Nonlinear Acoustic NDT: Approaches, Methods, and Applications

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Conventional (linear) acoustic NDT:

- Amplitude and phase variation of input wave

Nonlinear acoustic response

- Frequency conversion

Nonlinear NDT (NNDT)
Outline

- Introduction: Classical nonlinear NDT & frequency conversion in homogeneous materials

- „Mechanical diode“ and nonlinear resonance approaches to nonlinear spectra of planar defects

- Experimental nonlinear spectra of cracked defects

- Methods & case studies of multi-frequency NNDT

- Conclusions
Classical nonlinearity of homogeneous (intact) materials

Lattice anharmonicity

\[ \sigma(\varepsilon) = C^{II} (1 - \beta_2 \varepsilon - \beta_3 \varepsilon^2 - \ldots) \varepsilon \] - generalized Hooke’s law

Nonlinearity: Stiffness depends on strain

\[ c(\varepsilon) = c_0 (1 - \beta_2 \varepsilon - \frac{3}{2} \beta_3 \varepsilon^2 - \ldots) \] velocity depends on local strain due to stiffness modulation
Classical NNDT & frequency conversion in homogeneous materials

Waveform distortion and higher harmonic generation is a measure of material nonlinearity

Power-law dynamics:

\[ U_{2\omega} \sim U_{\omega}^2; \quad U_{3\omega} \sim U_{\omega}^3 \]

\[ \varepsilon \approx 10^{-5} - 10^{-4} \]

Second harmonic

\[ \frac{U_{2\omega}}{U_{\omega}} \leq 1\% \]

Classical NNDT is a "second harmonic" NDT
Nonlinear spectrum of a delamination in CFRP

Planar defects exhibit specific nonlinearity: Contact Acoustic Nonlinearity (CAN)
Nonlinear spectra of planar defects

“Clapping” CAN mechanism:
Asymmetrical stiffness modulation

Mechanical diode model

Pulse-type stiffness modulation
Clapping mechanism: higher harmonic generation by defects

\[ \Delta C(t) = H(\varepsilon) \cdot \Delta C = H(\varepsilon_0 \cos \nu_0 t - \varepsilon^0) \cdot \Delta C \]

\[ \sigma^{NL}(t) = \Delta C(t) \varepsilon = 2 \Delta C \left( \frac{\tau}{T} \right) (\varepsilon_0 \cos \nu_0 t - \varepsilon^0) \sum_{n=1}^{\infty} \text{sinc} \left( \frac{n \tau}{T} \right) \cos n \nu_0 t \]

\[ \sigma^{NL}(t) = \sum_{N=1}^{\infty} A_N \cos N \nu_0 t. \]

CAN non-classical features (clapping mechanism):

- Amplitude threshold
- Higher-order NLT
- Sinc-type spectrum modulation
- Both odd- & even HH
- Non-power dynamic behavior
- Rectified nonlinear waveform
Higher harmonics via micro-slip

Symmetrical stiffness modulation (tangential clapping)

\[ \sigma^{NL}(t) = \Delta C(t) \varepsilon = \sum_{N=0}^{\infty} A_{2N+1} \cos((2N+1)\nu_0 t) \]
Higher harmonics via micro-slip

\[ \sigma^{NL}(t) = 2\Delta C\varepsilon_0 \left( \frac{\tau}{T} \right) \sum_{n=1}^{\infty} \frac{2n\tau}{T} \left[ \cos(2n+1)v_0t + \cos(2n-1)v_0t \right] \]

\[ \equiv \sum_{N=0}^{\infty} A_{2N+1} \cos(2N+1)v_0t. \]

\[ A_{2N+1} = 2\Delta C\varepsilon_0 \left( \frac{\tau}{T} \right) \left[ \text{sinc} \left( \frac{2N\tau}{T} \right) + \text{sinc} \left( \frac{2(N+1)\tau}{T} \right) \right] \]

Nonclassical features (micro-slip mechanism):

- Symmetrical waveform distortion
- Only odd harmonics
- Sinc-type spectrum modulation
- Non-power dynamics
Nonlinear friction mechanism

Stick-and-slip: Symmetrical nonlinearity

Spectrum: \( \sigma_{NL}(t) = \sum_{N=0}^{\infty} B_{2N+1} \cos((2N+1)\nu_0 t). \)

\[
B_N = C_s \varepsilon_0 [\Delta \tau_f (\text{sinc}(N - 1)\Delta \tau_f + \text{sinc}(N + 1)\Delta \tau_f) - 2\Delta \varepsilon \text{sinc}(N\Delta \tau_f) - 0.5(1 - \Delta \varepsilon) \text{sinc}N/2], \quad N = 2n + 1
\]
Summary of CAN mechanisms

CAN is due to local mechanical constraint of vibrations

Strong step-wise stiffness variation:

- **Clapping**
  - \( C_{\varepsilon} \)
  - \( C_{II} \)
  - \(-\varepsilon \leq \varepsilon \leq \varepsilon \)

- **Micro-slip (asperities)**
  - \( C_{\varepsilon} \)
  - \( C_{II} \)
  - \(-\varepsilon \leq \varepsilon \leq \varepsilon \)

- **Stick-and slide**
  - \( C_{\varepsilon} \)
  - \( C_{II} \)
  - \(-\varepsilon \leq \varepsilon \leq \varepsilon \)

**CAN:**
- strong, high-order
- local
- no accumulation
- specific nonlinear waveforms
- non-power dynamics

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Experimental spectra of planar defects

Impact in CFRP

Clapping mechanism

20 kHz excitation
Laser vibrometer detection

0 50 100 150 200 250
Time (μs)
Experimental spectra of planar defects

Friction (hysteretic) mechanism

Intact wood

Damaged wood

Friction (hysteretic) mechanism
Frequency conversion via CAN: subharmonic mode

Equation of motion

\[ \ddot{X} + \omega_0^2 X = f(t) + F^{NL}(X) \]

\[ X = X^{(1)} + X^{(2)} + \ldots \]

\[ X^{(1)} = A \cos \omega_0 t + B \cos \nu t \]

\[ X^{(2)} + \omega_0^2 X^{(2)} = F^{NL}(X^{(1)}) \]

\[ F^{NL} \approx \cos(\nu - \omega_0) t \]

\[ \nu - \omega_0 \approx \omega_0 \]

\[ \omega_{out} = \omega_0 \approx \nu / 2 \]

Higher-order NL terms

\[ m\nu - n \omega_0 \approx \omega_0 \]

\[ \omega_0 \approx m\nu / 2 \]

Ultra-subharmonics (USB)
Frequency conversion via CAN: frequency pair modes

Combination resonance

\[ \nu - \omega_\alpha \approx \omega_\beta \]

\[ \nu - \omega_\beta \approx \omega_\alpha \]

Higher-order NL terms

\[ F^{NL}(\omega) \sim \sum_{m,n,p} (n\nu + m\omega_\alpha + p\omega_\beta) \]

Resonance decay instability

Ultra-frequency pairs (UFP) (forerunner of chaos)

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Dynamics of frequency conversion

Bifurcation of USB - UFP decay

Chaos

Ultra-frequency pairs (UFP)

Ultra-subharmonics (USB)

Higher harmonics (HH) and frequency mixing

Threshold instability modes

Excitation frequency

Excitation amplitude

velocity amplitude, mm/s

Driving amplitude, µm

Frequency, kHz

Velocity amplitude, mm/s

Driving amplitude, µm
Non-harmonic frequency conversion: examples

- Ultra-subharmonic spectrum
- Ultra-frequency pair spectrum
- Transition to chaos

Impact in GFRP

Crack in PMMA

Delamination in C/C-SiC composite

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Defect-selective localization of nonlinear vibrations

A crack in polystyrene: Higher harmonic spectrum

**CAN frequency conversion is defect-selective**
CAN applications: Defect-selective imaging via Nonlinear Laser Vibrometry (NLV)

- Scan area
- Defect (Crack)
- Scan Tool
- Laser-Interferometer
- Excitation (Piezo-Stack)

FFT of local acoustic field

Delamination in C-C/SiC ceramic
Applications: Higher harmonic defect-selective imaging

Gear clapping in commercial cutting tool

Oval delamination in GFRP

Fatigue crack in riveted aviation component
CAN applications: Ultra-subharmonic NDE

5µm-fatigue crack in Ni-base super-alloy

Delaminations in Glass fibre reinforced aluminum laminate (Glare®)

6%-plastic tensile deformation in a steel component

35th ultra-subharmonic image
Ultra-frequency pair (UFP) NDE of impacts

Impact in multi-ply GFRP

UFP (198.8 kHz) mage

UFP- spectrum

Delamination area in GFR-concrete slab (15x30x1.5cm)
Nonlinear Air-Coupled Emission (NACE)

$M = 8 \cdot 10^{-5}$

Higher harmonics

$M = 10^{-4}$

USB + UFP

$M = 1.2 \cdot 10^{-4}$

UFP

40 kHz

½” condenser mic

Spectrum analyzer

Airborne ultrasound radiated by nonlinear defect vibrations
Evidence for NACE via Air-Coupled Vibrometry

Input voltage at frequency $\Omega$

FFT of output signal $\rightarrow$ Image of NACE field
Evidence for NACE

Damaged CFRP rod

NDT & defect imaging via “nonlinear listening”?
NACE setup for defect imaging

Input frequencies \( \Omega = 20, 40 \text{ kHz} \)

ACU output frequency \( \sim 400 \text{ kHz} \)

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NACE vs Nonlinear Laser Vibrometry

Impact damage in multi-ply (+45° -45°) CFRP

Second harmonic NLV image

9th – 11th harmonic NACE image

Quite similar sensitivity and resolution
Applications: NACE for nonlinear NDE of defects (metals)

50µ-crack in steel plate

NACE-image

Hammer peening area in steel
Applications: NACE for nonlinear NDE of defects (composites)

10 x 40 mm simulated subsurface delamination in 3mm-thick CFRP

ACU-transmission image

9th-11th NACE image

Delamination in GFRP
Applications: NACE for nonlinear NDE of constructional materials

Surface-cutting crack in wooden plate

Laser welding line in steel

Zoom-in: folded structure
Conclusions

• Highly nonlinear contact dynamics provides anomalously efficient frequency conversion by planar defects

• Nonlinear defects are localized sources of new frequency components that makes nonlinear NDT defect-selective

• NLV & NACE are sensitive and robust methods for NNDT and defect-selective imaging

• Analysis of multi-frequency spectra of planar defects improves reliability and quality of NDT
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