A Phase Synthesis Based Time Reversal Focusing Method for Impact and Damage Imaging of Complex Composite Structures

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Abstract. The growing use of composite structures in aerospace structures has attracted much interest to structural health monitoring (SHM) for localization of impact and damage positions due to their poor impact resistance properties. Propagation mechanism and frequency dispersion characteristics of Lamb wave signals on complex composite structures are more complicated than that on simple composite plates. Recently, much attention has been paid to the research of time reversal focusing method because this method shows a promising advantage to give a focusing image of the structural damage, improve the signal-to-noise ratio and compensate the dispersion of Lamb wave signals. In this paper, aiming at developing a practical method for on-line localization of impact and damage on aircraft composite structures which can take advantage of time reversal focusing and does not rely on the transfer function, a new phase synthesis based time reversal focusing method is proposed. In this method, complex Shannon wavelet transform is adopted to extract frequency narrow-band signals of piezoelectric transducers (PZTs) array at a special time-frequency scale and measure phase and group velocity of the signals. Impact and damage images are given out directly through time reversal focusing based on phase synthesis process of the signals. A SHM demonstration system is built on a composite panel of an aircraft wing box with many bolt holes and stiffeners using the phase synthesis based time reversal focusing method. The demonstration results show that this method can estimate the positions of impact and damage efficiently with a low sensitivity of velocity errors and a high anti-jamming capability. According to the demonstration results, when the velocity error is up to 26.7%, the impact and damage localization errors are less than 4.0cm and 3.0cm in the monitoring area of 410mm×450mm and 460mm×800mm, respectively.

Introduction

Advanced composites are increasingly used on aircraft where weight and performance are a great concern. Composites are very sensitive to impact which can cause inner damages in the composite, such as delamination. Early detection and on-line monitoring of impact and damage position can greatly help to prevent catastrophic failure and reduce the ground maintenance time of aircraft. Much attention has been paid to the research of time reversal focusing method [1] in Lamb wave based structural health monitoring (SHM) [2] because this method shows a promising advantage to give a focusing image of the structural damage and can improve the signal-to-noise ratio of the propagating waves. 

Recently, several researchers reported their contributions in the impact position localization taking advantage of the time reversal focusing methods [3-6]. Most of these
researches were done on metal structure or glass plate. Meo et al reported the research on composite structure [6]. In recent damage monitoring researches, Wang investigated the feasibility of this time reversal focusing process applied in guided waves of plate-like structures and discussed a damage imaging method based on the time reversal concept [7]. Based on this research, Wang and Yuan studied a time reversal focusing method for baseline free damage imaging [8]. Park and Sohn [9] developed a time reversal focusing based damage identification method by comparing differences of focused signal and original input signal at the excitation point. The time reversal focusing process was realized by hardware and this method was validated on a simple plate-like composite structure. Some researchers conducted further researches to use software realization of the time reversal focusing process [10-12]. To realize the process, transfer functions of the propagation of the signals on structures are obtained firstly and stored in computers, and the virtual realization based on the transfer functions of the time reversal focusing process is conducted in software. A key issue in these methods is to obtain the transfer functions of the wave propagation between excitation element and the sensing element. Two typical methods are studied to obtain the transfer functions: (1) theoretic modelling of the structure [10-11]; (2) measuring the transfer functions through experiments [6, 12]. The first method is difficult to be achieved in complex composite structures. For real applications of the impact localization method on aircraft composite structure, this method is not permitted.

Impact and damage can be considered to be two typical Lamb wave sources of structure. In this paper, aiming at developing a practical method for on-line localization of impact and damage on aircraft complex composite structure which can take advantage of the time reversal focusing method, a PZT sensors array based phase synthesis time reversal Lamb wave source imaging method for composite structure is proposed.

1. The Phase Synthesis Time Reversal Focusing Lamb Wave Source Imaging Method

According to the time reversal focusing concept, a signal can be focused at the original source point if some sensed signals recorded at different points are reversed in the time domain and re-emitted back to the original source point. This time reversibility is based on the spatial reciprocity and time-reversal invariance of linear wave equations [1]. Figure 1 shows the time reversal focusing process. $E_C(\omega)$ is the signal excited at the original source. $H_{Ci}$ is the frequency response of transfer function of the signal propagating from $C$ to the PZT $i$. The output sensing signal of the PZT $i$ can be represented as:

$$E_i(\omega) = H_{Ci}(\omega, r)E_C(\omega)$$  \hspace{1cm} (1)

When the sensed signal of each PZT sensor is time reversed and re-emitted back to the original source position. A synthesis signal at the source $C$ is obtained by:

$$E'_C(\omega) = \sum_{i=1}^{n} H_{Ci}(\omega, r)E_i(\omega)^*$$  \hspace{1cm} (2)

where the superscript ‘*’ denotes complex conjugate. It means in the frequency domain, time reversal of a signal is equivalent to phase conjugation. According to the spatial reciprocity of linear wave equation, there is $H_{Ci}=H_{Ci}$. By substituting Eq. (1) into Eq. (2), the synthesis signal at $C$ can be represented as:

$$E'_C(\omega) = \left( H(\omega, r)H(\omega, r)^* \right)E_C(\omega)^* = |H(\omega, r)|^2 E_C(\omega)^*$$ \hspace{1cm} (3)

where $H(\omega, r)$ is the transfer functions matrix of the signals. The term of $|H(\omega, r)|^2$ is a real positive even function. Depending on the basic time reversal theory of Lamb wave propagating on plate-like structure, the modulus value of the synthesis signal reaches the maximum at the position $C$ because all the time reversed signals arrive at the source point at
the same time and add together because of the spatial reciprocity and time-reversal invariance of linear wave equations [7, 9, 13].

To a frequency narrow-band Lamb wave signal at low frequency, only $A_0$ mode and $S_0$ mode exist and the amplitude of $A_0$ mode is much higher than that of $S_0$ mode. Thus, the Lamb wave can be considered to be a frequency narrow-band single-mode Lamb wave [14] and the transfer function $H(\omega, r)$ can be simplified to:

$$H(\omega, r) \approx a_k(\omega, r)e^{-jk_0(\omega)r}$$  \hspace{1cm} (4)

where $a_k(\omega, r)$ and $k_0(\omega)$ are the amplitude term and the phase term of the transfer function, respectively. $k_0(\omega)$ denotes the wave number of $A_0$ mode. $k_0 = \omega/C_A \cdot C_A$ denotes phase velocity of $A_0$ mode. By substituting Eq. (4) into Eq. (3), the synthesis signal at the position $C$ can be changed to:

$$E'_c(\omega) = \left( H(\omega, r)H^*(\omega, r) \right)^{1/2} E_c(\omega) = \sum_{i=1}^n a_k(\omega, r_i)E_i(\omega)$$ \hspace{1cm} (5)

where the distance from $C$ to all the PZT sensors are denoted as $r_{iC}$, $i=1,2,\ldots,n$.

Using Eq.(5), if the transfer functions of $A_0$ mode can be obtained beforehand, the time reversal focusing process can be realized in software. This is the ordinary software based virtual time reversal method mentioned in the introduction.

![Figure 1. Illustration of time reversal focusing process of a Lamb wave source](image1.png)

![Figure 2. Schematic diagram of phase synthesis process](image2.png)

Regarding the Lamb wave source imaging of composite structure, to apply above method, two difficulties must be solved. First, broadband frequency Lamb wave signals induced by impact have to be processed. Frequency narrow-band signal has to be extracted. Second, the transfer functions of $A_0$ mode of composite material are difficult to obtain. A method not to rely on the transfer functions should be presented. How to solve the first problem is discussed in detail in section 2. A phase synthesis method is put forward to solve the second problem in this section.

To introduce the proposed phase synthesis time reversal focusing imaging method in more detail, a plate-like structure with $n$ PZT sensors arranged is adopted as an example to explain the method shown in Figure 2. In the monitoring area, a random position $D$ is chosen whose coordinate is $(x, y)$. The distances from the position $D$ to all the PZT sensors are denoted as $r_1, r_2, \ldots, r_n$. $E_i(\omega)$ is the frequency response of the frequency narrow-band Lamb wave response signal of the PZT $i$. The output synthesis signal at the position $D$ can be represented as Eq.(6) by time reversing $E_i(\omega)$ and applying the phase term of the transfer function $e^{j\phi(\omega)}$ to $E_i(\omega)$ at the same time.

$$V_S = \sum_{i=1}^n E_i(\omega) e^{-jk_0(\omega)r_i}$$  \hspace{1cm} (6)
The modulus value of the inverse Fourier transform of Eq.(6) can be represented as

$$|v_s(t)| = \left| \frac{1}{2\pi} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} E_i(\omega) e^{-jk_0(\omega)t} e^{j\omega t} d\omega \right|$$

(7)

where $v_s(t)$ denotes the phase synthesis signal. The phase delay factor $e^{-jk_0(\omega)t}$ in Eq.(6) can be represented as:

$$e^{-jk_0(\omega)t} = e^{-j\omega t/C_{n0_k}}$$

(8)

Considering that the $E_i(\omega)$ is frequency narrow-band, Eq.(7) can be changed to:

$$|v_s(t)| = \left| \frac{1}{2\pi} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \left( \sum_{\omega_0} E_i(\omega_0) e^{-j\omega_0 t/C_{n0_k}} \right) e^{j\omega t} d\omega \right| = \frac{1}{2\pi} \sum_{n=1}^{\infty} \left[ \sum_{\omega_0} \epsilon_i(\tau - t + r_i/C_{n0_k}) \right]$$

(9)

where $E_i(\omega_0)$ is a frequency component of $E_i(\omega)$ when the frequency is $\omega_0$. $\epsilon_i(t)$ is the corresponding time domain signal. $C_{n0_k}$ denotes the phase velocity of the frequency component. The $\omega_0$ is in the frequency range of $[\omega_l, \omega_h]$. The frequency band is defined as $\omega_b = \omega_h - \omega_l$ and the center frequency is $\omega_c$.

When imaging the results, the pixel value corresponding to position D is calculated by Eq.(9) representing the modulus value of the focusing signal at position D. It is known that the real focusing only happened at the original source which means here the source position. The pixel value calculated only at the Lamb wave source position is the highest. Thus, the whole monitored area can be divided into many small areas, by a searching algorithm to calculate the pixel value of each of these small areas, the source position can be decided by choosing the highest pixel value position or higher pixel value area.

Since the phase velocities of signals at different frequencies are different, combining with that for composite structure, the phase velocities of waves propagating at different directions are also different, in the searching process, if the phase velocities of different frequencies and propagation directions are used directly, a large amount of calculation has to be performed. To avoid this, some approximations are made to solve this problem.

Depending on the dispersion nature of Lamb wave and the superposition principle, the frequency narrow-band Lamb wave can be considered to be consisted of a finite numbers of sine waves. Thus, Eq.(9) can be represented as Eq.(10) approximately:

$$|v_s(t)| \approx \sum_{i=1}^{n} \epsilon_i \left( \tau - t + \frac{r_i}{C_{n0_k}} \right)$$

(10)

where $e$ denotes the frequency narrow-band signal extracted from the impact responded signal of the PZT sensor $i$.

The implementation process of the PZT sensors array based phase synthesis time reversal imaging method of Lamb wave source is shown in Figure 3.
2. Frequency Narrow-Band Signal Extracting Method

A software processing method based on continuous complex Shannon wavelet transform is proposed taking advantage of its high time-frequency resolution to extract the frequency narrow-band signals corresponding to the $A_0$ mode of impact response signals.

Eq.(11) shows the complex Shannon wavelet function adopted:

$$\Psi(t) = \sqrt{f_b \text{sinc}(f_b t)} e^{2\pi \phi t}$$

The terms of $f_b$ and $f_c$ are the frequency band and center frequency of the wavelet, respectively. The function of sinc is represented as:

$$\text{sinc}(x) = \begin{cases} 1 & x = 0 \\ \frac{\sin(\pi x)}{\pi x} & x \neq 0 \end{cases}$$

The Fourier transform of Eq. (12) can be represented as:

$$\Phi(\omega) = \begin{cases} \frac{2\pi}{\omega_b} & \frac{\omega_b}{2} < \omega \leq \omega_c + \frac{\omega_b}{2} \\ \frac{2\pi}{\omega_c} & \omega_c - \frac{\omega_b}{2} < \omega \leq \omega_c \\ 0 & \text{Others} \end{cases}$$

where $\omega_b = 2\pi f_b$, $\omega_c = 2\pi f_c$, $\omega_c > \omega_b/2$. Eq.(11) indicates that the centre time of complex Shannon wavelet function is at $t=0$. The centre frequency according to Eq. (13) is at $\omega_0 = \omega_c$. The frequency band is limited in the range of $[\omega_c - \omega_b/2, \omega_c + \omega_b/2]$. Therefore, the centre time of complex Shannon wavelet transform $\Psi((t-b)/a)$ is at $t=b$. The centre frequency is at $\omega_0 = \omega_c/a$. The complex Shannon wavelet transform of the signal $x(t)$ is a frequency narrow-band signal which represents the time-frequency components at $t=b$, $\omega_0 = \omega_c/a$ and the frequency band is $(\omega_c/a - \omega_b/2a, \omega_c/a + \omega_b/2a)$.

Figure 4 gives out a typical impact response signal of a PZT sensor and its frequency response. The sampling rate is 10MHz. It shows that most of the signal energy distributes in the frequency range of 0Hz to 60KHz. Because of this, the frequency of 50KHz is selected to be the centre frequency of the frequency narrow-band signals. Depending on the complex
Shannon wavelet transform, the frequency narrow-band signal of the centre frequency $f_c = 50$KHz and the frequency band $f_b = 0.4f_c = 20$KHz is extracted from the impact responded signal. The real part of the signal and the frequency response are shown in Figure 5.

![Impact response signal and frequency response](image1.png)

**Figure 4.** Impact response signal and the frequency response

3. Validation Experiment of Impact and Damage Imaging

The validation experimental setup shown in figure 6 is consists of the ISHMS [15] and top panel of an aircraft wing box specimen with many bolt holes and stiffeners. The dimension of the wing box top panel is 1000mm×1800mm (width × height). 24 PZT layers [16] are used to construct the PZT sensors array. The PZT 1, 7, 19, 20, 22, 16, 4 and 3 are used to fulfil the impact imaging task. The sizes of impact and damage monitoring areas constructed by the PZT layers are 410mm×450mm and 460mm×800mm, respectively.

Figure 7 gives out the measured group velocity around the PZT 10 and the PZT 8, respectively. It shows that the group velocities are of great difference at each direction. Compared Figure 7 (a) with (b), the group velocity is different even at the same direction. Thus, the average group velocity is adopted. According to the measured group velocity of the 34 actuator-sensor channels, the average velocity is 1455m/s approximately. The maximum error of the group velocity in each direction to the average group velocity is 26.7%.

![Extracted frequency narrow-band signal](image2.png)

**Figure 5.** Extracted frequency narrow-band signal, $f_c = 50$KHz and $f_b = 0.4f_c$
In impact imaging validation, the energy of the impact signal is concentrated in the frequency range of 0Hz to 100kHz. The propagation mode is much more complicated and the energy is much lower of the signal at higher frequency. Because of this, the signal of centre frequency of 50kHz is selected to be the frequency narrow-band signal. Many impacts are applied to the composite panel, respectively. Figure 8 shows two impact imaging results. 16 PZTs are showed in each image to illustrate the relative position of the impacts. The highest pixel point of the impact image is estimated to be the centre of the impact. Table 1 shows the position comparison between the centre of actual impacts and the estimated centre of impacts depending on the impact images. It indicates that the distance error of impact localization by the impact imaging method is less than 4cm. Considering that the size of the impact monitoring area is 410mm×450mm and the velocity errors in each direction, the localization errors are reasonable.

In damage imaging validation, the centre frequency of Lamb wave is also 50kHz. A kind of solid adhesive is used to simulate the damage. Many simulating damages are applied to the composite panel, respectively. Figure 9 shows two damage imaging results. The point with the largest pixel value of the damage image is estimated to be the centre of the damage. Table 2 gives out the position comparison between the centre of actual damage and the estimated centre of damage depending on the damage images. It indicates that the distance
errors of damage localization by the damage imaging method are less than 3.0 cm. Considering that the size of the damage monitoring area is 460 mm × 800 mm and the velocity errors in each direction, the localization errors are reasonable.

Figure 8. Impact imaging results

Figure 9. Damage imaging results

Table 1. Localization results of impacts

<table>
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<th>Impact</th>
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<th>Estimated impact position (mm, mm)</th>
<th>Error (cm)</th>
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</tr>
<tr>
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<td>(0, -20)</td>
<td>2.0</td>
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<tr>
<td>4</td>
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</table>

Table 2. Localization results of damage

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<th>Estimated damage position (mm, mm)</th>
<th>Error (cm)</th>
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<tbody>
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<td>(125, 136)</td>
<td>1.5</td>
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<tr>
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<td>(-100, 0)</td>
<td>(-102, 16)</td>
<td>1.6</td>
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4. Conclusion

This paper proposes a phase synthesis time reversal focusing method to achieve the time reversal focusing of impact response signals and damage scattered signals on complex composite structure. The imaging method does not require any work on the modelling or the measuring of the transfer functions of signal propagation. Impact and damage images are given out directly through time reversal focusing. The complex Shannon wavelet transform is adopted to extract the frequency narrow-band signals of impact response signals. A validation experiment is performed on composite of many bolt holes and stiffeners. When the group velocity error is 26.7%, the impact and damage localization errors are less than 4.0cm and 3.0cm in the monitoring area of 410mm×450mm and 460mm×800mm, respectively.

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References