An Inverse Problem for Grain Size in Low Carbon Steels and Ultrasonic Measurements

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Abstract
Low carbon steels are considered in this article. It is often the case in practice to determine the average size of the grains \( D \) in them [1]. The non-destructive evaluation /NDE/ of \( D \) [2] is interest.

For a sample from normalized carbon steel, \( \overline{D}, \text{mm} = 0.022 \pm 3\% \) by metallographic analysis is determined.

For it, they are measured \( (V_L; V_T) \) [2,4]. With the ZEROIN [5] program, the NDE of \( \overline{D}, \text{mm} \) by measured \( (V_L; V_T) \) is calculated.

Keywords: Inverse Problem, Grain Size, Low Carbon Steel, Ultrasonic Measurement

1. Introduction

The carbon steels in machine building are the most commonly. It is known [1,3,4] that the relationship \( \sigma_{YS}(E) \) exists, where \( (E; \sigma_{YS}; \ldots) \), where \( E = E(V_L; V_T) \) – Young modulus \( \sigma_{YS} = \sigma_{YS}(\overline{D}) \) – yield stress. Here \( (V_L; V_T) \) – velocities of longitudinal and transversal ultrasonic waves, \( \overline{D} \) – average of ferrite grains. This is the direct problem.

An interest is the inverse problem, namely “Determining the average size of the grains \( \overline{D} \) in carbon steels by measuring of \( (V_L; V_T) \)”. 

2. Theory

2.1. Relationship \( \sigma_{YS}(D) \)

The semi-empirical relationship of Hall-Petch is considered [3]

\[
\sigma_{YS} = \sigma_0 + K_y (\overline{D})^{-1/2}
\]

where \( \sigma_{YS} \) – low limit of yield stress (\( \sigma_0 = 72MPa; K_y = 23.9MPa\text{mm}^{1/2} \) [3] for low carbon steels).

2.2. Regression model \( \sigma_{YS}(E) \)

For carbon steel with a carbon content (0.10 to 0.35) % C, dependence \( \sigma_{YS}(E) \) was obtained. The graphics is shown in Fig.1. It is shown 95% confidence intervals for the probability measurements.
Fig. 1. Polynomial regression for carbon steels, where $YS\, MPa = \sigma_{YS} MPa$

It can be seen, that the trend of experimental data is approximated by polynomial regression (PR), (2).

$$\sigma_{YS} = \sum_{K=0}^{2} \beta_{K} E^{K}$$

(2)

3. Inverse Problem for $\overline{D}, mm$

After equalizing (1) and (2), the equation $\overline{D}, mm$ is obtain

$$K_{y} (\overline{D})^{n} + \psi (V_{L}; V_{T}) = 0$$

(3)

where $n = -1/2$; $\psi (V_{L}; V_{T}) = \sigma_{0} - \sum_{K=0}^{2} \beta_{K} E^{K}$; $E = \frac{3 - 4(V_{T}/V_{L})^{2}}{1 - (V_{T}/V_{L})^{2}}$ [4].

The parameters in equation (3) are explained in dependence (1) and Fig.1. The parameters in (3) $\{K_{y}; \sigma_{0}; \beta_{0}; \beta_{1}; \beta_{2}\}$ are explained in dependence (1) and Fig.1. The equation (3) is non-linear. An effective method for its solution is the method of the bisection (Newton, 1669). The method is implemented with the algorithm ZEROIN (Brent, 1973) [5]. This algorithm combines the reliability of slotting with the asymptotic speed of the the souls method. The number of iterates for the implementation of the algorithm is $N \sim \log_{2}[(b-a)/TOL]$ where $TOL \sim 10^{-6}$ – uncertainty and $(a;b)$ is the root search interval of the equation (3).

4. Equipment

For ultrasonic measurements the following equipment shall be used: Digital ultrasonic flaw detector SITESCAN 150S, Sonatest, England, transducers 5 MHz, with X-cut and Y-cut of piezo-plates, Panametrics, USA. Calibration block $(V'_{L} = 5.93 \, mm, \, \mu s)$, Sonatest, England, Digital micrometer Digimatic, Mitutoyo, Japan. With this micrometer, measurements are made with accuracy $\pm 0.5 \, \mu m$. 

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5. Experiment

A sample of steel grade 20, normalized, with dimensions \( h = 25 \text{ mm}, a = 20\text{mm}, b = 150\text{mm} \) are measured. The velocities \((v_L; v_T)\) in the sample according to [4] are measured. The results are \((v_L = 5.92 \pm 0.03; v_T = 3.25 \pm 0.015 \text{ mm/μs})\). Transducers with frequency 5MHz, X-cut and Y-cut of piezoplastins are used. The value \(D_{mm}\) is determined by the measured velocity \((v_L; v_T)\) and the solution of (3), received by the ZEROIN program, at a given root interval \([a=10^{-6}; b=0.05]mm\).

A metallographic shliph is made. The reference value \(D_{mm}\) for the metallographic microscope at 100x magnification is determined. The results of measured are given in Table 1.

<table>
<thead>
<tr>
<th>The calculated value / NDT evaluation /</th>
<th>The reference value / Metallographic evaluation /</th>
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<tbody>
<tr>
<td>(D_{mm} = 0.020218)</td>
<td>(D_{mm} = 0.022 \pm 3%)</td>
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</table>

The calculated value for \(D_{mm}\), is by five real orders because in ZEROIN program the uncertainty is \(TOL \sim 10^{-6}\). The number of iterations is \(\sim 117\) (in this case the time of work of the program ZEROIN is \(\sim 5\)s). The error of NDE of \(D_{mm}\) is 0.68%.

6. Conclusion

The article formulated and solved the inverse problem of NDE the average of grain size – \(D_{mm}\) in a low carbon steel by using ultrasonic measurements – \((v_L; v_T)\). The solution for \(D_{mm}\) is obtain. It is decided by the ZEROIN algorithm. In this case \(D_{mm} = 0.020218\) is obtained. The error in NDE, by measuring velocities \((v_L; v_T)\), is less than 1%.

Reference