**Damage Identification Method by Basis Pursuit Method and PSO Algorithm**

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**Abstract**

A three-stage method is presented using Basis Pursuit (BP) and the Particle Swarm Optimization (PSO) technique. Recently, optimization techniques have been employed to solve the damage detection problems. The proposed method needs the first few modal responses of structure. First, a system of equations which relates the damage extent to the structural responses is solved using the BP method. The results of this stage are used as the initial population of PSO algorithm. To reduce the search space of PSO, a micro search operator is embedded in the algorithm to eliminate low damage values. To overcome the noise present in structural responses, a method known as Basis Pursuit De-Noising (BPDN) is also used. The efficiency of the proposed method is investigated by numerical models of a pipe rack and a portal plane frame. The simulation results demonstrate the accuracy and efficiency of the proposed method in detecting multiple damage cases and exhibit its robustness regarding noise.

**Keywords:** Structural damage detection, Basis Pursuit, Particle Swarm Optimization, Micro Search, Modal analysis

**1- Introduction**

In recent decades, numerous non-destructive test data have been used in structural damage detection [1]. Most non-destructive methods developed use the changes in system’s modal parameters such as; natural frequencies, mode shapes [2, 3] and modal strain energy [4]. Recently, optimization techniques have been employed to solve the damage detection nonlinear inverse problem [5]. Genetic algorithm (GA) is a global optimization technique which has been applied to obtain a reliable solution. Some improved GA methods and GA hybridized with other meta-heuristic methods have been used to solve damage detection problems [7–10]. Particle swarm optimization (PSO) is also a useful optimization technique that becomes very popular because of its simplicity and convergence speed. Recently, Particle swarm optimization (PSO) has been used by a number of researchers in solving damage detection problems [11–14]. A two stage damage detection method using a modal strain energy index (MSEBI) and the PSO method was proposed by Seyedpoor [5]. In his method, the damage location was firstly detected by MSEBI and then the damage extent was determined using PSO. The numerical results indicated the accuracy and efficiency of the proposed method. Also a two stage method was proposed by Fallahian and Seyedpoor [18] which used the neural network at the first stage and the PSO at the second one. The method converges to the exact values but many sample model of each case study was required for the first stage.

In reality, only a few structural elements within a structure may be damaged. Therefore, the vector of elemental damage extent is sparse and damage detection is a sparse problem. Recent theoretical works have focused on orthogonal matching pursuit (OMP) and basis pursuit (BP) methods for solving sparse problems with and without noise [19–22]. Liu et al. [23] also employed matching pursuit to decompose a vibration signal in to time-frequency atomics. The vibration signature was then extracted for detection of localized bearing defects. The proposed method was reported to be more flexible than the bilinear time frequency distribution. Gerist et al. [25] proposed a two stage method using the basis pursuit method and continuous genetic algorithm (CGA). In their method, the damage residual is formed using the frequencies of damaged and healthy structure to set a system of equations. The system of equations is then linearized in different ways and is solved by basis pursuit method. Finally, the continuous genetic algorithm is used to improve the first stage results. The proposed method converged to the results accurately and rapidly comparing to the methods found in the literature.

In the present paper, a new three stage algorithm is proposed to identify the damage location and extent. In the first stage, more probable damage locations are predicted and intact elements are excluded using the basis pursuit method. In the second stage, the damage extents of the sited elements in the first stage are determined by the PSO. Multiple damage location assurance criterion (MDLAC) has been formulated as objective function of PSO. Finally, after several generations, a micro search (MS) operator eliminates the low value variables of elites. Basis pursuit de-noising method is also utilized to overcome the noise of damage responses. To demonstrate the efficiency of the algorithm, three numerical case studies are solved and the results are compared with those from other algorithms.
2- Damage Detection

In structural damage detection techniques, the structural responses such as natural frequencies, mode shapes and static deflections or a combination of these may be used [26]. The dynamic properties of a damaged structure such as its natural frequencies and mode shapes have been widely used to solve the damage detection problem. To detect the damage locations and extents, two parameter vectors are defined as follows:

\[
\begin{align*}
\Delta_{Rd} &= \frac{R_h - R_d}{R_h} \quad \text{(1)} \\
\Delta_R(X) &= \frac{R_h - R(X)}{R_h} \quad \text{(2)}
\end{align*}
\]

where, \( R_d \) is the vector of \( m \) structural responses of measured damaged structure, \( R_h \) is response vector of the healthy structure, \( X = (x_1, x_2, \ldots, x_n) \) contains the design variables and vector \( R(X) \) represents \( m \) responses of analytical model. A correlation index based on the two parameter vectors, called the multiple damage location assurance criterion (MDLAC), is expressed in the following form [24]:

\[
\text{MDLAC} = \frac{(\Delta_{Rd}^T \times \Delta_R(X))^2}{(\Delta_{Rd}^T \times \Delta_R(X) \times \Delta_R(X) \times \Delta_{Rd})}
\]

To solve the structural damage detection problems, a set of damage variables should be found to equalize the analytical and measured responses of the structure in an optimal way. Thus, the problem is solving a set of equations considered as follows:

\[
R_d = R(X) \Rightarrow X = ?
\]

where, \( x_i \) is the damage ratio of the \( i^{th} \) element. The damage ratio is expressed as the ratio of the damaged element stiffness to the stiffness of the intact element.

2-1 Basic Pursuit (BP)

In classical theory of linear algebra, when measurements of an equation are fewer than the unknowns, the problem is underdetermined and the solution is generally not unique. The mathematical expression of the set of linear equations is as follows:

\[
A \times x = b
\]

where, \( b \) is a vector of interest, \( b \in \mathbb{R}^m \) and \( x \) is a subspace of \( \mathbb{R}^n \). Also, \( A \) is a matrix of \( \mathbb{R}^{m \times n} \) named dictionary matrix columns of which are normalized. The aim is to identify a solution using sparse representation. To solve Equation (5) a model can be found when \( x \) is known to be S-sparse for some \( 1 \leq S \leq n \), which means that at most \( S \) coefficients of \( x \) can be non-zero. In principle, only \( S \) measurements are required to reconstruct \( x \) rather than \( n \). Three criteria are related to the notion of sparsity: the \( l_0 \), \( l_1 \) and \( l_2 \) norms of \( x \). The \( l_0 \) norm is the unique sparsest solution which is the number of non-zero values of \( x \).

Unfortunately, the \( l_0 \) sparse approximation is at least as hard as a general constraint satisfaction problem even with no restrictions on \( A \) and \( b \). In fact, the problem is in general NP-hard because \( l_0 \) minimization is not a convex optimization problem and its complexity exponentially increases with the number of dictionary matrix columns [27, 28]. The \( l_2 \)-norm is in fact the least square solution of problem. Minimization of \( l_2 \) is generally less time-consuming than \( l_0 \). But the result of \( l_2 \) minimization doesn’t match that of \( l_0 \). A new convex and tractable approach has been recently presented as Basis Pursuit (BP) which uses \( l_1 \) minimization to solve the problem [29]. The \( l_1 \)-norm is the summation of the absolute value of \( x \) components. The \( l_1 \) minimization can be solved in polynomial time. BP is closely related to linear programming (LP) and it works whenever the dictionary matrix is sufficiently incoherent.

2-2 Particle Swarm Optimization (PSO)

In order to make the paper self-explanatory, the PSO characteristics are briefly explained. The PSO algorithm was first put forward by Kennedy and Eberhart [31] motivated by fish schooling and bird flocking [32]. The PSO algorithm randomly generates the initial population called particles. A velocity is assigned to each particle which is dynamically adjusted and updated according to the flying experience of its own and its companions in a g-dimensional search space. The initial velocities are also generated randomly using uniform probability distribution function. The objective
function is evaluated for the particles and their positions are updated based on the best positions of individual particles, \( p_{best} \), in each iteration and the previous best solution of the entire swarm, \( g_{best} \). The position and velocity of each particle are updated using Equations (15) and (16), respectively, as follows:

\[
\begin{align*}
    x_i(t+1) &= x_i(t) + v_i(t+1) \\
    v_i(t+1) &= w \cdot v_i(t) + c_1 \cdot r_1(\Delta p(t)) + c_2 \cdot r_2(\Delta g(t)) - x_i(t)
\end{align*}
\]

where, \( t \), \( x_i \) and \( v_i \) are the iteration number, the position and velocity of the \( i \)th particle, respectively; \( w \) is inertia weight, \( r_1 \) and \( r_2 \) are uniformly distributed random numbers in the range of (0,1) and \( c_1 \) and \( c_2 \) are the acceleration constants which update the velocity respectively toward \( p_{best} \) and \( g_{best} \).

2- The proposed algorithm: BP-PSO-MS

Practically, using an optimization approach is computationally very intensive. Hence, in this study, the search space of optimization process is reduced by basis pursuit method. Also, a micro-search which eliminates low value parameters is embedded to the optimization algorithm to decrease the computational process. In the following, the multi-stage method is presented to detect the damage locations and severities.

At the first stage, the sensitivity matrix of the structure is first calculated by the Finite Difference Method. For an element, each column of the matrix is then evaluated by a random finite difference interval as follows:

\[
S^i = \frac{R^i - R_{th}}{FDI^i}
\]

in which, \( S^i \) is the \( i \)th column of the sensitivity matrix and \( FDI^i \) is the random finite difference interval for the \( i \)th element. \( R_d \) is the vector of structural responses when the damage ratio of the \( i \)th element is equal to \( FDI^i \) and the other elements are intact. The sparse damage vector is recovered using the BP method. This process is carried out at some different random intervals to obtain a set of damage vectors. In second stage, PSO algorithm improves the initial damage vectors to the best solution. The set of damage vectors obtained from stage one is used as the first population of PSO algorithm, i.e. each damage vector is a particle of the first population. The objective function, \( OF \), of the PSO algorithm is formulated as:

\[
OF = \min (\text{Error}) \quad \text{Error}(i) = 1 - \text{MDLAC}(i)
\]

where, \( i \) is the particle number and MDLAC elements are evaluated by Equation (3). At the last stage, the damage ratios of the best particle which are less than 10% are set to zero to reduce the search space and running time and considered intact. The flowchart of the proposed algorithm is shown in Figure 1. \( N_i \) is the total number of iterations.
3- Case Study
A portal frame investigated previously by Veizaga [29] is selected. The length and height of the portal frame are \( L = 2.4 \text{m} \) and \( H = 1.6 \text{m} \), respectively. All frame elements have identical cross sectional dimensions of \( h = 0.24 \text{m} \) and \( b = 0.14 \text{m} \). The material density is assumed to be \( 2.5 \times 10^3 \text{kg/m}^3 \) and elasticity modulus is taken as \( 2.5 \times 10^{10} \text{N/m}^2 \). The finite element representation of this frame is shown in Figure 2. Each node has two translational and one rotational degrees of freedom. In this section, the proposed method is verified through a case study. In the following case study, damage is simulated by a reduction in Young’s modulus of the elements. The elements whose damage ratios exceed 10% are considered as damaged elements in the MS operator. To demonstrate the convergence of the proposed method efficiently, the objective function is multiplied by 500.

![Figure 2. Plane portal frame finite element model](image)

Three different damage scenarios given in Table 1 are considered to test the proposed method.

<table>
<thead>
<tr>
<th>Scenario no.</th>
<th>Damage element no.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E7=10</td>
</tr>
<tr>
<td>2</td>
<td>E24=30</td>
</tr>
<tr>
<td>3</td>
<td>E40=10</td>
</tr>
</tbody>
</table>

Table 2 shows the results of the algorithms in detecting the damage elements of all the scenarios using the first seven frequencies. The PSO and PSO-MS methods are unable to diagnose the damage locations in all the scenarios. The use of MS in PSO brings down the number of false damage elements. The detected damaged elements by the BP method are element 7 in scenario 1; element 24 in scenario 2 and elements 40 and 44 in scenario 3. The damage parameters are approximately predicted by the BP method. The exact damage parameters of scenarios 1 and 2 are detected by the BP-CGA method. The element 24 in scenario 3 is diagnosed as damage location but the damage extent is not exactly identified by the BP-CGA method. The modal sensitivity analysis and GA methods investigated in ref [30] predict two damage locations in all the scenarios, where one of the locations is incorrect. The scenarios 1 and 3 are also investigated by the MSSEBI and SSEBI methods proposed in ref [31]. The methods are able to detect the damage locations correctly, but the identified damage extents are incorrect. The damage extents of scenarios 1 and 3 are exactly predicted by the BP-PSO and BP-PSO-MS methods. All the investigated methods are unable to evaluate the exact damage extent of scenario 2. But the best detected damage extent is proposed by the BP-PSO and BP-PSO-MS methods.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Detected damage elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP</td>
<td>E7=10 E24=4 E40=2 E44=4</td>
</tr>
<tr>
<td>BP-CGA</td>
<td>E7=10 E24=21 E44=10</td>
</tr>
<tr>
<td>MSEBI</td>
<td>E7=22 - E44=23</td>
</tr>
<tr>
<td>SSEBI</td>
<td>E7=22 - E44=23</td>
</tr>
<tr>
<td>BP-PSO-MS (Proposed)</td>
<td>E7=10 E24=23 E44=10</td>
</tr>
<tr>
<td>Real damage</td>
<td>E7=10 E24=30 E44=10</td>
</tr>
</tbody>
</table>
Figure 3 shows the convergences of the best and the mean values of the objective function in damage scenario 1. The mean and cost values of the proposed method converge to acceptable accuracy after 20 and 10 iterations respectively. The use of MS operator in the PSO algorithm increases the convergences speeds of the best and the mean values of objective function.

![Convergence history](image)

![Damage extents and locations](image)

Figure 3. The damage detection results of portal plane frame, scenario 1 (a) convergence history (b) damage extents and locations

The algorithm converges to the exact damage parameters. A total of 1,000,000 and 162,00 objective function evaluations are respectively carried out by the proposed approach of [30] and [3] to obtain each of their results. But in the present study, the objective function of the PSO algorithm is evaluated 10,248 times for each damage scenario. Therefore, the computation cost of the proposed approach is considerably less than [30] and [3]. The algorithm results of scenario 2 is shown in Figure 4.
The proposed method converges to correct damage location and extent of scenario 2. The results of scenario 3 are shown in Figure 5.

The algorithm converges to the exact results of scenario 3. Therefore, the algorithm could identify the damage with various locations and extents accurately.

5- Results and Discussions
A portal plan frame is used to verify the proposed algorithm. Three damage scenarios which was defined are used and the results are expressed. Based on them, the proposed method could converge to the exact damage parameters even with low values. In compare with other methods, the proposed algorithm is much faster and do much less analysis.

6- Conclusions
A three stages hybrid method to solve the sparse problem of damage detection, termed BP-PSO-MS, was presented. The potentially flawed elements of damaged structure are first located using basis pursuit (BP). The results are then improved using the particle swarm optimization (PSO) method. Finally, the design variables with low values are set to zero after a number of iterations by the micro search (MS) operator embedded in the PSO algorithm. The objective function of the PSO method is multiple damage location assurance criterion (MDLAC) which compares the frequencies of real and modeled damaged structure. The performance of the proposed algorithm was investigated considering noisy modal responses of real damaged structure.
References