EFFICIENT CALCULATION OF GUIDED WAVE PHASED ARRAY WAVE FIELDS BASED ON SYMMETRY PRINCIPLES

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Abstract. In the present paper we extend the formerly introduced EFIT-PSS technique, a combination of the elastodynamic finite integration technique (EFIT) with transient point-source-synthesis (PSS), to the problem of Lamb wave propagation in plate-like structures. By using this new hybrid technique 3-D ultrasonic wave fields of nearly arbitrary transducer configurations can be efficiently calculated and analysed in the time-domain. Various numerical results for the radiation characteristics of circular, rectangular, and array transducers obtained by the new EFIT-PSS technique are discussed. The results for rectangular transducer elements are compared with the corresponding 3-D EFIT calculations of the same problem demonstrating the accuracy of the new approach. The proposed technique represents a significant extension of the EFIT framework since it broadens the range of applications to the problem of high-frequency 3-D guided wave fields and allows a fast and reliable design and optimization of Lamb wave transducer configurations.

Keywords. Elastodynamic finite integration technique, Transient point-source-synthesis, Guided waves, EFIT-PSS

INTRODUCTION

Recently we introduced the EFIT-PSS technique as a combination of numerical EFIT and transient point-source-synthesis (PSS) in order to calculate high-frequency ultrasonic wave fields of various transducer configurations [1]. This hybrid technique represents the first successful example of a combination of a purely numerical method (EFIT) with a technique formerly known from analytical calculations (PSS). In the original paper [1] a spatial PSS was described but the possibility for a full spatio-temporal synthesis was already discussed. Meanwhile the technique has been successfully expanded to spatio-temporal point-source-synthesis allowing the treatment of more complex angle-beam and phased array transducers [2]. In the present paper we consequently expand our approach to the problem of Lamb wave transducer configurations in plate-like structures.

One important possibility for structural health monitoring (SHM) and non-destructive evaluation (NDE) of plate- and shell-like structures is based on the application of guided ultrasonic waves generated and detected by arrays of connected or distributed transducers. For design purposes and POD (probability of detection) studies, fast and reliable modelling tools for the calculation of phased array wave fields are needed. While semi-analytical methods for wave field synthesis are very fast but often suffer from several approximations like neglect of near-field effects and mode conversions, purely numerical methods include all relevant wave phenomena but are characterized by significantly larger computation times and memory requirements, especially in 3-D. In the following a new hybrid method is presented which combines the Elastodynamic Finite Integration Technique (EFIT, [3, 4]) with easy but powerful
symmetry principles allowing for transient point-source-synthesis. As a result transient 3-D Lamb wave fields generated by arrays of connected and distributed transducers can be calculated efficiently. We present various examples of application in an isotropic plate but also discuss how the proposed technique can be expanded to multi-layers, anisotropic media and cylindrical shells.

1. THE BASIC LAMB WAVE FIELD OF A POINT SOURCE

A common starting point of the proposed technique is the wave field of a normal force point source acting on a linearly elastic isotropic plate with stress-free boundary conditions. For this problem the axisymmetric version of the elastodynamic finite integration technique based on cylindrical coordinates is a valuable tool. This so called CEFIT technique can be used for arbitrarily layered systems using a fully numerical framework [5]. Due to the curvilinear coordinate system, axisymmetric 3-D problems like normal force point sources can be effectively reduced to 2-D models in the r/z-plane with very small memory and computation time requirements.

In Fig. 1 the B-Scan data of the elastic wave field due to a normal point source (center frequency = 200 kHz) acting on a 4 mm thick aluminum plate with stress-free boundary conditions is shown. The fast symmetric Lamb wave S0 and the slower antisymmetric A0 mode can clearly be identified. The spatio-temporal data for the \( v_r \) and \( v_z \) velocity components along the top surface of the plate form the basis for the subsequent point-source-synthesis.

![Figure 1](image1.png)

**Figure 1:** B-Scan data of an axisymmetric EFIT simulation of a Lamb wave field generated by a normal point source acting on a 4 mm thick aluminum plate (\( c_P = 6260 \) m/s, \( c_S = 3080 \) m/s, \( \rho = 2700 \) kg/m\(^3\)). On the left: radial component of particle velocity \( (v_r) \); on the right: axial component of particle velocity \( (v_z) \).

In Fig. 2 the corresponding time domain signals obtained at \( r = 170 \) mm are shown. One can see that the point source generates a strong A0 but only a weak S0 wave.

![Figure 2](image2.png)

**Figure 2:** A-Scan data at \( r = 170 \) mm of axisymmetric EFIT simulation of a Lamb wave field generated by a normal point source acting on a 4 mm thick aluminum plate. On the left: radial component of particle velocity \( (v_r) \); on the right: axial component of particle velocity \( (v_z) \).
From the B-Scan data in Fig. 1 the complete wave field of the point source along the surface of the plate in a Cartesian coordinate system can be calculated. In Fig. 3 the $v_x$, $v_y$, (in-plane) and $v_z$ component (out-of-plane) as well as the absolute value of the particle velocity vector, $|\mathbf{v}|$, after $t = 94.08 \, \mu s$ are shown.

**Figure 3:** In-plane components $v_x$, $v_y$ (top), out-of-plane component $v_z$ (bottom left) and absolute value of the particle velocity vector $|\mathbf{v}|$ (bottom right) along the surface of the plate at $t = 94.08 \, \mu s$ calculated from the B-Scan data in Fig. 1. The spatio-temporal Cartesian point source wave field serves as a basis for the transient point-source-synthesis (PSS). The initial wave field can be obtained within one second on a conventional PC.

2. TRANSIENT POINT-SOURCE-SYNTHESIS FOR CIRCULAR AND RECTANGULAR APERTURES

After performing the axisymmetric CEFIT calculation of the elementary point source, the resulting wave field of a circular aperture can simply be calculated by summing up the contributions of various point sources lying inside the finite transducer aperture. The result of this EFIT-PSS combination for a circular aperture with a radius of 5 mm is shown in Fig. 4. The absolute value of the particle velocity vector shows that the S0 wave is significantly stronger
compared to the point source in Fig. 3 (bottom right). However the A0 wave is still dominant due to the normal force excitation used here.

![Diagram 4](image)

**Figure 4:** EFIT-PSS result for a circular transducer element with a radius of 5 mm. The picture displays the absolute value of the particle velocity vector, $|v|$. 

In Fig. 5 (left-hand side) a rectangular (square) transducer element with a lateral size of 15 mm has been used for the transient point-source synthesis. As a result the A0 wave is predominantly excited in the x and y directions while a characteristic gap appears in 45° directions. In contrast to the A0 wave the S0 mode is not significantly affected due to its larger wavelength. The corresponding wave field is obviously isotropic.

![Diagram 5](image)

**Figure 5:** Comparison of EFIT-PSS (left-hand side) and 3-D EFIT results (right-hand side) for a square transducer element with a lateral size of 15 mm showing a very good agreement. However, the EFIT-PSS calculation needs 600 times less computer memory and is 100 times faster than the corresponding 3-D EFIT calculation.
In order to test the accuracy of the EFIT-PSS procedure the wave field of the rectangular aperture was compared with the results of a 3-D EFIT calculation of the same problem. Since the 3-D EFIT code has been successfully validated in the past [6] it can serve as a reference for the new method.

The right picture of Fig. 5 shows the result of the 3-D EFIT calculation. The agreement between the two calculations is very good. Slight differences in the A0 wave front are caused by its extremely short wavelengths and can be avoided by a finer discretization of the 3-D EFIT and/or the EFIT-PSS reconstruction grid. However, it is important to point out that the EFIT-PSS calculation needed approximately 600 times less computer memory and was approximately 100 times faster than the corresponding 3-D EFIT calculation.

Fig. 6 shows another example in which the wave field of a rectangular transducer element with a size of 30 mm in x- and 15 mm in y-direction is shown. In this case the \( v_y \) velocity component is displayed and the agreement between 3-D EFIT and EFIT-PSS calculation is excellent demonstrating the accuracy of the transient point source synthesis.

3. TRANSIENT POINT SOURCE SYNTHESIS FOR ARRAY WAVE FIELDS

If a number of transducer elements is combined to a multi-channel array the resulting Lamb wave fields can be simply varied by changing the spatio-temporal excitation of the single elements. In the following example we calculated the Lamb wave field of a linear array of 16 circular transducer elements with a radius of 3.3 mm and a distance of 6.6 mm to each other. In order to produce both, a strong S0 and a strong A0 wave, radially vibrating transducers were used in this case. In order to further speed up the point-source-synthesis the wave field of a complete circular transducer element instead of a single point source has been calculated by the CEFIT technique and used as the reference wave field for the synthesis. The corresponding B-Scan data and an example of an A-Scan calculated at \( r = 170 \) mm is shown in Figs. 7+8.
Fig. 7: B-Scan data of axisymmetric EFIT simulation of a Lamb wave field generated by a radially vibrating circular transducer with a radius of 3.3 mm. On the left: radial component of particle velocity ($v_r$); on the right: axial component of particle velocity ($v_z$).

Fig. 8: A-Scan data at $r = 170$ mm of an axisymmetric EFIT simulation of a Lamb wave field generated by a radially vibrating circular transducer with a radius of 3.3 mm. On the left: radial component of particle velocity ($v_r$); on the right: axial component of particle velocity ($v_z$).

Fig. 9 shows four different wave front snapshot for the case in which no time delay is used between the single transducers. Due to the orientation of the linear array along the x-axis strong S0 and A0 waves are excited in $\pm y$-direction.

By using specific time delays between the single transducer elements the wave fields of S0 and A0 modes can be steered and focused to nearly arbitrary directions or focal points. In Fig. 10 a constant time delay of $\Delta t = 0.904 \, \mu s$ has been used leading to a strong S0 wave front in a direction of approx. 40° and an A0 wave front in approx. 70° direction with respect to the array axis.

In Fig. 11 another example with a larger time delay of $\Delta t = 1.24 \, \mu s$ is shown. In this case the strongest S0 wave is propagating in 0° direction while the A0 wave is steered in a direction of approx. 60° with respect to the array axis.
Figure 9: EFIT-PSS results for a linear array of 16 radially vibrating circular transducers with a radius of 3.3 mm and a distance of 6.6 mm to each other. No time delay is used between the single elements.
Figure 10: EFIT-PSS results for a linear array of 16 radially vibrating circular transducers with a radius of 3.3 mm and a distance of 6.6 mm to each other. A constant time delay of $\Delta t = 0.904 \mu s$ is used between the single elements.

Figure 11: EFIT-PSS results for a linear array of 16 radially vibrating circular transducers with a radius of 3.3 mm and a distance of 6.6 mm to each other. A constant time delay of $\Delta t = 1.24 \mu s$ is used between the single elements.
4. FURTHER APPLICATIONS AND EXTENSIONS

By using the EFIT-PSS technique Lamb wave fields of nearly arbitrary transducer configurations like linear, matrix, circular and distributed arrays can be calculated. For normal force excitation and radially vibrating circular transducers the axisymmetric CEFIT can be used to obtain the initial wave field. For other kinds of sources like shear-horizontal excitation the initial CEFIT calculation must be replaced by a single 3-D EFIT calculation with appropriate boundary conditions. This data can then be used to calculate the wave field of various transducer geometries without the need for further numerical 3-D calculations.

The initial 3-D calculation is also necessary if guided wave propagation in cylindrical shells is investigated. In this case no axisymmetric source is available. However, the wave field of different transducer configurations can be reconstructed by transient point-source-synthesis using a similar procedure as explained in this paper for plates. For both plates and shells the formalism can also be extended to multi-layered systems as exemplary described in [1] for body waves.

5. CONCLUSIONS AND OUTLOOK

In the present paper the hybrid EFIT-PSS technique that combines the advantages of the numerical EFIT technique and the well-known point-source-synthesis has been extended to the problem of Lamb wave propagation in plate-like structures. With this time-domain approach, 3-D Lamb wave fields for various transducer and array configurations can be easily calculated with only a fraction of the computational effort needed for full numerical 3-D calculations. The typical gain in computer memory requirements lies between 500 and 1000%, while the computational speed-up factor amounts to 100 at least.

The EFIT-PSS results can be presented as wave front snapshots for different points in time, wave front animations, time-domain signals at arbitrary positions (A-, B-, C-Scan representations) and the well-known intensity plots. The proposed technique is not restricted to axisymmetric isotropic problems since the basic point source data can also be calculated by using a single 3-D calculation of an arbitrarily layered isotropic or anisotropic plate or cylindrical shell. This initial data can then be used to calculate the wave field of various transducer geometries without the need for further numerical 3-D calculations. As a conclusion one can summarize that the proposed EFIT-PSS technique represents a significant extension of the EFIT framework establishing a powerful time-domain tool for the design and optimization of Lamb wave transducer set-ups.

With the EFIT-PSS method free Lamb wave fields without any interaction with potential defects can be calculated. However, for the more complex problem of wave-defect interaction other efficient techniques based on reciprocity principles are possible. By using this sophisticated framework the interaction of the different wave modes with various defects can be modelled with smallest possible computational effort. Due to space limitations this approach cannot be described here and thus, will be published elsewhere.
REFERENCES


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