A design strategy for highly directional piezoelectric transducers in Lamb waves inspections

Luca DE MARCHI\textsuperscript{1}, Nicola TESTONI\textsuperscript{1}, Alessandro MARZANI\textsuperscript{2}

\textsuperscript{1} Electrical, Electronic and Information Eng. Dept., University of Bologna, Viale del Risorgimento 2, 40136, Bologna, Italy \texttt{l.demarchi@unibo.it}, \texttt{nicola.testoni@unibo.it}
\textsuperscript{2} Civil, Chemical, Environmental and Material Eng. Dept., University of Bologna, Viale del Risorgimento 2, 40136, Bologna, Italy, \texttt{alessandro.marzani@unibo.it}

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Abstract
Drastic hardware simplification and cost reduction of Guided Waves (GWs) based systems can be achieved by using shaped transducers that present inherent directional capabilities when generating and sensing elastic waves. Directional transducers for GW generation and sensing are achieved by patterning the piezoelectric material lay-out and the electrodes. The peculiar electrodes' shape produces a spatial filtering effect which is frequency-dependent, so that a direct relationship can be established between the direction of propagation (wavenumber) and the spectral content of the transmitted/received signal. This kind of transducer has been named Frequency Steerable Acoustic Transducers (FSATs). In this work, a transducer's shape design strategy is presented which is able to enhance the accuracy of the desired Directivity function approximation. The proposed design strategy is based on Dithering techniques. The effectiveness of the novel transducer design methodology is shown through a numerical validation with application to defect detection in an aluminum plate.

1 INTRODUCTION
Guided Waves (GW) inspection is a popular methodology employed by many Structural Health Monitoring (SHM) systems. GW inspection in plate-like components is typically achieved through phased arrays featuring a large number of piezoelectric transducers. The weight penalty, the complex circuitry, and maintenance concerns associated with wiring a large number of transducers have to be addressed for the widespread field deployment of SHM systems.

Drastic hardware simplification and cost reduction of Guided wave (GWs) based systems can be achieved by using shaped transducers that present inherent directional capabilities when generating and sensing elastic waves. This kind of devices can be realized by patterning the shape of the piezo- transducer. A noticeable example is the so called Frequency Steerable Acoustic Transducer (FSAT) proposed by M. Ruzzene and coauthors [1,2,3]. However, in the practical realizations of these devices presented in literature, just a relatively rude approximation of the desired directivity has been achieved, resulting in a wave generation/detection even in directions other than the desired one.

In this work, to overcome such limitations, a transducer's shape design strategy is presented which is able to enhance the accuracy of the desired Directivity function
approximation. The proposed design strategy is based on Dithering techniques. Dither is an intentionally applied form of noise used to randomize quantization error. In our approach, Dither is used to reshape the spectral energy of quantization error in the piezoelectric load distribution, so that signal and noise bands are separated completely. In practice, such effect is obtained by imposing a local variation in the density of the piezoelectric material over the transducer area. It will be shown that undesired side lobes in the transducer Directivity function are considerably reduced. Such effect has a positive impact on the radiation pattern: the reduction of secondary side lobes will enhance the angular resolution performance of Lamb inspection methods.

The effectiveness of the novel transducer technology is shown through a numerical validation with application to defect detection in an aluminum plate.

2 THE FREQUENCY STEERING CONCEPT

Following the theory presented in [1], the voltage measured by a piezoelectric transducer and due to an incident guided wave mode m can be written as:

$$V_m(\omega, \theta) = j U_m(\omega, \theta) H(\theta) k_m(\omega, \theta) D(k_{1,m}(\omega, \theta), k_{2,m}(\omega, \theta))$$  \hspace{1cm} (1)

where $\theta$ is the direction of arrival of the incident wave mode m, $U_m$ represents the amplitude of the wave mode at the considered angular frequency $\omega$, H is a function of the piezo-structure system, $k_m$ is the angular wavenumber for the considered wavemode (while $k_{1,m}$ and $k_{2,m}$ are its components along the orthogonal directions 1 and 2), and D is a function (namely, the Directivity function) of the wavenumber which can be computed as:

$$D(k_1, k_2) = \int_{\Omega} e^{j(k_1 x_1 + k_2 x_2)} f(x_1, x_2) \, dx_1 \, dx_2$$  \hspace{1cm} (2)

The Directivity function D is the 2D-Fourier Transform of the weight function (or load distribution function) $f(x_1, x_2)$, which represents the effect of the piezoelectric patches shapes and polarizations.

Being D and f a Fourier pair, it has been shown in Ref. [3] that it is convenient to design the desired directivity function in D and then apply the Fourier transformation to obtain the piezo-transducer geometry which actually produces such directional behaviour. In this paper, the directivity function is shaped as a 180°-spiral, as schematically depicted in Fig. 1. It has been shown in Ref. [5] that this solution allows to overcome the limitation of the first generation FSATs which excite and sense elastic waves contemporarily in one direction and in the opposite direction (180° ambiguity). It is worth noting that just a limited approximation of a regular spiral can be achieved, because of the limited spatial extension of the transducer.
More specifically, the Directivity function shown in Fig. 1 is given by the following formula:

\[ D(k_1, k_2) = \sum_{n=1}^{N} \text{Ker}(k_1 - \gamma_{1,n}, k_2 - \gamma_{2,n}) \]  

(3)

where \( N \) is the number of kernel functions \( \text{Ker}(k_1 - \gamma_{1,n}, k_2 - \gamma_{2,n}) \) replica used to synthesize the Directivity function. In the proposed approach, Gaussian kernels are used:

\[ \text{Ker}(a1, a2) = 2\pi b \left( \frac{b}{2} \right)^2 \exp \left( - \left( \frac{b}{2} \right)^2 (a1^2 + a2^2) \right) \]  

(4)

where \( b \) is an arbitrary parameter, while \( \gamma_{1,n} \) and \( \gamma_{2,n} \) are given by:

\[
\begin{align*}
\gamma_{1,n} &= k_{1,m}(\omega_n, \theta_n) \\
\gamma_{2,n} &= k_{2,m}(\omega_n, \theta_n)
\end{align*}
\]

(5)

with \( \omega_n \) and \( \theta_n \) selected so that:

\[
\begin{align*}
\theta_n &= \frac{2\pi}{N} n \\
\omega_n &= \frac{\omega_{\text{max}} - \omega_{\text{min}}}{N} n + \omega_{\text{min}}
\end{align*}
\]

(6)
The radiation pattern which corresponds to the directivity function of eq. (3) is a function of the excited pulse spectrum. In Fig. 2, the radiation patterns which correspond to sinusoidal bursts with central frequencies equal to 115 kHz, 205 kHz, 320 kHz and 380 kHz are represented.

![Radiation patterns](image)

**Fig. 2.** Radiation patterns corresponding to the Directivity function depicted in Fig. 1 for sinusoidal bursts with central frequencies equal to 115 kHz (top left), 205 kHz (top right), 320 kHz (bottom left) and 380 kHz (bottom right).

### 2 PRACTICAL REALIZATION OF FSATS

The load distribution $f$ which corresponds to the Fourier Transform of eq. (3) is a complex function which cannot be realized in practice. A complex thresholding function is used to determine the shapes of the actual piezoelectric transducer electrodes. Such procedure is schematically depicted in Fig. 3 where each point of the spatial domain of the load distribution is associated to one of the 4 different electrodes E1-E4, more specifically:

- when $\text{Re}(f(x_1, x_2)) > |\text{Im}(f(x_1, x_2))| + \delta$ the point is attributed to E1
when \( \text{Re}(f(x_1, x_2)) < -|\text{Im}(f(x_1, x_2))| - \delta \) the point is attributed to E2
when \( \text{Im}(f(x_1, x_2)) > |\text{Re}(f(x_1, x_2))| + \delta \) the point is attributed to E3
when \( \text{Im}(f(x_1, x_2)) < -|\text{Re}(f(x_1, x_2))| - \delta \) the point is attributed to E4

where \( \delta \) is an arbitrary positive value.

Fig. 3. Quantization strategy for the complex valued load distribution function \( f \) (a); resulting second generation of FSAT electrodes shapes (b).
It is worth noting the effect of the quantization process on the Directivity function (see Fig. 4) as well as on the radiation pattern for sinusoidal bursts in input (Fig. 5). In particular, the quantization produces some undesired sidelobes which will result in secondary peaks in the FSAT frequency response. Such sidelobes can be reduced by applying more sophisticated thresholding procedures in order to enhance the angular resolution performance of FSAT-based inspection methods. In particular, Dithering techniques can be used to enhance the accuracy of the Directivity function approximation. Dither is an intentionally applied form of noise used to randomize quantization error (see Fig. 6). In our approach, Dither is used to reshape the spectral energy of load distribution quantization error so that signal and noise bands are separated completely. Such effect is very effective in the considered application, as
can be seen by looking at the directivity function achieved with the dithering procedure in Fig. 7. Indeed, such directivity function is very similar to the one achieved with the ideal continuously modulated load distribution depicted in Fig 1. Consequently, the radiation pattern achieved with dithering is also very close to the ideal one (see Fig. 8), having narrow beams and weak sidelobes in the whole considered angular range [0-180°].

**Fig. 6.** Dither-based load quantization strategy.

**Fig. 7.** Directivity function after dither-based quantization.
5 CONCLUSIONS

In this paper a novel strategy to realize Frequency Steerable Transducers was presented. Thanks to this strategy, a load distribution closer to the ideal one and consequently enhanced sensor directivity can be achieved, attenuating the detrimental effect of quantization. In particular, the presented FSAT design procedure is based on the local variation of piezoelectric material density and on the implementation of a complex piezoelectric load distribution which is capable to steer the acoustic beam over the whole (360°) angular range.

More specifically, dithering techniques have been used to enhance the accuracy of the Directivity function approximation w.r.t. simple thresholding procedures. The simulated results show that the undesired side lobes in the spiral shaped directivity function can be considerably reduced. Such effect has a positive impact on the radiation pattern: the reduction of secondary sidelobes, and the decrease of the width of the primary lobe will enhance the angular resolution performance of FSAT-based inspection methods.
REFERENCES


