Damage Detection of Structures with Detrended Fluctuation and Detrended Cross-Correlation Analyses

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Abstract
Recently, fractal analysis has shown its potential for damage detection and assessment in fields such as biomedical and mechanical engineering. Because of its practicability in interpreting irregular, complex, and disordered phenomena, a structural health monitoring (SHM) system based on detrended fluctuation analysis (DFA) and detrended cross-correlation analysis (DCCA) is proposed. First, damage conditions can be swiftly detected by evaluating ambient vibration signals measured from a structure through DFA. Damage locations can then be determined by analyzing the cross correlation of signals of different floors by applying DCCA. A damage index is also proposed based on multi-scale DCCA curves to improve the damage location accuracy. To verify the performance of the proposed SHM system, a four-story numerical model was used to simulate various damage conditions with different noise levels. Furthermore, an experimental verification was conducted on a seven-story benchmark structure to assess the potential damage. The results revealed that the DFA method could detect the damage conditions satisfactorily, and damage locations can be identified through the DCCA method with an accuracy of 75%. Moreover, damage locations can be correctly assessed by the damage index method with an improved accuracy of 87.5%. The proposed SHM system has promising application in practical implementations.

1 INTRODUCTION
Preventing considerable loss of human life and property engendered by catastrophic events is a major growing concern; structural health monitoring (SHM) is thus aimed at providing reliable information concerning possible damage to structures, after which actions such as maintenance or retrofitting can be executed. SHM involves two main approaches for identifying the level of damage, namely global and local health monitoring methods.

The global health monitoring method is used to diagnose an entire structure, and in this procedure, the presence of damage is indicated by a change in the global dynamic properties of a structure such as mass, damping, or stiffness. Several examples of well-known damage detection techniques are the natural frequency-based method [1], mode shape-based method [2], and mode shape curvature (MSC)-based method [3, 4]. The local health monitoring method is used to measure the extent of damage following the application of the global monitoring method. After a potential damage location or critical area is predetermined, local assessment techniques are employed to track the damage size. Compared with global monitoring approaches, local monitoring techniques are more difficult to execute because they are time consuming, costly, and require high levels of detail. Therefore, combining both
methods is essential for achieving an exemplary SHM system.

In 1983, Mandelbrot studied the shapes of nature objects and discovered fractal phenomena. The shape of rocks or other shapes of nature show similar structures with the initial stage when zoomed at different scales [5]. Fractal analysis was first applied to time series by following the work of Hurst to the hydrological approach [6, 7]. When a time series is viewed at different scale levels, the fractal scaling behavior can be generated.

On the basis of the fractal concept, Peng et al. presented a method called detrended fluctuation analysis (DFA) for scaling the long-term correlation of nonstationary signals [8]. Spurious detections induced by trends of different orders were eliminated [9]. Therefore, DFA has been demonstrated to be a robust method for determining the scaling behavior of noisy data. It has been successfully applied to identify various features of time series such as heat rate variability [8,10], human gait signals [11], economic trend [12], weather records [13], gearbox and bearing fault diagnosis [14,15], and seismic analysis [16].

Podobnik and Stanley proposed a new method called detrended cross-correlation analysis (DCCA), which is a generalization of DFA based on detrended covariance. The cross correlation between two distinct signals, namely air humidity and temperature, and sleep disorder breathing and cardiovascular disease were both analyzed to obtain the power-law cross correlations between two time series [17]. Similarly, DCCA has been extensively used in physics for analyzing the basic relation between the correlation properties of an original signal and its magnitude fluctuations [18].

Although studies on DFA and DCCA have been widely conducted in numerous fields, little information is available on the application of these approaches in SHM. Consequently, the main objective of this study is to exploit the advantages of DFA and DCCA for damage condition and location detection. The remainder of this paper is organized as follows. First, the basic concept and mathematical expressions of DFA, DCCA, and the proposed damage index are introduced in Section 2. In Section 3, numerical evaluation is conducted on a four-story steel structure, and the effect of input noise is also examined. A practical experimental evaluation involving a seven-story steel benchmark structure located at the National Center for Research on Earthquake Engineering (NCREE) is described in section 4. The results of a series of ambient vibration experiment to verify the performance of the proposed system are presented in Section 5. Finally, a summary is provided, and conclusions are drawn in the final section.

2 THE PROPOSED SHM SYSTEM

DFA is applied to detect the damage condition of a structure by measuring the structural response from a single sensor. Furthermore, a DCCA-based algorithm is developed for locating the damaged floor of the structure by analyzing the signals from different floors. A damage index is also proposed to improve the accuracy of the damage location procedure. A detailed derivation of each component of the proposed SHM system is expressed as follows.

2.1 Detrended Fluctuation Analysis (DFA)

As an extension of the classical fluctuation analysis (FA), which corresponds to the Hurst rescaled range analysis, this random-walk-based method facilitates determining correlation properties on large time scales. In contrast to the direct calculation of correlation (autocorrelation) functions, DFA has advantages of identifying long-range correlations of nonstationary time series as well as removing underlying trends in time series with different orders of detrending polynomials. The DFA procedures comprise several steps as follows.
1. Considering the time series \( X_i \) of length \( N \), with \( i = 1, 2, \ldots, N \). A new “profile” \( Y(i) \) is formed by subtracting the mean value \( X_{ave} \) of series from the whole time series. This new “profile” is called random walk.

\[
Y(i) = \sum_{k=1}^{i} [X_k - X_{ave}] 
\]  

(1)

2. The profile \( Y(i) \) is divide into \( N_s \) non-overlapping segments of equal length \( s \), where \( N_s = \text{int}(\frac{N}{s}) \). To avoid omitting the short segment at the end of the profile, the same procedure is repeated from the opposite end of time series. Hence, \( 2N_s \) segments are obtained in total.

3. The existence of different polynomial orders in local trends change the capability of eliminate the trends in the series. Therefore, this order enables estimating the type in the series. The local trend \( y_v^m(i) \) for each of the \( 2N_s \) segment is determined by a least-square fitting of order \( m \) for each segment \( v \). Furthermore, the variance is calculated by

\[
F_s^2(v) = \frac{1}{s} \sum_{i=1}^{s} [Y[N-(v-N_s)s+i] - y_v^m(i)]^2 
\]  

(2)

where \( v = N_s + 1, \ldots, 2N_s \).

Finally, for each of the \( 2N_s \) segments, the DFA fluctuation function is obtained by taking the square root of the average over all segments as

\[
F_{DFA}(s) = \left[ \frac{1}{2N_s} \sum_{i=1}^{2N_s} F_s^2(v) \right]^{1/2} 
\]  

(3)

4. The Hurst exponent \( H \) is used as a measure of long-term memory of time series; therefore, the final procedure of DFA entails estimating \( H \) from a single time series. The scaling behavior of fluctuation function can be determined as

\[
F_{DFA}(s) \propto s^{H} 
\]  

(4)

The slope of the scatter plot of the relation between \( \log(F_{DFA}(s)) \) and \( \log(s) \) is denoted as the Hurst exponent \( H \). It is related to the autocorrelation of the time series, and the rate decreases as the lag between pairs of values increases. \( H = 0.5 \) indicates a short-range correlation or no correlation at all in the time series; \( H < 0.5 \) signifies a negative signal correlation[19]; and \( H > 0.5 \) represents a positive long-range correlation . The Hurst exponent \( H \) is treated as an index for damage condition detection.

2.2 Detrended Cross Correlation Analysis (DCCA)

Based on a similar concept derived from DFA, DCCA is used to quantify the presence of long-range correlations between different signals. The DCCA procedures are described as follows.

1. Considering the time series \( X_i \) and \( Y_i \) of length \( N \), with \( i = 1, 2, \ldots, N \). New profile \( X(i) \) and \( Y(i) \) are formed by subtracting the mean value \( X_{ave} \) and \( Y_{ave} \).

\[
X(i) = \sum_{k=1}^{i} [X_k - X_{ave}] 
\]  

(5)
\[ Y(i) = \sum_{k=1}^{i} [Y_k - Y_{ave}] \]  

(6)

2. The profile \( X(i) \) and \( Y(i) \) are divided into \( N_i \) non-overlapping segments of equal length \( \delta \), where \( N_i = \text{int}(\frac{N}{s}) \). According to the second step of the DFA analysis, \( 2N_i \) segments are obtained.

3. The local trend \( x^m(v)(i) \) and \( y^m(v)(i) \) for each of the \( 2N_i \) segments are determined by a least-square fitting of order \( m \) for each segment \( v \), and the covariance is calculated as

\[ F_s^2(v) = \frac{1}{s} \sum_{i=1}^{s} \{X[N - (v - N_i)s + i] - x^m(v)(i)\} \{Y[N - (v - N_i)s + i] - y^m(v)(i)\} \]  

(7)

where \( v = N_i + 1, \ldots, 2N_i \); \( x_v(i) \) and \( y_v(i) \) are the fitting polynomial in segment \( v \).

The preceding procedures are repeated for various scales. Finally, the DCCA covariance is derived by taking the square root of the average over all segments as

\[ F_{DCCA}(s) = \left[ \frac{1}{2N_i} \sum_{v=1}^{2N_i} F_s^2(v) \right]^{1/2} \]  

(8)

2.3 The Proposed Damage Index

To improve the efficiency and accuracy of the SHM system for damage location, a damage index is further proposed. As mentioned, signals measured from the ground floor of a structure are cross-correlated with those from other floors, resulting in a total of \( N \) cross-correlation curves. The detrended covariance of a healthy case is set as the reference value, and the detrended covariance associated with different scales of damage cases is then processed to obtain the damage index.

For the structure with \( N \) floor, the DCCA index under conditions involving damage and no damage can be expressed as

\[ \text{DCCA}_{\text{undamaged}} = \{H_1, H_2, H_3, \ldots, H_N\}^T \]  

(9)

\[ \text{DCCA}_{\text{damaged}} = \{D_1, D_2, D_3, \ldots, D_N\}^T \]  

(10)

After the individual area between the DCCA curves of the damaged and undamaged structures is derived, the proposed damage index for a specific floor is obtained. To specify the location clearly, the damage index is represented as a comparison between the damaged floor and the reference (undamaged case). Therefore, the damage index of a specific floor can be expressed as

\[ \text{DI}_F = \sum_{n=1}^{q} (F_{\text{DCCA}}^{q}_{H_F} - F_{\text{DCCA}}^{q}_{D_F}) \]  

(11)

where the subscript \( F \) is the floor number for damage evaluation; \( F_{\text{DCCA}}^{q}_{H_F} \) is the DCCA curve of undamaged case, and \( F_{\text{DCCA}}^{q}_{D_F} \) is the DCCA curve of damaged case.

As indicated in equation 11, the final damage index \( \text{DI}_F \) is evaluated by summing the difference between the DCCA curve of the undamaged structure and that of the damaged structure. For a specific floor, a negative damage index value indicates the existence of damage on the floor, whereas a positive value indicates a lack of damage on the floor.
3 NUMERICAL EVALUATION

A series of numerical analyses was conducted to verify the performance of the proposed SHM system. A four-story numerical steel structure was simulated by the finite element method on the basis of the layout presented. The structure comprised four identical stories, and the height, length, and width of each floor are 160, 200, and 200 cm, respectively. To reflect practical structural characteristics, a 120-kg mass block was simulated on each story.

Regarding the damage simulation strategy, the damage was initiated by the removal of the bracing, engendering a loss of stiffness, and the resulting damage was classified into 11 cases constituting 5 groups for verification. The damage groups were categorized as follows: (1) undamaged, involving no damage to the structure; (2) slight damage, involving damage occurring on only one floor of the structure; (3) moderate damage, involving damage occurring on any two floors; (4) severe damage, involving damage occurring on any three floors, and (5) Complete damage, involving damage occurring on all floors.

3.1 Damage Condition

The damage database was analyzed using DFA to detect the possible damage condition on the structure. Having been widely used in the field of biomedical engineering, geophysics, and physiology, parameters used in previous research were adopted as the basis to analyze the dynamic features. Input white noise time series of 8000 samples were generated for 40 s at a sampling rate of 200 Hz. The minimum and maximum scales (s) were set to 16 and 512, respectively. Moreover, the fitting polynomial (m) was set to 1.

As shown in Figure 1(a), the DFA procedure was initially executed for the healthy structure (undamaged). The Hurst exponent H was determined by the slope value of the fluctuation function at different scales. The original ground floor signal generated in the numerical model had a lower amplitude than the signals of the other floors; a baseline correction approach was thus adopted and applied to the ground signal from the input (white noise) to resemble ambient vibration in the experiment. Consequently, the ground floor curve had a higher H value than those of the other floors.

The other two damage cases are analyzed using similar procedure as shown in Figure 1(b) (c). For instance, the healthy structure had the lowest H value, and the presence of damage was represented by an increase in H. The damage on the first floor, which would inherently have a greater contribution to the global potential failure of the structure, was associated with a higher value compared with the second floor. Similar trends could also be observed for the other damage cases.

3.2 Damage location

Following the DFA procedure, DCCA was employed to identify the damage location.
Cross-correlation between floor signals is employed in the DCCA analysis. Five DCCA curves, namely G-G, G-1F, G-2F, G-3F, and G-4F, were generated from the four-story structure. Furthermore, to ensure the practical effectiveness of the DCCA method in situations involving noise, the numerical evaluation of DCCA subjected to noise effect was also examined for different signal-to-noise ratios (SNRs). Since the effect of noise added into output is higher than into input as the noise is not filtered by the structure [20].

Figure 2 shows the numerical DCCA diagram for an SNR of 60 dB. As shown in Figure 2(a), the healthy structure was first executed. The area between curves, calculated by subtracting the curve of the floor below from that of the floor above, was assigned as a reference of DCCA curve. Figure 2(b) shows the case of damage on the third and fourth floors. Compared with the reference, the area between the third (blue) and fourth (red) curves was larger. Figure 2(c) shows a numerical DCCA diagram for damage on all floors. Slight differences in area were observed between all damaged and healthy structures, and revealed the necessity of substantial improvement in all damage cases.

![Figure 2: The numerical DCCA diagram for SNR = 60 dB](image)

### 3.3 Damage Index

The damage index proposed based on DCCA curve areas was employed to improve the damage location assessment. The damage index can be calculated by comparing the DCCA curve areas of healthy and damaged structures. Figure 3(a) shows the case involving damage on the third and fourth floors; significant negative values were derived on both the third and fourth floors. In addition, a minor error was observed on the second floor, indicating a slight misclassification of the damage indices. For all damage case, the negative values determined for all floors (Figure 3(b)) indicated the accuracy of the damage-index in detecting damage locations, even in the presence of noise. A slight error was observed only for the high SNR ratio (SNR = 10 dB).

The accuracy of the two proposed methods was further compared, as presented in Table 1. At different noise levels, the accuracy of DCCA was approximately 60%. Moreover, the accuracy of damage location could be enhanced to 80% through the proposed damage index at noise interference levels ranging from 60 to 20 dB. For a higher noise level (SNR = 10 dB), the accuracy slightly dropped to 70%.

![Table 1: Accuracy of DCCA and damage index at different SNRs](image)
Table 1: The accuracy of DCCA and damage index method for different noise levels

<table>
<thead>
<tr>
<th>Noise Level</th>
<th>SNR = 60</th>
<th>SNR = 40</th>
<th>SNR = 20</th>
<th>SNR = 10</th>
</tr>
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<tbody>
<tr>
<td>Case</td>
<td>DCCA</td>
<td>Damage Index</td>
<td>DCCA</td>
<td>Damage Index</td>
</tr>
<tr>
<td>Damage 1F</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
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<tr>
<td>Damage 2F</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Damage 3F</td>
<td>F(4F)</td>
<td>F(4F)</td>
<td>F(4F)</td>
<td>F(4F)</td>
</tr>
<tr>
<td>Damage 4F</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Damage 1&amp;2F</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Damage 2&amp;3F</td>
<td>F(2F)</td>
<td>F(2F)</td>
<td>F(2F)</td>
<td>F(2F)</td>
</tr>
<tr>
<td>Damage 3&amp;4F</td>
<td>F(3F)</td>
<td>C</td>
<td>F(3F)</td>
<td>C</td>
</tr>
<tr>
<td>Damage 1&amp;2&amp;3F</td>
<td>F(4F)</td>
<td>C</td>
<td>F(4F)</td>
<td>C</td>
</tr>
<tr>
<td>Damage all</td>
<td>F(4F)</td>
<td>C</td>
<td>F(4F)</td>
<td>C</td>
</tr>
</tbody>
</table>

Accuracy (%): 60%  80%  60%  80%  60%  80%  60%  70%

4 EXPERIMENTAL VERIFICATION

4.1 Experimental Setup

The ambient vibration experiment was conducted on a seven-story benchmark structure at National Center for Research on Earthquake Engineering (NCREE). Detail of the experimental specimen is shown in Figure 4(a). The height, length, and width of each story are 110, 150, and 110 cm, respectively. To simulate the actual structural characteristics, an additional mass of 500 kg was installed on each story. An L-shaped steel angle measuring 65 × 65 × 6 mm that provided the bracing; Figure 4(b) shows the installation of the bracing.

Data acquired from sensors in the longitudinal direction for 17 damage cases were stored at a sampling rate of 200 Hz. The recording for each damage case lasted 20 min, and the recording process was divided into four segments (5 min for each).

(a) Seven-story benchmark structures  (b) Bracing and non-bracing

4.2 Damage Location

In the follow-up phase of the study, DCCA was employed to determine the damage location. From the seven-story structure, eight DCCA curves, namely G-G, G-1F, G-2F, G-3F, G-4F, G-5F, G-6F, and G-7F, were generated through the crossing strategy between the ground and other output signals. The healthy structure was first analyzed. The area between curves, calculated by subtracting the curve of the floor below from that of the floor above, was established as a reference in the DCCA analysis as shown in Figure 5(a).
Figure 5(b) presents the case involving damage on the third and fourth floors (34F). Damage was demonstrated by the decline of curves G-2F and G-3F, which clearly resulted in larger areas than the reference. These trends evidenced the existence of damage on the third and fourth floors.

In the case of all floors damage (AD) as shown in Figure 5(c), the seven curves were distributed in more complex manners compared with the reference curve for the healthy structure, consequently resulting in misclassifications.

4.3 Damage Index

Although most of the damage locations could be identified accurately by the DCCA technique, misclassifications were still encountered. Therefore, the proposed damage index was employed to improve the reliability and practicability of the SHM system. The damage index values for five selected damage cases are provided in Figure 6. Trends were observed for the cases of damage on the third and fourth floors, where the proposed DCCA damage index accurately detected damage locations (Figure 6(a)). Negative values were observed on the third and fourth floors.

Misclassifications occurred for the damage on all floors. In contrast to the expectation, negative index values did not appear on all floors. Only the first, second, sixth, and seventh floors had negative values, whereas the rest of the floors remained positive.

The DCCA method was compared with commonly adopted methods in the field of SHM. A total of 16 damage cases were evaluated using the MSC, multiscale transfer entropy (MSCE) [21], and DCCA techniques, and the results are listed in Table 2. As indicated, the accuracy of MSC was 53.3%; the accuracy rates of the MSCE approach and MSCE damage index were 60% and 86.7%, respectively. Furthermore, the accuracy rate of DCCA was 75%. After the proposed damage index was applied, the accuracy was improved to 87.5%, and only two damage cases were misclassified. Hence, a more reliable result can be provided by the proposed SHM system compared with the MSC and MSCE methods.
<table>
<thead>
<tr>
<th>Damage Location</th>
<th>Mode Shape Curvature</th>
<th>Multi Scale Cross-Entropy (MSCE)</th>
<th>Damage Index (MSCE)</th>
<th>Detrended Cross-Correlation Analysis (DCCA)</th>
<th>Damage Index (DCCA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damaged 1F</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Damaged 2F</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Damaged 3F</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Damaged 4F</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Damaged 5F</td>
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<td>F(1F)</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Damaged 6F</td>
<td>C</td>
<td>F(1F)</td>
<td>C</td>
<td>F(2&amp;6F)</td>
<td>F(5&amp;6F)</td>
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<tr>
<td>Damaged 7F</td>
<td>C</td>
<td>F(1F)</td>
<td>C</td>
<td>F(1&amp;2&amp;3F)</td>
<td>C</td>
</tr>
<tr>
<td>Damaged 1&amp;2F</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
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<td>Damaged 3&amp;4F</td>
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<td>C</td>
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</tr>
<tr>
<td>Damaged 5&amp;6F</td>
<td>F(5&amp;6&amp;7F)</td>
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<td>C</td>
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<td>C</td>
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<tr>
<td>Damaged 1&amp;3&amp;5F</td>
<td>F(1&amp;3F)</td>
<td>-</td>
<td>-</td>
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<td>C</td>
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<td>Damaged 4&amp;5&amp;6F</td>
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<td>F(1F)</td>
<td>F(1F)</td>
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<td>C</td>
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<tr>
<td>Damaged 1&amp;2&amp;3&amp;4F</td>
<td>F(4F)</td>
<td>C</td>
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<td>Damaged 4&amp;5&amp;6&amp;7F</td>
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<td>F(1F)</td>
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</tr>
<tr>
<td>Damaged all</td>
<td>F(Non)</td>
<td>C</td>
<td>C</td>
<td>F(1&amp;2&amp;3&amp;4F)</td>
<td>F(2&amp;3&amp;4&amp;7F)</td>
</tr>
</tbody>
</table>

Table 2: Damage location comparison for different methods

5 SUMMARY AND CONCLUSION

A SHM system based on DFA and DCCA is proposed in this study. DFA was employed to determine the damage condition in the structure. By adopting fractal analysis, which has shown its practicability in interpreting irregular, complex, and disordered phenomena, DFA was employed to determine the damage condition of structures. The Hurst exponent (H) value, the main parameter in DFA, was used as an indicator of damage. DCCA was then used to localize the damage location in the structures. The detrended covariance, calculated from the DCCA technique, was used to identify the damage location.

A four-story numerical model was simulated to verify the performance of the proposed SHM system. The effect of different noise levels was also evaluated in the numerical analysis. The results of this evaluation indicated that the DCCA approach could determine the damage location under the interference of different noise levels through the comparison between the damage and healthy structures, which served as a reference. Furthermore, the damage location could be efficiently determined by the proposed damage index, thus demonstrating the feasibility of the proposed SHM system.

On the basis of the DFA method, the damage condition could be accurately assessed through H. The damage location could be localized using the DCCA technique with an accuracy of 75%. Using the proposed damage index further improved the accuracy to 87.5%. As only ambient vibration is required in a set of initial reference measurements, the proposed system offers an easy and alternative strategy for practical SHM.

REFERENCES


