scan occurs. Just as the above discussion, the distance X-ray passing through the object calculated by forward projection is not valid any more.

Considering the case of local scan in dental CT system, this paper proposed a method to resolve the beam hardening correction problem. The method was also available for industry CT. A polymethyl methacrylate (PMMA) cylindrical phantom with 145mm diameter was designed. According to geometrical character of cylinder, the true distance X-ray passing through the object could be calculated analytically and used for polynomial fitting.

2. X-ray beam hardening correction based on forward projection

The nonlinear relation between intersection length and polychromatic projection can be written as:

$$L = \sum_{i=0}^{N} a_i P^i(L)$$

Where $L$ is the intersection length; $a_i$ is the polynomial coefficient which needs to be determined; $P(L)$ is polychromatic projection and $N$ is the order of the polynomial.

If we use $M(L)$ as monochromatic projection, it follows:

$$M(L) = \mu(E_0) \cdot L = \mu(E_0) \cdot \sum_{i=0}^{N} a_i P^i(L)$$

Figure 1. Local scan schema
Where $\mu(E_0)$ is the attenuation coefficient at $E_0$. $\mu(E_0)$ can be calculated as the first derivative of $P(L)$ at $L=0$.

$$
\mu(E_0) = P'(0) = 1/L'(0) = 1/a_i 
$$

(3)

A set of data points labeled by $(p(L), L)$ should be obtained for polynomial fitting. Polychromatic projection is just the real data measured by detector, and intersection length can be calculated by ray-driven method. It is the sum over the pixels along a specified path in a reconstruction image which has been converted to binary one by the threshold segmentation.

2. X-ray beam hardening correction in local scan

As mentioned above, the key point of polynomial fitting is how to acquire the distance X-ray passing through object. If the FOV cannot cover the whole object completely, intersection length computed by forward projection in reconstruction domain is not valid. The relationship between the path length and polychromatic projection would no longer reflect the real attenuation law. As shown in fig.2, circle $O_0$ with $R_0$ radius is the circumcircle around the object. In addition, circle $O_1$ with $R_1$ radius is the FOV. Reconstruction image can only recover the region indicated by circle $O_1$. Supposing a ray passing through object, the length which is calculated by forward projection in a reconstruction image is $|BC|$. It is easy to find that $|BC|$ deviates from the real intersection length indicated by $|AD|$.

![Figure 2. Intersection length computed by forward projection deviates from the real distance X ray passing through the object](image)

Considering the application background of local scan, a special phantom is designed to establish the nonlinear relationship between polychromatic projection and the intersection length. As shown in left one of fig.3, the phantom composed of PMMA is cylindrical. The diameter and height of the phantom is 145mm and 150mm respectively. In the center of phantom, a cylindrical cavity which has the same center as phantom is designed. Here the diameter and height of the cavity is 22mm and 10mm respectively. If the diameter of FOV satisfies: $22mm < \Phi < 145mm$, the true intersection length can be calculated analytically according to the geometrical character of the cylinder.
Here, we just discuss the cone beam CT, considering which provides volume images. As the diameter of cavity is only 22mm, all of the reconstruction images of cavity can be obtained. Indicated as \((x_0, y_0)\), the coordinate of cavity’s center can be also got. Because the cavity has the same center as PMMA phantom, so the coordinate of cavity’s center can be approximated as the coordinate of the whole phantom’s center. The role of cavity here is just marking the center of the cylinder. Solid PMMA phantom is used to establish an attenuation model. As shown in right one of fig.3, projecting the phantom onto horizontal plane, \(R_0\) , \(R_2\) indicates the radius of phantom and cavity respectively, and dashed line indicates the range of FOV. Given a ray \(\overline{SP}\) , it is defined by the source spot \(S\) and individual detector bin \(P\). As the geometry of CT is known, given a ray \(\overline{SP}\) , its equation can be determined as \(Ax + By + C = 0\). So the distance from the center of phantom to the line can be written as the following:

\[
|O_0M| = \frac{Ax_0 + By_0 + C}{\sqrt{A^2 + B^2}}
\]

(4)

It is easy to get the intersection length the ray passing through the phantom:

\[
|AD| = 2\sqrt{R_0^2 - |O_0M|^2}
\]

(5)

Once get the intersection length given ray passing through the phantom, a set of sample data can be determined, and then attenuation model illustrating the relationship between polychromatic projection and intersection length can be established by polynomial fitting.

3. Experiment

Implementing steps of method proposed in this paper were summarized based on actual experiment.

Place the phantom in the center of FOV. Then adjust the altitude of phantom in order that the
cavity and a part of solid PMMA phantom at least could be contained in the FOV. A spirit level was used to adjust the phantom as level as possible for keeping the center’s coordinates \((X, Y)\) at variant heights the same as cavity’s center.

### Table 1 imaging parameter

<table>
<thead>
<tr>
<th>FOV/mm</th>
<th>PixelSize/mm</th>
<th>voltage/kV</th>
<th>current/mA</th>
<th>matrix</th>
<th>VoxelSize/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.254</td>
<td>85</td>
<td>10</td>
<td>560×560</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Select suitable imaging parameters and get reconstruction images. Related parameters were shown in table 1.

As shown in fig.4, one slice image containing cavity was used. The column-row coordinate of the cavity’s center was \((293.5, 315.5)\) in chosen image. That is:

\[
x_0 = \left(293.5 - \frac{560}{2}\right) \times 0.16\text{mm} = 2.16\text{mm}
\]

\[
y_0 = \left(315.5 - \frac{560}{2}\right) \times 0.16\text{mm} = 5.68\text{mm}
\]

It should be noted that cavity’s diameter was approximately 139 pixels measured in reconstruction image. In other words, the measured diameter of cavity was 22.24mm, getting close to the manufactured size.

As the phantom and cavity had the same center, the coordinate of cavity’s center could be approximated as coordinate of phantom’s center. Given a ray \(\overrightarrow{SP}\), corresponding equation \(Ax + By + C = 0\) could be calculated. Contributed to the known of CT’s geometry, supposing the source spot coordinate \((x_s, y_s)\) and one individual detector bin coordinate \((x_p, y_p)\), we could get:

\[
A = \frac{y_p - y_s}{x_p - x_s}, \quad B = -1
\]

\[
C = y_s - \left(\frac{y_p - y_s}{x_p - x_s}\right) x_s
\]

The distance from the center of phantom to the line could be computed by (4), and then the intersection length \(|AD|\) that the given ray \(\overrightarrow{SP}\) passes through object could be computed by (5).
Once the data sets containing polychromatic projections and intersection lengths were got, the attenuation model could be established by polynomial fitting:

\[
L = 2.254760 P^3 - 7.965450 P^2 + 37.746468 P
\]  

(7)

It should be noted that the range of intersection length was \( L \in [L_{\text{min}}, L_{\text{max}}] \) in local scan, and the data information contained in the range of \( (0, L_{\text{min}}) \) was lost. Since \( L(P = 0) = 0 \), it was beneficial to fit the constant coefficient correctly in polynomial fitting if adding a data point \((0,0)\) to the original data sets. The left one of fig.6 showed the original reconstruction image. The right one of fig.6 showed the horizontal profile of the corresponding image along central line. It was easy to find that cupping artifacts were obvious. The reconstruction image with beam hardening correction proposed in this paper was shown in fig.7. Compared with fig.6, cupping artifacts were suppressed effectively.

The attenuation model established in equation (7) could be applied to dental CT. The original reconstruction image and the beam hardening corrected image were shown in fig.8. Comparing with the two images, cupping artifacts and streak artifacts due to beam hardening were suppressed.

4. Discussion

A beam hardening correction method for local scan was proposed in this paper. This method was also effective under the condition of data truncation, such as dental CT equipped with flat panel detector. According to the experiment result, the proposed method could reduce beam hardening artifact. It illustrated that the attenuation model between polychromatic projection and intersection length was reasonable and reliable. However, there were some limitations about this method:

1) During the scanning process, phantom should be positioned as level as possible, so as to keep the cavity’s center the same as the whole phantom.

2) Intersection length calculation was on the base of the geometry of circle. A phantom with high precision was preferred.

3) The accuracy of voxel size was also important. Furthermore the assembling precision of CT should be met some requirements.
However, polynomial fitting which was an optimized method based on least square had anti-noise ability itself. The influence on the final reconstruction result by errors listed above should be studied further.

The correction method proposed here could also be extended to industry CT. There were some changes that the material and size of phantom should be redesigned according to the FOV of CT, the material of object to be scanned.

Figure 6. Original reconstruction image

Figure 7. Reconstruction image with beam hardening correction
Figure 8. Left: Original reconstruction image. Right: Reconstruction image with beam hardening correction
References


