NEAR–ZONE FIELDS SCATTERED BY HIGHLY CONDUCTING PERMEABLE OBJECTS IN THE FIELD OF AN ARBITRARY LOOP; DETECTION AND CHARACTERIZATION OF METALLIC BURIED OBJECTS

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Abstract: The problem of near-zone electromagnetic fields scattered by three-dimensional lossy objects is of practical interest in many disciplines, such as geophysics, ordnance detection, biomedical applications and nondestructive examination. In these applications a current loop is usually used to generate an incident field. The scattered field by an object is detected by another loop. The behavior of the scattered field will vary with the shape, orientation, size, depth and material of the scattering object. In this paper, a numerical solution based on the integral equation approach combined with the method of moments is proposed to compute efficiently the near-zone scattered field due to an arbitrarily shaped highly conducting object. The incident field is generated by a current loop of arbitrarily shape and orientation. The concept of impedance boundary conditions is used to simplify the integral equation formulation. For highly conducting objects, the impedance boundary conditions are reasonable eliminating the need for the inclusion of the interior fields, expressing the magnetic surface current in terms of the electric surface current. Numerical results were validated for steel spheres at low frequency in air and buried in soil, including the effect of ground conductivity and the air-ground interface.

Keywords: near zone field, scattering, arbitrary sources, metallic buried objects,

1. Introduction

The problem of near-zone electromagnetic fields scattered by three-dimensional lossy objects is of practical interest in many disciplines, such as geophysics [1], ordnance detection [2], [3], nondestructive evaluation [4] and biomedical applications [5]. In these applications a current loop is usually used to generate an incident field. The scattered field by an object is detected by another loop. The behavior of the scattered field will vary with the shape, orientation, size, depth and material of the scattering object, being an important issue in the design of detectors.

In this paper, a numerical solution based on the integral equation approach [6] combined with the method of the moments [7], is proposed to compute efficiently the near-zone scattered field due to an arbitrarily shaped highly conducting object. The incident field is generated by a current loop of arbitrary shape and orientation.

For an object of arbitrary shape, exact analytic solutions are usually not feasible and numerical solutions must be obtained. The problem becomes even more difficult when the object is neither perfectly conducting nor perfectly dielectric. In such cases, the integral equation approach is usually utilized to generate useful formulation.

In this paper, the concept of impedance boundary conditions [6] is used to simplify the integral equation formulation and thus reduce the computational efforts. For highly
conducting objects the impedance boundary conditions are reasonable, eliminating the need for inclusion of the interior fields, expressing the magnetic surface current in terms of the electric surface current, and thus, the computation time.

2. Scattering model

Let be a conductive object, supposed to have revolution symmetry, electrical conductivity \( \sigma = \sigma(r) \), magnetic permeability \( \mu = \mu(r) \) and electric permittivity \( \epsilon = \epsilon(r) \) where \( \vec{r} \) is the position vector of a certain point on the surface of the object. To this object, a Cartesian coordinate system XOYZ is attached, named global system.

Let be an arbitrary source of field, circulated by a current with temporal dependence \( e^{j \omega t} \) where \( j = \sqrt{-1} \) and \( \omega \) is angular frequency.

The field scattered outside the surface \( S \) which delimit the object [8] will be

\[
\vec{H}^s = - \int_S \vec{J}(\vec{r}') \times \nabla G_0 dS' - \frac{j}{kZ_0} \int_S \vec{M}(\vec{r}') \tilde{G}(\vec{r}, \vec{r}') dS' \tag{1}
\]

where \( \vec{J} \) and \( \vec{M} \) are the current density, respective magnetic density.

\[
G_0(\vec{r}, \vec{r}') = \frac{1}{4\pi|\vec{r} - \vec{r}'|} e^{jk|\vec{r} - \vec{r}'|}, \quad k = \frac{2\pi}{\lambda}
\]

with \( \lambda \) -wavelength of the field created by a source in free space, \( \vec{r}, \vec{r}' \in S \) represents the observation point and respective the source point, \( Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \) is the characteristic impedance of the free space,

\[
\tilde{G}(\vec{r}, \vec{r}') = (k^2 I + \nabla\nabla) G_0, \quad I = \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}.
\]

The geometry of scattering problem is presented in Figure 1.

![Figure 1. The definition of global and local coordinate system attached to the scattering problem](image)

To each source point and observation point on \( S \), a local coordinate system is attached, \( \hat{n}, \hat{\phi}, \hat{t} \) where \( \hat{n} \) is the versor of the exterior normal to \( S \) in the considered point, \( \hat{\phi} \) the
versor of azimuthally direction and the versor of the tangent to the curve obtained from the intersection of the object surface with a plane containing the Z axis and which passes by the considered point.

The relationship between the local and global systems vectors is

\[
\begin{bmatrix}
\hat{n} \\
\hat{\phi} \\
\hat{t}
\end{bmatrix} = \begin{bmatrix}
\cos \phi \cos \phi & \cos \phi \sin \phi & -\sin \phi \\
-\sin \phi & \cos \phi & 0 \\
\sin \phi \cos \phi & \sin \phi \sin \phi & \cos \phi
\end{bmatrix} \begin{bmatrix}
\hat{x} \\
\hat{y} \\
\hat{z}
\end{bmatrix}
\] (2)

According to [5], the surface impedance of an object with high electrical conductivity is

\[
Z_s(\vec{r}) = \sqrt{\frac{j \omega \mu(\vec{r})}{j \omega \varepsilon(\vec{r}) + \sigma(\vec{r})}}, \vec{r} \in S
\] (3)

Noting \(\vec{E}^i\), \(\vec{H}^i\) the incident field and \(\vec{E}^S\), \(\vec{H}^S\) scattered fields [8]

\[
\hat{n}(\vec{r}) \times (\vec{H}^i(\vec{r}) + \vec{H}^s(\vec{r})) = \vec{J}(\vec{r}), \vec{r} \in S
\]

\[
\hat{n}(\vec{r}) \times (\vec{E}^i(\vec{r}) + \vec{E}^s(\vec{r})) = -\vec{M}(\vec{r}), \vec{r} \in S
\] (4)

According to [9], the relation between the magnetic current density and the electrical one can be written

\[
\vec{M}(\vec{r}) = Z_s(\vec{r})\vec{J}(\vec{r}) \times \hat{n}(\vec{r}), \vec{r} \in S
\] (5)

The total field in a certain point from space is written as

\[
\vec{E}(\vec{r}) = \vec{E}^i - j \omega \mu_0 \int_S \vec{J}(\vec{r}') G_0(\vec{r}, \vec{r}') dS' + \frac{1}{j \omega \varepsilon_0} \int_S [\nabla \cdot \vec{J}(\vec{r}')] \nabla G_0(\vec{r}, \vec{r}') dS'
\]

\[
+ \int_S \vec{M}(\vec{r}') \times \nabla G_0(\vec{r}, \vec{r}') dS'
\]

\[
\vec{H}(\vec{r}) = \vec{H}^i - j \omega \varepsilon_0 \int_S \vec{M}(\vec{r}) G_0(\vec{r}, \vec{r}') dS' + \frac{1}{j \omega \mu_0} \int_S [\nabla \cdot \vec{M}(\vec{r}')] \nabla G_0(\vec{r}, \vec{r}') dS'
\]

\[
- \int_S \vec{J}(\vec{r}') \times \nabla G_0(\vec{r}, \vec{r}') dS'
\] (6)

Introducing (6) in (4) and effectuating the calculation, is obtained

\[
- \frac{1}{2} \vec{M}(\vec{r}) = \hat{n} \times \vec{E}^i(\vec{r}) + \hat{n} \times \int_S \vec{M} \times \nabla G_0 dS' - j \omega \mu_0 \hat{n} \times \nabla \vec{J}(\vec{r}) dS'
\]

\[
+ \frac{1}{j \omega \varepsilon_0} \hat{n} \times \int_S (\nabla \cdot \vec{J}) \nabla G_0 dS'
\]

\[
\frac{1}{2} \vec{J}(\vec{r}) = \hat{n} \times \vec{H}^i(\vec{r}) - \hat{n} \times \int_S \vec{J} \times \nabla G_0 dS' - j \omega \varepsilon_0 \hat{n} \times \nabla \vec{M} dS'
\]

\[
+ \frac{1}{j \omega \mu_0} \hat{n} \times \int_S (\nabla \cdot \vec{M}) \nabla G_0 dS'
\] (7)

The two equations from (7) can be decoupled taken into account the eq. (5). The equations (7) are known in decoupled form, as Electric Field Integral Equation (EFIE) and respective Magnetic Field Integral Equation (MFIE).

To evaluate the scattered field we will use EFIE which becomes
\[
\frac{1}{2} \bar{J}(\bar{r}) = \hat{n} \times \bar{H}'(\bar{r}) - \int_S \hat{n}(\bar{r}) \times \left[ \bar{J}(\bar{r}') \times \nabla G_0(\bar{r}, \bar{r}') \right] dS'
\]

\(- j \omega \epsilon_0 \int_S \hat{n}(\bar{r}) \times \left[ Z_s(\bar{r}') \bar{J}(\bar{r}') \times \hat{n}(\bar{r}) \right] G_0(\bar{r}, \bar{r}') dS' \)

\(+ \frac{1}{j \omega \mu_0} \int_S \hat{n}(\bar{r}) \times \left[ Z_s(\bar{r}') \bar{J}(\bar{r}') \times \hat{n}(\bar{r}) \right] \nabla' G_0(\bar{r}, \bar{r}') dS' \)

(8)

Effectuating all the calculation in local coordinates, the eq.(8) is

\[
\frac{1}{2} J_\phi \hat{\phi} + \frac{1}{2} J_t \hat{\psi} = - \left[ H_x' \sin \psi \cos \phi + H_y' \sin \psi \sin \phi + H_z' \cos \psi \right] \hat{\phi} + \left[ - H_x' \sin \phi + H_y' \cos \phi \right] \hat{\psi} - 
\]

\(- \int_S \frac{jkR - 1}{4\pi R^3} e^{jkR} \left[ \sin \left( \phi' - \phi \right) \left[ \rho \sin \psi' \cos \psi - \rho \sin \psi \cos \psi' \right] + \right. \left( z' - z \right) \sin \psi \sin \psi' \right] J_t' \)

\(+ \left[ \cos \left( \phi' - \phi \right) \left[ \left( z' - z \right) \sin \psi + \rho \cos \psi \right] - \rho \cos \psi \right] J_\phi' \) \]

\(- \frac{1}{j \omega \mu_0} \left[ Z_s(\bar{r}') \left[ \sin \left( \phi' + \phi \right) J_t' + \sin \psi \left( \sin \phi' \cos \phi - \cos \phi' \cos \psi \right) \right] \right] dS' \)

\(- j \omega \epsilon \int_S \left[ Z_s(\bar{r}') \left[ \sin \left( \phi' + \phi \right) J_t' + \sin \psi \left( \sin \psi \cos \phi' \cos \psi' \right) \right] \right] dS' \)

\(- j \omega \epsilon \int_S \left[ Z_s(\bar{r}') \left[ \cos \left( \phi' - \phi \right) J_t' - \sin \psi \sin \left( \phi' - \phi \right) \right] \right] dS' \)

\(+ \frac{1}{j \omega \mu_0} \left[ Z_s(\bar{r}') \frac{k^2}{4\pi} e^{ikR} \left[ - \sin \left( \phi' + \phi \right) J_t' + \sin \psi \left( \sin \psi \cos \phi' \cos \psi' \right) \right] \right] \hat{\phi} + 
\]

\(+ \frac{1}{j \omega \mu_0} \left[ Z_s(\bar{r}') \frac{k^2}{4\pi} e^{ikR} \left[ \cos \left( \phi' - \phi \right) J_t' - \sin \psi \sin \left( \phi' - \phi \right) \right] \right] \hat{\psi} + 
\]

(9)

The integral equation (9) has not analytical solution. To can solve it we will use the method of moments which transform the integral equation into a system of algebraic equations having as unknowns the electric current densities in the center of the mesh cells in which the surface of scatter was split.

Considering that the surface of scatter was discretized in N cells, small enough, with \(\Delta S_n\) area, so that the density of electric current can be considered constant in any point of cells.

The electric current density on the scatter surface \(\bar{J}(\bar{r})\) can be write as

\[
\bar{J}(\bar{r}) = \sum_{n=1}^{N} \left( J_{\phi_n} \hat{\phi}(\bar{r}_n) + J_{t_n} \hat{\psi}(\bar{r}_n) \right) f_n(\bar{r})
\]

(10)

\(f_n(\bar{r})\) is named basis function [7] and we choose basis function as pulse

\[
f_n(\bar{r}) = \begin{cases} 
1, & r \in \Delta S_n \\
0, & \text{in rest} 
\end{cases}
\]

(11)
We consider the test function $w_m$, as Dirac functional

$$ w_m = \delta(\vec{r} - \vec{r}_m) $ \hspace{1cm} m=1, \ldots, N \tag{12} $$

This variant of the method of the moment is named point matching [7]. Introducing (11) in (9) and calculating the moments through intern product with the test function in both members [6]

$$ \sum_{n=1}^{N} J_{\phi n} \int_{S} f_n w_m dS + \sum_{n=1}^{N} J_{I n} \int_{S} A_1 f_n w_m dS' + \sum_{n=1}^{\infty} J_{\phi n} \int_{S} B_1 f_n w_m dS' = \int_{S} C_1 w_m dS \tag{13} $$

$$ \sum_{n=1}^{N} J_{\phi n} \int_{S} f_n w_m dS + \sum_{n=1}^{N} J_{I n} \int_{S} A_2 f_n w_m dS' + \sum_{n=1}^{\infty} J_{\phi n} \int_{S} B_2 f_n w_m dS' = \int_{S} C_2 w_m dS \tag{14} $$

The expression of $A_1$, $B_1$, $C_1$, $A_2$, $B_2$, $C_2$ are complicated, being presented in [10].

3. Results

Considering a conductive sphere with $\sigma=10^7$ S/m having 0.2m diameter, placed in the center of the global coordinate system. The incident field is created by a circular loop having 0.6m diameter and is circulated by an alternative electric current with frequencies varying between 100Hz and 1GHz. The center of the loop is placed in the point (-0.5, 0, 0).

The sphere was discretized in 120 triangular surface elements.

In Figure 2 is presented the distribution of the currents $J_\phi$ on the sphere for 1MHz frequency.

Figure 2. Distribution of the currents $J_\phi$ on the sphere for 1MHz

In Figure 3 is presented the dependence by frequency of the amplitude of magnetic field, scattered on sphere.

In Figure 4 is presented the dependence by frequency for the case of an ablate spheroid having electrical conductivity $\sigma=10^7$ S/m, minor axis 0.2m and major axis 1m. The spheroid was discretized in 420 triangles.
Examining comparative the data form Figure 3 and Figure 4, it can be observed that the amplitude of scattered field depends by the shape of scatter. This can allow the solving of the inverse problem in the case of scattering.

4. Conclusions

The problem of near-zone fields scattered by 3D highly conducting objects initially theoretical developed has start to have practical applications, in the same time with the development of calculation techniques and measurement equipments. By scanning a region and solving inverse problem is possible to determine the position and the shape of scatter. The possibility that the proposed method shall be used also at low frequency,
thus very big wavelengths, allows the detection of buried objects, becoming a complementarily method for ground penetrating radar which functions at lower wavelength so that the scatter are in the far field region.

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