An Efficient Eddy-Current Method for Online Determination of the Radius of a Finite-Length Conductive Rod

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Abstract
In this paper, an efficient method is proposed to determine the radius of a cylindrical conductive rod of finite length, using eddy current data. In this method, first, the output signals of an eddy current probe in the presence of the rod is simulated using a finite element solver. A fast regression method is then introduced for online calibration of the probe, relating the probe output signal to the rod radius for various probe standings. The validity of the proposed method is demonstrated using various simulation and experimental results.

Keywords: Eddy Current Probe, Cylindrical Conductive Rod, Finite Length, Edge Effect, Radius Measurement, Regression method, COMSOL

1- Introduction
Eddy current testing is a widely used nondestructive method for on-line characterization of conductive materials [1]. When the probe approaches a test object, interrogating eddy currents are produced in the test object by an exciting coil, and the resultant perturbation electromagnetic field cause the coil impedance to change. The impedance change in the coil is analyzed to acquire information about the test object [2].

The theoretical modeling of eddy current problems plays an important role in the proper design of probe and inspection procedure as well as producing data to calibrate probe for defect sizing, material characterization, and dimensional control measurement. In cases where the inspection is to be done in the vicinity of the edge of a test object, the eddy currents are displaced by the edge, affecting the probe output signal [2]. To avoid erroneous results, one needs to consider the edge effect when calibrating the eddy current probe [3].

In this paper, we simulate the output signal of an eddy current probe in the vicinity of a cylindrical conductive rod of finite length using COMSOL Multiphysics software [4]. The so-obtained probe data are used to determine the rod radius in automated online applications by an efficient regression algorithm. The proposed method can play an important role in online inspection of finite metallic shafts and pins. It can also be extended for use in quality control of a variety of automotive parts, including gudgeon pin, tappet, and objects with complex geometries. As a supplementary tool, the proposed method can be used in conjunction with an eddy current defect detection probe for monitoring the dimensional tolerance of a test object.

The paper is organized as follows. The theory modeling of the problem is outlined in Section 2 and the proposed regression-based method for sizing the rod radius is described in Section 3. The validity of the proposed method is demonstrated in Section 4 where the predicted values of several rod radii are compared with their actual values in both simulated and experimental test environments

2- Theoretical Modeling

![Image](https://via.placeholder.com/150)

Figure 1: A current-carrying coil in the vicinity of a finite length conductive rod.
The problem is schematically depicted in Figure 1. An eddy-current probe, consisting of a horizontal $N$-turn coil of length $l$ and inner radius $r_2$ and outer radius $r_2$, whose center is placed at $z = \tau$ in the vicinity of a conductive cylindrical rod of length $c$ and radius $\rho_a$ with conductivity $\sigma$ and permeability $\mu$. The coil is excited by an ac current of amplitude $I$ and frequency $f$. The objective is to obtain the impedance variation of the coil for various values of the rod radius when scanning the along the $z$-axis of the rod.

Derive from the governing Maxwell equations, the magnetic vector potential, $\mathbf{A}$, can be obtained as follows [5],

$$\nabla^2 \mathbf{A} - j\omega \mu \sigma \mathbf{A} = -\mu \mathbf{J}_s; \quad (1)$$

where, $J_s = \frac{NI}{i(r_2 - r_2)}$ is the coil current density.

To solve the boundary problem describe above, we adopt the finite element (FE) method. The problem is treated using the well-known using COMSOL Multiphysics FE solver.

Having determined the values of $\mathbf{A}$, the induced current density, $\mathbf{J}_{\text{ind}}$, inside the rod can be computed as follows,

$$\mathbf{J}_{\text{ind}} = -j\omega \sigma \mathbf{A} \quad (2)$$

Also, the coil impedance (probe output signal), $Z = R + j\omega L$ can be obtained, using joule loss $(JL)$ and magnetic energy $(W_m)$, respectively, i.e.,

$$R = \frac{2jL}{I^2} = \frac{1}{I^2} \int_V \mathbf{J} \cdot \mathbf{J} \sigma \, dV \quad (3)$$

$$L = \frac{2W_m}{I^2} = \frac{1}{I^2} \int_V \mathbf{B} \cdot \mathbf{B} \mu \, dV \quad (4)$$

where, $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic flux density, $\mathbf{J} = \mathbf{J}_{\text{ind}} + \mathbf{J}_s$ denotes the current density, and $V$ represents the volume surrounding the coil.

3- Proposed Method

To derive the relationship between the eddy current probe data ($\Delta X$ or $\Delta R$) for various probe location ($\tau$) and the rod radius ($\rho$), we adopt a third polynomial regression algorithm. The algorithm utilizes the least square error method to fit a set of $n$ values of $\Delta X$ to $n$ values of $\rho_a$, using a third polynomial, i.e.,

$$\rho_a = a_1 + a_2 \Delta X + a_3 \Delta X^2 + a_4 \Delta X^3 \quad (5)$$

The can be done by adopting the method of Vandermonde matrix [6], relating

$$Y = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{bmatrix}_{N \times 1} \quad (6)$$

to

$$X = \begin{bmatrix} \Delta X_i \\ \Delta X_i^2 \\ \Delta X_i^3 \end{bmatrix} \quad (7)$$

where $\rho_i (i = 1: N)$ represents the rod radius.

Applying the pseudoinverse transportation, the unknown coefficients, $a = [a_0, a_1, a_2, a_3]^T$, can be determined as follows,

$$a = (X^T X)^{-1} X^T Y \quad (9)$$

4- Results and Discussion

To examine the validity of the proposed method, we present the results of several simulated and experimental tests. The specifications of the eddy current probe (coil) and sample rods in these tests are given in Table 1.
4-1- Simulation Results

The boundary value problem, describing the electromagnetic interaction of the coil-rod (specified in Table 1), has been solved by COMSOL Multiphysics software. Fig. 3 shows the distributions of eddy currents within the rod for two locations of the coil, namely, \( \tau = 0 \) and \(-5 \text{ mm}\). As can be seen in this figure, the induced eddy currents are severely disturbed as the coil approaches the edge of the rod. This phenomenon can drastically affect the probe output signal (coil impedance) as can be seen in Fig. 4 where variations of the real (\( \Delta R \)) and imaginary (\( \Delta X \)) parts of the coil impedance are plotted for various locations of the probe (\(-20 \text{ mm} < \tau < 10 \text{ mm}\)). A study of the results shown in this figure indicates that one cannot adopt the conventional calibration process where long reference rods are used to relate the probe data to the radius of a sample rod. In other words, the location of the probe with respect to the sample plays an important role for achieving accurate measurements.

Now, we provide a quantitative comparison of the actual and predicted radius, \( \rho_{ae} \), based on the regression method described in Section 3. Referring to Table 1, COMSOL Multiphysics is used to create four data sets which are applied to the proposed regression algorithm. Each set is associated with the normalized value of the coil impedance at \( \tau = -5 \text{ mm} \) where the normalizing factor is the coil inductance in free space. The selected data sets constitute rods with different radius \( \rho_a = [1; 0.5; 5] \) and coil location at \( \tau = -5 \text{ mm} \). Having determined the polynomial relating \( \Delta X \) to \( \rho_a \), the radii of three rods (\( \rho_a = 1.25 \text{ mm} \), 3.05 mm and 4.13 mm) are obtained. A comparison of the predicted values and their actual counterparts given in Table 2 clearly demonstrate the validity of the proposed method.

<table>
<thead>
<tr>
<th>Table 1: Eddy-current probe (coil) and finite rod parameters</th>
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<tbody>
<tr>
<td><strong>Coil</strong></td>
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<td>Inner radius ( (r_1) )</td>
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<tr>
<td>Outer radius ( (r_1) )</td>
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<tr>
<td>Length ( (l) )</td>
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<tr>
<td>Number of turns ( (N) )</td>
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<td>Lift-off ( (l_0) )</td>
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<td>Frequency ( (f) )</td>
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<td>Current ( (I) )</td>
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![Figure 2: Induced current density norm (\(|\mathbf{J}_{\text{ind}}|\)) when the coil center position (\(\tau\)) is at a) zero and b) \(-c/2\).](image-url)
Figure 3: Variations of the simulated and experimental probe (coil) output signals when scanning the rod specified in Table 1 with radius $\rho_a = 5 \text{mm}$ along its axis ($z$ axis) at a lift-off distance $l_o=1\text{mm}$ and different probe locations, $\tau$; (a) the normalized real part, $\Delta R/\mathcal{X}_0$, and (b) the normalized imaginary part, $\Delta \mathcal{X}/\mathcal{X}_0$, of the probe output signal.

Table 2: Comparison of the predicted values of rod radius and their actual values, using the simulated probe (coil) data.

| Coil Location, $\tau$ (mm) | Normalized Coil Impedance, $Z$ | Actual Rod Radius, $\rho_a$ (mm) | Predicted Rod Radius, $\rho_{ap}$ (mm) | Relative Error $\left|\rho_a - \rho_{ap}\right| \times 100$ |
|---------------------------|-------------------------------|---------------------------------|--------------------------------------|----------------------------------|
| -5                        | 0.0117 - 0.0088i               | 1.25                            | 1.29                                 | 3.4                              |
| -5                        | 0.0188 - 0.0255i               | 3.05                            | 3.04                                 | 0.1                              |
| -5                        | 0.0207 - 0.0298i               | 4.10                            | 4.13                                 | 0.8                              |

4-2- Experimental Results

To further verify the proposed method, we use the set-up shown in Fig. 5. It comprises a commercial eddy current instrument [7], an eddy-current probe, and three samples. The instrument employs an auto-balance bridge circuit to perform tests over the frequency 5 Hz to 5MHz. The probe is a multi-turn coil (specified in Table 1) and is equipped with a specially designed nonmetallic fixture (Fixture I in Fig. 5) to maintain a constant lift-off distance ($l_o=1\text{mm}$). The samples are three aluminum finite length rods (specified in Table 1) with $\rho_a=3\text{mm}$, 4.5mm and 5mm.

In the first test, the probe mounted on Fixture I is used to manually scan the sample rod with $\rho_a=5\text{mm}$ along its axis ($-20\text{mm} < \tau < 10\text{mm}$). A study of the probe output signals shown in Fig. 4 indicates a good agreement between the simulated and experimental results. It also emphasizes the importance of the probe location on accurate measurement of the rod radius.

For radius measurements, tests are performed on the other two sample rods ($\rho_a=3\text{mm}$, 4.5 mm). In these tests, the probe mounted on Fixture I is placed into a second nonmetallic fixture (Fixture II in Fig 5), ensuring that it is located at the middle of the sample ($\tau=\ell/2$). Using the polynomial relating $\Delta \mathcal{X}$ to $\rho_a$ (obtained in Section 4.1) is used to determine the radii of the two sample rods. The results are given in Table 3 where a comparison of the predicted and actual values further substantiate the accuracy of the proposed method.
Figure 5: The experimental set-up: the commercial eddy-current instrument [7], the eddy-current probe (coil), Fixture I to maintain the probe lift-off at \( l_0 = 1\) mm, Fixture II to ensure the probe location at \( r = c/2\), and a sample rod of radius 4.5.

Table 3: Comparison of the predicted values of rod radius and their actual values, using the experimental probe (coil) data.

| Coil Location, \( r \) (mm) | Normalized Coil Impedance, \( Z \) | Actual Rod Radius, \( \rho_a \) (mm) | Predicted Rod Radius, \( \rho_{ap} \) (mm) | Relative Error \( \frac{|\rho_a - \rho_{ap}|}{\rho_a} \times 100 \) |
|-----------------------------|----------------------------------|------------------------------------|------------------------------------|----------------------------------|
| -5                          | 0.013 - 0.012i                    | 3.0 mm                             | 3.06                               | 2.2                              |
| -5                          | 0.016 - 0.020i                    | 4.5 mm                             | 4.43                               | 1.5                              |

5- Conclusions

An efficient method has been proposed to predict the radius of a finite conductive rod in an online inspection task. The proposed method adopts a third polynomial regression algorithm that relates the output signal of an eddy-current probe to the rod radius for various positions of the eddy current probe. The algorithm is trained using simulated data using finite element electromagnetic solver. The validity of the proposed method has been demonstrated by comparing the predicted and actual values of rod radius, using several simulations and experimental test results.

References


