Digital model for prediction and analysis of ultrasonic guided waves propagation in structural health monitoring systems

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Objective

To develop a digital model which simplifies interpretation and analysis of the guided wave signals captured on objects under investigation in structural health monitoring (SHM) systems.

Object

Various composite structures.
Structural Health Monitoring

*Real-time* damage detection and characterization strategy which helps to improve the safety and the durability of critical structures by combining an array of *embedded* transducers for capturing of the data from wide variety of engineering components.

Basic example of SHM system
Comparison between NDT and SHM, using guided waves

<table>
<thead>
<tr>
<th>Feature</th>
<th>NDT</th>
<th>SHM</th>
</tr>
</thead>
<tbody>
<tr>
<td>During exploitation</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Surface scan ability</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Amount of measurement positions</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Duration of investigation</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Accuracy, identification of defect position</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Quickness of defect detection</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>System calibration, technique verification</td>
<td>+</td>
<td>-</td>
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</tbody>
</table>

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Advantages and disadvantages of SHM based on guided waves

**Advantages**

+ Guided waves propagate long distances with small attenuation;
+ Enables to investigate large objects and detect remote defects;
+ Guided wave parameters are directly related to elastic properties of materials.

**Expected problems**

- Detection of defects using limited amount of the measurement positions;
- Verification of the monitoring technique (coverage, reference samples, sensitivity to defect);
- Complicated interpretation and analysis of signals captured during the measurements.
Motivation

Propagation of low frequency guided waves in composite structures is complicated due multiple reflections, mode conversion and low attenuation. For these reasons signals captured from object under investigation contains data which represents a set of reflected modes. The problem is to identify which segment in the acquired signal correspond to a particular mode. The computer model helping to analyze and to interpret such signals has to be developed.
## Requirements for the model

**The model should take into account**

- The properties of object under investigation: *(shape, dimensions)*;
- Material properties: *(anisotropy, group velocity)*;
- Transducers characteristics: *(directivity patterns, position on object, orientation angle)*.

**The model should allow**

- To calculate the time of flight different modes propagating along different paths;
- To analyze reflections and to retrace wave propagation paths;
- To predict the waveform of received signal.
Concept of the model: transmitter

The model generates the wavefront in all 360° with amplitudes corresponding to transmitter directivity pattern.

Initial model information

Transmitter

Central coordinates

Directivity pattern

Orientation angle

Transmitter

Receiver

\[ (x_e, y_e) \]

\[ D_e(\alpha_e) \]

\[ (x_r, y_r) \]

\[ D_r(\alpha_r) \]

\[ x_p \]

\[ y_p \]

\[ l_1 \]

\[ l_2 \]
Concept of the model: receiver

The wavefront which crosses the receiver area is “received” by it.

Initial model information

- Directivity pattern
- Central coordinates
- Length and width
- Orientation angle

The wavefront which crosses the receiver area is “received” by it.
Concept of the model: the object

The generated wavefront propagates with velocities defined separately for each of the directions, so the anisotropy can be taken into account.

Initial model information

The object

Length and width

Group velocity

Transmitter

Receiver

\[ (x_e, y_e) \]

\[ D_e(\alpha_e) \]

\[ c_g(\alpha_m) \]

\[ D_r(\alpha_r) \]

\[ (x_r, y_r) \]

\[ l_1 \]

\[ l_2 \]
The main steps of the model performance

The operation of the model could be divided into five main steps:

- Calculation of wave front position at different time instances;
- Calculation of the wavefront reflection from the object boundaries;
- Calculation of the time of flight of the waves propagating along different paths. The pulse response of the object for particular guided wave mode is obtained;
- Calculation of the expected waveform of the received signal;
- Analysis of waves propagation paths.
Calculation of wavefront position on the object

Simulated wavefront is created as a set of wavefront positions for each initial angle \( \alpha_m \) at some time instance. The wavefront position on object under investigation is defined by variables \( x_F, y_F \).

\[
x_F(t_k, \alpha_m) = c_g(\alpha_m) \cdot t_k \cdot \cos(\alpha_m) + x_e,
\]

\[
y_F(t_k, \alpha_m) = c_g(\alpha_m) \cdot t_k \cdot \sin(\alpha_m) + y_e,
\]

\[
t_k = (k - 1) \cdot \Delta t + t_p, \quad k = 1 \div K, \quad K = \frac{t_g - t_p}{\Delta t} + 1,
\]

\[
\alpha_m = (m - 1) \cdot \Delta \alpha, \quad m = 1 \div M, \quad M = \frac{2\pi}{\Delta \alpha} + 1,
\]

where \( c_g(\alpha_m) \) is the group velocity of the wave under analysis; \( t_k \) is the current time instance; \( \Delta t \) is the step time domain; \( t_p, t_g \) are the time instances of the simulation start and the end; \( \alpha_m \) is the initial angles of wave propagation directions; \( \Delta \alpha \) is the step for angle determination; \( x_e, y_e \) are the coordinates of the transmitter center.
Estimation of the reflection from the object boundaries

The wavefront reflections from object boundaries are simulated by recalculating coordinates of wavefront parts which are outside the object.

\[ x_{F,m}(t_k) = |x_{F,m}(t_k)| \forall m, x_{F,m}(t_k) < 0, \]
\[ x_{F,m}(t_k) = x_p - (x_{F,m}(t_k) - x_p) \forall m, x_{F,m}(t_k) > x_p, \]
\[ y_{F,m}(t_k) = |y_{F,m}(t_k)| \forall m, y_{F,m}(t_k) < 0, \]
\[ y_{F,m}(t_k) = y_p - (y_{F,m}(t_k) - y_p) \forall m, y_{F,m}(t_k) > y_p, \]

where \( x_{F,m}(t_k), y_{F,m}(t_k) \) is the wavefront points, which are outside the object; \( t_k \) is the current simulation time instance; \( x_p, y_p \) are the length and the width of the object.
“Reception” of the wavefront and prediction of the waveform of the signal

The wave front amplitude $A_e(t_k)$ which is received on time $t_k$ is calculated by integrating wave energy which passes receiver area $L$.

The wavefront amplitudes $A_e(t_k)$ are registered until wavefront passes receiver area. The set $A_e(t_k)$ is called the object pulse response $h_0(t)$, because it determines transmission function transmitter-object receiver in the case of pulse excitation:

$$h_0(t) = \{A_e(t_{k_1})...A_e(t_{k_N})\}.$$

The waveform of received signal $u(t)$ can be predicted using convolution between excitation signal $u_{ref}(t)$ and calculated pulse response $h_0(t)$:

$$u(t) = u_{ref}(t) \otimes h_0(t).$$
Calculation of wave propagation paths

At the end of simulation model accumulates an array $\mathbf{M}$ with wavefront positions $x_F, y_F$ at any simulation time instance $t_k$, so the wave propagation path can be reconstructed by selection of the values corresponding to the particular initial angle from the array $\mathbf{M}$:

\[
\mathbf{M} = \left[ t_k, X_F(t_k, \alpha_m), Y_F(t_k, \alpha_m), \alpha_m \right]
\]

where $X_F(t_k, \alpha_m), Y_F(t_k, \alpha_m)$ – the matrices of wavefront positions.

Evaluation of wave front propagation path by wave front nodes positions in various time moments
The result of the modelling: the pulse response

The example of the pulse response $h_0(t)$ for $S_0$ mode obtained using proposed model is presented below.

Such pulse response can be simulated to different guided wave modes. It shows time instances when simulated wavefront passes receiver. Also it takes into account the amplitude regarding to directivity patterns of ultrasonic transducers. Using pulse responses of different guided wave modes it can be identified, of course with some limitation, which segment on experimental signal corresponds to which of the guided waves mode.

\[ h_0(t) = \{ A_c(t_{k1})...A_c(t_{kN}) \} \]
The result of the modelling: the waveform of the signal

The example of the waveform $u(t)$ for $S_0$ mode obtained using proposed model is presented below.

![Waveform of the signal](image)

$$u(t) = u_{\text{ref}}(t) \otimes h_0(t).$$

Such prediction works good for one mode, however integration of the signals of different modes is complicated as the relative efficiency of generation of the different modes is not strictly determined.
The result of the modelling: the wave propagation paths

The example of the wave propagation path for $S_0$ mode determined using proposed model is presented below.

The model allows to retrace received waves propagation paths by selecting time interval in pulse response. In the example we can see that in time range 363÷400μs receiver receives wave fronts traveling in four different routes.
The result of the modelling: the wave propagation paths

The example of the wave propagation path for $S_0$ mode determined using proposed model is presented below.

Wave front propagation paths can be plotted in different colors. These colors relates to the amplitude of the waves propagating along particular path.
Usage of the model for analysis of experimental data

The bending experiments of GFRP plate (1300x1000mm) in TWI UK:

*Transducers positions on test sample*

*Test sample in bending machine*
The model corresponding to the investigated GFRP plate

- Assigned group velocities of guided waves:
  - $S_0$ mode 3.45 km/s;
  - $A_0$ mode 1.4 km/s;
  - SH mode 1.9 km/s.

- Duration of simulation:
  - 2000 $\mu$s

- Assigned directivity patterns to ultrasonic transducers:
Comparison between experimental and simulated results

- Signal received during bending experiments:

![Graph showing signal received during bending experiments]

- Signals simulated with the model:

![Graph showing signals simulated with the model]
Conclusions

1. Investigation showed that the main advantage of the created model is possibility to take into account anisotropy of group velocity and complicated directivity patterns of the ultrasonic transducers;

2. Comparison of experimental and theoretical results showed that the proposed model predicts the propagation time and paths of different guided wave modes and their reflection. However in order to predict more accurately the waveforms of the signals the relations between efficiency of generation of different modes should be known and attenuation should be taken into account.
Acknowledgement

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