Nonlinear Acoustics for Structural Health Monitoring
– Classical vs. Non-classical Approaches

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Abstract
The past two decades have marked significant research interest in nonlinearities produced in micro-cracked and cracked solids. These investigations have been based on the analysis of nonlinear static stress-strain characteristics and various ultrasonic wave interactions. Crack-induced elastic, thermo-elastic and dissipative-type nonlinearities have been observed within various material scales and for various strain levels. The paper gives an overview of classical and non-classical nonlinear acoustic damage detection methods. Modelling, numerical simulation and damage detection applications are briefly reviewed. Some examples of recent research developments – related to damage localisation, nonlinear guided waves and enhancement techniques are demonstrated.

1 INTRODUCTION
Nonlinear vibration and ultrasonic/acoustic phenomena have been used for many years to detect material defects and structural damage. It is well known that all engineering structures are nonlinear to some extent with respect to structural dynamics [1]. Nonlinearities can be attributed not only to material behavior, structural joints, geometry or boundary conditions but also to structural damage. Therefore nonlinear phenomena have been also exploited for the diagnosis of damage based on vibration analysis in structures [2]. A fatigue crack that opens and closes under dynamic loading and alters natural frequencies is the best known example of nonlinear vibration phenomenon in vibration/modal analysis. Vibration-based damage detection methods are relatively well understood but not sensitive to detect small defects. This is mainly due to the fact that the vast majority of these methods rely on the analysis of global vibration responses. In contrast, ultrasonic and acoustic nonlinear phenomena have demonstrated better sensitivity to damage detection [3-5]. It is both remarkable and difficult to explain why relatively small defects exhibit strong nonlinearities resulting from relatively small strain amplitudes. Although, physical understanding of many non-classical nonlinear phenomena is still not clear and needs further theoretical and applied research investigations, recent years have shown a fast growing interest in ultrasonic/acoustic nonlinear methods for structural damage detection.
Nonlinear ultrasonic/acoustic phenomena involve various classical and non-classical effects. Classical nonlinear effects in ultrasonic wave propagation have been investigated for decades [6-7] and also explored for damage detection [8-9]. These effects - related to various material imperfections (e.g. intrinsic nonlinearities due to anharmonicity or imperfections in atomic lattices) - contribute to accumulated distortion of propagating waves, leading to higher harmonic generation. This effect is enhanced when additional imperfections – such as localized fatigue cracks or distributed micro-cracks - are present in material. Research work in this area also involves higher harmonic generation of Lamb waves used for the detection of material nonlinearity [10] and damage [11]. Recent years have demonstrated a growing interest of theoretical and applied research related to various non-classical effects in ultrasonic wave propagation [3-5,12-19]. These effects result from various crack-wave interactions that exhibit sub-harmonic generation, stress-strain hysteresis, amplitude-dependent non-classical (i.e. non-frictional and non-hysteretic) dissipation, acoustic equivalent of the Luxemburg-Gorky (LG) effect, vibro-acoustic modulations and other phenomena that are often not easy to investigate and explain. All these non-linear effects are remarkably enhanced in the presence of contact-type and small-severity defects in materials. Various damage detection techniques based on frequency mixing, nonlinear modulations, slow dynamics and reverberation analysis have been developed for the last two decades.

This paper briefly reviews some of the recent developments in the field, highlighting future trends and challenges. The major focus is on modelling, numerical simulation, damage detection and localization. Examples of methods that involve higher-harmonic generation, vibro-acoustic modulations and Lamb waves are presented.

2 MODELLING AND NUMERICAL SIMULATIONS

There are three major difficulties associated with nonlinear ultrasonic/acoustic phenomena when used for damage detection. Firstly, physical mechanisms behind nonlinear ultrasonic/acoustic crack-wave interactions are scale-dependent, strain-dependent and involve different types of elastic and dissipative effects, as explained in [20]. Nonlinear effects have been observed not only in macroscopic scales of metals for large \((10^{-4} \div 10^{-2})\) strain levels (e.g. closing-opening fatigue crack) but also in atomic scales of homogeneous solids for very small \((10^{-10} \div 10^{-8})\) strain levels (e.g. anharmonicity of interatomic potential). The former relates to damage whereas the latter results from material nonlinearity. Secondly, similar nonlinear effects used as damage indicators can be manifested by different physical mechanism and vice versa. For example, energy dissipation may result from frictional, hysteretic or non-classical LG phenomena. Hysteresis in turn involves both elasticity and dissipation, and could be linear or nonlinear. Thirdly, various non-linear effects can be observed experimentally when nonlinear ultrasonic/acoustic methods are used for damage detection, as illustrated in [18]. It is often very difficult – if not impossible – to separate all these effects involved.

Physical understanding of nonlinear mechanisms involved in classical and non-classical methods used for damage detection is very important to separate reliably damage-related nonlinearities from other nonlinear effects (e.g. material nonlinearity, measurement chain). Modelling and numerical simulations can play an important role in this effort. Research studies of classical and non-classical phenomena in solids have a long history and involved various physical models, as reviewed in [20]. Altogether, these models can be classified into five major groups: (1) classical nonlinear elasticity; (2) classical nonlinear crack and crack-wave models; (3) hysteretic models; (4) classical contact models; (5) non-classical
dissipation. A summary of these models is given in Table 1.

<table>
<thead>
<tr>
<th>Model Groups</th>
<th>Model Examples and Remarks</th>
<th>Examples of References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical nonlinear elasticity</td>
<td>- Higher order elements in the Hook’s law (nonlinear stress-strain, cubic and quadratic stiffness nonlinearity; used to model mainly material nonlinearity; not able to explain some nonlinear phenomena in rocks – e.g. stress-strain hysteresis)</td>
<td>[21-22]</td>
</tr>
</tbody>
</table>
| Classical nonlinear crack and crack-wave models | - Bi-linear stiffness  
- Breathing crack  
- “Mechanical diode” – acoustic waves rectification  
- clapping  
- Contact Acoustic Nonlinearity (CAN) (relates to stiffness asymmetry; simplified models from the physical point of view; can take a form of an oscillator or relaxator – different force to close and open cracks; can be used to model adhesive bond defects) | [1]  
[23-25]  
[26]  
[27-28]  
[26] |
| Hysteretic models                   | - Quasi-static (frequency-independent) model – dependence of the modulus on the strain-rate sign  
- Phenomenological models – e.g. the Preisach-Mayergoyz (PM) model (always associated with energy dissipation; generates odd harmonics; can explain amplitude-dependent frequency shift and modulations) | [22]  
[29-30] |
| Classical contact models            | - Hertzian contact  
- Rough-surface contact (involves contact of crack faces; can be used to model partial contact; can involve adhesion forces) | [31]  
[31-32] |
| Non-classical dissipation           | - solid containing high-compliant inclusions  
- The Luxemburg-Gorki effect (observed mainly in mesoscopic materials; often an addition to the hysteretic nonlinearity) | [33]  
[4] |

Table 1: Nonlinear crack-wave interaction models used for damage detection based on ultrasound.

Modelling and numerical simulations are used to ease the physical understanding of various classical and non-classical crack-wave interaction mechanisms. It appears that the majority of these investigations has involved mainly analytical, one-dimensional models or local FE approaches to explain classical nonlinearities. More recently, a non-local approach based on peridynamics has been used to explain nonlinear vibro-acoustic wave modulations.
Very little effort has been used to explain non-classical nonlinearities. Recent work in this area relates to the explanation of the LG effect through the nonlinear coupling between the strain field and thermal field and thermal dissipation (due to high level of temperature gradient) at crack faces [4]. Some experimental evidence of this phenomenon can be found in [18], where a clear link between nonlinear vibro-acoustic modulation intensity and local increase of temperature at crack faces has been demonstrated. A semi-nonlocal modelling approach has been also used to demonstrate the enhancement of the thermo-elastic effect due to crack [35].

3 DAMAGE DETECTION USING NON-CLASSICAL APPROACHES

Classical nonlinear ultrasonic/acoustic methods – mainly based on higher harmonic generation - are relatively well established. Numerous examples that involve the analysis of amplitude can be found in the literature. Research investigations related to non-classical nonlinear ultrasonic/acoustic methods have led to various approaches that can be used for damage detection. This includes methods based on frequency mixing, vibro-acoustic-modulations, slow dynamics and reverberation [3,5,12,14-19]. The method that involves nonlinear vibro-acoustic wave modulations [3,5] – illustrated in Figure 1 – is by far the most widely used approach for damage detection. When a monitored structure is excited modally ($f_L$ - low-frequency excitation), an ultrasonic wave ($f_H$ - high-frequency excitation) is introduced. Then ultrasonic responses are utilised for damage detection. Intact (or undamaged) structures normally exhibit mainly two frequency components associated with the high- and low-frequency excitations. In contrast damage exhibit additional vibro-acoustic wave modulations that can be observed as a pattern of sidebands in ultrasonic response spectra. Modulation sidebands can be used to detect damage and the modulation intensity – based on amplitudes of the ultrasonic carrier component and the sidebands – is the most widely used damage index. Alternatively, vibro-acoustic modulations can be analysed in the time domain through the instantaneous amplitude and frequency/phase [36] and in the combined time-frequency (or time-scale) domain [37].

![Figure 1: Nonlinear vibro-acoustic wave modulations used for damage detection.](image)
The majority of applications use ultrasonic transducers, piezoceramic transducers, speakers, electromagnetic shakers and impact hammers for excitation. More recently, noncontact laser vibrometry [19] and air-coupled transducers [38] have been applied to avoid intrinsic nonlinearities and to localize damage. Although the vast majority of applications are related to crack detection in metallic structures, some attempt have been made to detect damage in other materials, e.g. composites, concrete and bones. It appears that damage application papers are scattered in different physical journals and conference proceedings covering Non-Destructive Testing, Structural Health Monitoring and Applied Physics. Very few publications discuss monitoring strategy with respect to excitation frequencies, sensor location and damage severities [37,39]. Table 2 gives some application examples.

<table>
<thead>
<tr>
<th>Type of damage</th>
<th>Examples of References</th>
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<tbody>
<tr>
<td>Cracks in steel</td>
<td>[40-41]</td>
</tr>
<tr>
<td>Cracks in aluminum</td>
<td>[5,18,36,18]</td>
</tr>
<tr>
<td>Delamination in composites</td>
<td>[14,16-17,19,27,42]</td>
</tr>
<tr>
<td>Debonding and kissing bonds</td>
<td>[43-44]</td>
</tr>
<tr>
<td>Impact damage in sandwich structures</td>
<td>[45-47]</td>
</tr>
<tr>
<td>Cracks in glass</td>
<td>[48]</td>
</tr>
<tr>
<td>Cracks in concrete, ceramics</td>
<td>[13,41]</td>
</tr>
<tr>
<td>Cracks in bones</td>
<td>[49-50]</td>
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Table 2: Application examples of non-classical nonlinear ultrasonic/acoustic methods for structural damage detection.

4 RECENT DEVELOPMENT EXAMPLES

This section demonstrates some of the recent advancement in the field. The work reviewed and examples presented are related to damage localization using non-classical and classical nonlinear and ultrasonic/acoustic approaches, nonlinear Lamb waves and enhanced techniques that combine nonlinear acoustics and guided ultrasonic waves.

4.1 Damage localisation

The vast majority of non-classical nonlinear approaches detect damage but do not offer damage location. This is one of the major problems when non-classical nonlinear acoustics is used to monitor structures. The question is whether nonlinear features used for damage detection are enhanced in the vicinity of damage, allowing for damage localisation. Several approaches have been proposed for damage imaging based on nonlinear system responses, including the work on nonlinear scattering [51], spatial mapping of higher harmonics [52], local defect resonance [53] and time reversal [38].

Recent modelling and experimental work in [54] demonstrates a local increase of amplitude of higher harmonics near the crack location, i.e. the crack localisation effect. An
example from this work is presented in Figure 2, where the crack localisation effect is investigated in a 300 x 25 x 10 mm cantilever aluminium beam. Numerical simulations based on Finite Element modelling reveals that the coefficient of nonlinearity - based on the amplitude of the second harmonics – increases in the vicinity of the crack for three different locations investigated. Experimental validation of this work can be found in [54].

![Figure 2: Crack localisation effect in classical nonlinear acoustics based on higher harmonics generation: (a) schematic diagram of the cantilever beam investigated; (b) damage index based on the amplitude of the second harmonics for three different simulated crack positions.](image)

Following the work presented in [53], the example of damage location based on nonlinear acoustics is illustrated in Figure 3. A rectangular (300×150×2 mm) composite plate (carbon/epoxy prepreg) was impacted in the centre. The impact energy was equal to 3.9 J. The Monit SHM vibrothermographic system with the 35 kHz ultrasonic excitation column – was used to reveal butterfly-like delamination in the plate, after impact (Figure 3a).

![Figure 3: Delamination after 3.9 J impact revealed by: (a) vibrothermography; (b) non-classical nonlinear acoustics based on vibro-acoustic crack modulations.](image)
Following these investigations, a non-classical nonlinear acoustic test was performed. Low-profile, surface bonded transducers were used again for low- and high-frequency excitations. Once the plate was excited, ultrasonic responses were gathered. The plate was scanned using a 3-D laser vibrometer to analyse sideband amplitudes at various positions. The intensity of modulation, based on the spectral amplitudes of modulation sidebands, was calculated and mapped on the measured surface to reveal the same delamination in Figure 3b.

4.2 Nonlinear Lamb Waves

Nonlinear wave propagation is a subject of research investigations for decades in discrete [55] and continuous media [56], revealing the complexity of analysed wave fields. The majority of these investigations are limited to weakly nonlinear dependence of model parameters. Previous work investigated includes aspects related to higher harmonic generation, energy flux between the primary and secondary modes and synchronization. More recently internal resonance of frequency shifting has been also investigated [57]. Figure 4 demonstrates how the amplitude of excitation affects simulated dispersion curves when nonlinear Lamb wave field is investigated. Clear shifts of these curves can be observed with the increased amplitude. It is well known that changes of resonant frequencies in nonlinear systems influence spectral characteristics, resulting in amplitude-dependent wave speeds. This problem is not trivial since Lamb waves arise from multiple reflections of longitudinal and shear bulk waves from stress free surfaces. As a consequence the entire wave field is affected and application for damage detection not easy to implement. Application examples of nonlinear Lamb waves for damage detection can be found in [58-59].

![Figure 4: Dispersion curves of 1 mm thick aluminum plate for various excitation amplitudes (LN – linear wave field; NL – nonlinear wavefield). The cale factor of 10 was used in these numerical simulations.](image)

4.3 Enhanced nonlinear vibro-acoustic guided waves

The combined vibro-acoustic nonlinear modulation technique is based on continuous low- and high-frequency sine excitation. Some attempts have been made to replace the continuous sinusoidal high-frequency excitation with the synchronized localized (windowed) sinusoidal excitation to propagate guided waves in monitored structures and to enhance the nonlinear
A different damage detection technique that combines guided wave propagation with nonlinear acoustics has been proposed in [61]. Fatigue testing was used to introduce a crack in the mid span of a 300×20×10 mm aluminium beam. A guided ultrasonic wave (150 kHz) was introduced to the beam when the structure was modally excited (harmonic sinusoidal 10 Hz excitation). Then ultrasonic responses were gathered for two different scenarios of synchronised low-frequency excitation, i.e. when the beam was compressed (closed crack) and when the beam was in tension (open crack). The difference between both responses was then calculated, as illustrated in Figure 3a. These measurements and calculations were gathered for various positions on the surface of the beam using a 3-D scanning laser vibrometer. The RMS values for different measurements are shown in Figure 3b, were B-scan (measurements for various positions vs. time) are given for the cracked beam to reveal the position of the crack.

![Figure 3a](image1.png)  
![Figure 3b](image2.png)

**Figure 3**: Enhanced nonlinear acoustics based on guided ultrasonic waves used for crack detection in an aluminium beam: (a) ultrasonic responses for the synchronised compressive and tensile low-frequency excitations; the difference of both responses reveals the nonlinearity; (b) wave-filtered B-scan for the cracked beam exhibiting the location of the crack.

## 3 CONCLUSIONS

A brief overview of nonlinear ultrasonic/acoustic damage detection methods have been given. The major focus has been on modelling approaches, damage detection examples and recent developments related to damage localization and damage imaging. This short overview can be summarized with the following conclusions:

- Although substantial research effort has been put to model various classical approaches, non-classical nonlinear phenomena involved in ultrasonic/acoustic methods are still not well explained. Further research work is required to reveal and understand different physical mechanisms involved.
- Research investigations related to classical and non-classical nonlinear ultrasonic/acoustic phenomena are scattered in the literature ranging from Engineering, Solid Mechanics, Applied Physics, Geophysics to Seismology. Interdisciplinary effort of all these research communities could be beneficial for practical applications.
• Although nonlinear methods offer relatively good sensitivity with respect to damage severity, nonlinear effects used for damage detection can also result from various intrinsic non-damage related phenomena. Major research effort is needed to distinguish both effects for reliable damage detection.
• Very little studies have been performed to establish monitoring strategies with respect to excitation frequencies/amplitudes, sensor location, signal processing and uncertainty analysis.
• The vast majority of damage detection approaches leads to damage detection but does not offer damage localization. Further effort is required in this area of research.
• Non-classical damage detection methods based on ultrasonic wave propagation and wave interaction with damage have been used mainly in laboratory conditions. Field tests have not been performed. Although the methods are attractive engineering applications are not imminent.

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REFERENCES


