HIGHER ORDER STATISTICS IN MAGNETOACOUSTIC NDT

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Abstract
The present work suggests the application of the higher order statistics in signal analysis of the magnetoacoustic emission, in order to find some correlations with the size and distribution of defects in ferromagnetic samples.

Keywords: Kurtosis, Spectral Kurtosis, Magnetoacoustic emission, Nondestructive evaluation

1. Introduction

Kurtosis is a magnitude used in higher order statistics and can be evaluated by means of the fourth central moment of a distribution and the fourth power of standard deviation. If we analyze a time signal \( x(t) \) with a Gaussian distribution in time of its amplitude, the value of the Kurtosis is 3. Because many signals we meet in practice tend to have such a distribution of the amplitude, the Excess Kurtosis is used as well, computed by subtracting the constant 3 from the value of the Kurtosis, so we obtain a Kurtosis estimate, expected to be zero in the case of Gaussian distribution of experimental data, and non-zero for other distributions. This gives a very simple criterion to describe and separate Gaussian and non-Gaussian signals. However, when applied to a time domain signal, Kurtosis analysis may indicate only the presence of non Gaussian signals mixed in the full bandwidth of the analyzed signal, without any spectral discrimination. To achieve spectral channel discrimination, a frequency domain statistics may be employed based on the Spectral Kurtosis (SK) estimator, defined as the ratio of the second moment of the instantaneous Power Spectral Density (PSD) and the square of its first moment, which is expected to be 1 when derived from a Gaussian time domain signal by the means of a Fast Fourier Transform (FFT).

Time Domain Kurtosis (TDK) is mostly used in industrial diagnosis of some phenomena which can produce damages of the system. For example, Kurtosis was used for blind deconvolution separation of multiple sources mixed by mechanical systems, in fatigue analysis, or in a mixed method based on Kurtosis analysis and non-Gaussian projections to explore the clustering structure of the experimental data. Using supplementary computer simulations and Kurtosis estimation the crack detection of thin isotropic rectangular plates and the effect of the added noise are examined.

Our work proposes a computational method to obtain experimental data, to calculate SK and an intuitive representation of SK in accord to NDT requirement.
2. Spectral kurtosis – theoretical considerations

2.1 Cumulants and higher order moments

Many recently published papers show the importance of higher order correlations and moments in both theoretical and experimental studies. We can mention in this context the statistical description of fluctuations for systems in the presence of fields [1], critical phenomena and phase transitions etc. In these cases the higher order correlations refer to the considered random variables, e.g. energy, magnetization, magnetic induction, electrical current, voltage etc. The progress of the computational technique allows the complex calculation in real time, so this fact potentates data processing and taking the suitable decision during the processes.

For one non-gaussian process or a non-stationary signal, usually we calculate the higher order estimators, in order to accurately describe the studied phenomena. One tool in this regard is Kurtosis (K) and Spectral Kurtosis (SK), as statistical estimators.

Let $\Phi(t)$ be the characteristic function defined as the Fourier transform of the probability density function $\rho(x)$.

$$\Phi(t) = \int_{-\infty}^{\infty} e^{itx} \rho(x) dx \quad (1)$$

The cumulants of a random variable $x$ are defined by the logarithm of the characteristic function.

$$g(t) = \ln \Phi(t) = \sum_{n=1}^{\infty} k_n \frac{(it)^n}{n!} \quad (2)$$

The cumulants are given by the derivatives of $g(t)$ at zero:

$$k_n = g^{(n)}(0) \quad (3)$$

Let us denote by $\mu_n$ the central moments, i.e.

$$\mu_n = \left\langle (x - \langle x \rangle)^n \right\rangle \quad (4)$$

where $\langle x \rangle$ represents the expectation value of the random quantity $x$.

In terms of central moments we have

$$k_1 = \mu_1$$
$$k_2 = \mu_2$$
$$k_3 = \mu_3$$
$$k_4 = \mu_4 - 3\mu_2^2 \quad (5)$$
The Kurtosis is given by

\[ K = \frac{k_4}{k_2^2} \quad (6) \]

2.2 The definition of Spectral Kurtosis in the case of stationary signals

We consider a real discrete time random process described by the physical variable \( x(n) \). Let \( X(m) \) be the \( N \)-point Discrete Fourier Transform (DFT) of the variable \( x(n) \).

\[ X(m) = \sum_{n=0}^{N-1} x(n) e^{-\frac{2\pi inm}{N}} \quad m = 0,1,\ldots,N-1 \quad (7) \]

The Spectral Kurtosis (SK) of \( x(n) \) is defined as the Kurtosis of the complex random variable \( X(m) \) at each frequency bin \( m \) \([2,3]\):

\[ SK_x(m) = \frac{k_4 \left\{ X^+(m), X^+(m), X^+(m), X^+(m) \right\}}{\left[ k_2 \left\{ X^+(m), X^+(m) \right\} \right]^2} \quad (8) \]

where

\[ X^+(m) \in \{ X(m), X^*(m) \} \quad (9) \]

If we consider that \( x(n) \) is a stationary random process, then the DFT \( X(m) \) is a circular complex random variable at each frequency bin. So the only definition of SK that gives a non-null value is \([3]\)

\[ SK_x(m) = \frac{k_4 \left\{ X(m), X^*(m), X(m), X^*(m) \right\}}{\left[ k_2 \left\{ X(m), X^*(m) \right\} \right]^2} \quad (10) \]

In terms of expectation values the above relation can be transformed as follows \([2]\):

\[ SK_x(m) = \frac{\langle |X(m)|^4 \rangle - 2 \left[ \langle |X(m)|^2 \rangle \right]^2}{\left[ \langle |X(m)|^2 \rangle \right]^2} \quad (11) \]

We observe that in the above relation appears the factor 2 instead of 3 as in the usual definition of cumulants, because \( X(m) \) is a circular random variable \([3,4]\).
2.3 The definition of spectral kurtosis in the case of non-stationary signals

Let us consider stochastic non-stationary process. Using the Wold-Cramer decomposition this kind of process can be described mathematically as [4]

\[ Y(t) = \int_{-\infty}^{\infty} e^{2\pi i f} H(t, f, \omega) dX(f) \]  \hspace{1cm} (12)

We will assume that the transfer function \( H(t, f, \omega) \) is independent of the spectral process \( dX(f) \). So the process \( Y(t) \) will be stationary in general, but non-stationary for any particular outcome \( \omega \). Such a process is called conditionally non-stationary (CNS).

We will introduce the spectral moments by means of the relation

\[ S_{2nY}(f) = \left( \left| H(t, f) dX(f) \right|^{2n} \right) = \left( \left| H(t, f) \right|^{2n} \right) S_{2nY} \]  \hspace{1cm} (13)

where the averaging is done on many outcomes \( \omega \).

The Spectral Kurtosis is given by the relation [4]

\[ SK_Y(f) = \frac{S_{4Y}(f)}{[S_{2Y}(f)]^2} - 2 \]  \hspace{1cm} (14)

2.4 Estimation of Spectral Kurtosis

Let us consider that we divide our signal \( x(n) \) into \( M \) unoverlapped blocks, each of them having the length \( N \). We will perform an \( N \) -point Discrete Fourier Transform on each segment. This way we obtain the random variables \( X_i(m), (i = 1, 2, \ldots, M) \).

An unbiased estimator of the spectral kurtosis of \( x(n) \) is given by the relation [3]

\[ K_x(m) = \frac{M}{M-1} \left[ \frac{(M+1) \sum_{i=1}^{M} \left| X_i(m) \right|^4}{\left( \sum_{i=1}^{M} \left| X_i(m) \right|^2 \right)^2} - 2 \right] \]  \hspace{1cm} (15)

2.5 Transient signal detection by SK

The estimation of SK of complex signals which are obtained from transducers during the evolution of physical phenomena can be used in the detection of the transient phenomena. The criterion is the value of the SK which will be different for a transient stationary signal and other kind of signal. To illustrate this, we build a complex signal consisting from white noise combined in three time intervals with three stationary sine
signals with the frequencies 1 KHz, 10 KHz and 20 KHz. The graph with the value of the SK expressed in color scale is given in Figure 1.

![Figure 1 Detection by SK of three transient stationary signals](image)

3. **The application of the SK to study the magnetoacoustic emission**

   The magnetoacoustic emission (MAE) is an elastic phenomenon produced by wall movement of the magnetic domains in a ferromagnetic sample. Very often the walls of the magnetic domains are pinning around the defects from the ferromagnetic sample, therefore the existence of internal mechanical stress, crystal dislocations, defects or clusters of defects will change the magnetoacoustic signal.

   To measure the magnetoacoustic effect, the ferromagnetic sample is placed in an external continuous magnetic field and a slow alternative signal. The magnetoacoustic signal was detected by a noncontact method based on laser Doppler interferometry, the signal was amplified and acquired in the computer by an acquisition board NI DaqPAD 6015.

   To calculate Spectral Kurtosis (SK), the acquisition must have a special particular form by building clusters of subspectra which will be averaged. So, in our case we accumulated for example a number of 50000 spectra, each spectrum consisting of 500 subspectra with 1024 samples, the frequency of acquisition being 50000 Hz. For each subspectrum from one spectrum, we calculated Power Spectral Density (PSD) and the square of PSD, PSD². The SK algorithm was described previously by Nita et al.
To compute the SK estimator, one must accumulate sums of power and power-squared

\[ S_1 = \sum_{m=1}^{M} P_{k,m}, \quad S_2 = \sum_{m=1}^{M} P_{k,m}^2; \quad k = \text{frequency channel} \]  

(16)

The SK estimator for an accumulation over \( M = 500 \) samples is then:

\[ SK = \frac{M}{M - 1} \left( \frac{M}{S_1} \cdot \frac{S_2}{S_1^2} - 1 \right), \quad (\approx 1 \text{ for Gaussian noise}) \]  

(15’)

The variance of the SK estimator is \( \frac{4}{M} \), so with a criterion of \( \pm 3\sigma \) an accumulation in spectral channel \( k \) is Gaussian if it obeys the expression

\[ |SK - 1| < 3 \sqrt{\frac{4}{M}} \]

We studied the magnetoacoustic signal for three different materials. So, in Figure 2 is illustrated the value of SK versus time for a Ni polycrystalline sample. The first peak from figure corresponds to the signal of global movement of all magnetic walls at the external magnetic field application. The sample consists from a special sample of Ni with 99.9% purity. The sample has no defects and no crystalline distortions. In Figure 3 we analyzed the same phenomenon but for a sample of a common carbon steel material. There are 4 peaks in the SK evolution which indicate the existence of the impurities and crystal lattice distortions.
Figure 2. SK estimator versus time for a Ni polycrystalline sample

Figure 3. SK evolution for a carbon steel sample

In Figure 4 we illustrated the time evolution of the SK for a nonmagnetic sample from brass. Some low variations of the SK are due to the electronic noise of the measuring chain.
4. Conclusions

The present work demonstrates the utility to use in nondestructive evaluation of methods based on the higher order statistics in signal processing, in our case in the magnetoacoustic signal analysis. Such method can be used in designing of special control methods from ferromagnetic materials.

References