Wave scattering analysis in a delaminated cross-ply laminate due to incident $S_0$ wave

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Abstract

This paper investigates the effect of delamination on wave scattering due to incident $S_0$ mode. When composite laminate is subjected to ultrasonic guided wave, due to the wave-delamination interaction forward and backward wave appear which includes important cross-correlation terms that couple the propagating modes with the evanescent waves, and the forward waves with the backward waves. These cross-correlation terms can be used effectively to extract certain quantities of interest that give significant insight regarding the delamination characteristics. Reliable identification of scattering mechanism in this way can make it possible to confirm the presence and predict the location and geometric properties of delamination. In this paper various wave scattering behaviour is presented from analytical model as well as time domain spectral finite element simulation. Wave interaction with delamination in different interfaces is studied to understand the nature of wave near the damaged region and predict the response characteristics for different delamination interface in a laminate. $S_0$ wave produces no reflection for unidirectional laminates but for cross-ply laminate there is reflection if the delamination is not in mid-plane. It happens due to mismatch in the stiffness properties in sub-laminates which leads to $S_0$ wave to propagate in different velocities with phase differences, that causes reflection as well as mode conversions.

Keywords: Guided wave, delamination, mode conversion, wave scattering

1. Introduction

Composite has wide application in various industries especially in aerospace because of its high specific strength which offers scope to reduce the weight of the structure. Composites are sensitive to various unwanted loading it experience during service life or maintenance. One of the most common types of damage found in composite is delamination. Difficulty in detection and characterization of delamination remains challenging tasks due to growth of damage in interfacial plane and direction dependent and non-homogeneous properties of composite laminate. There are various methods existing in non-destructive method which includes through transmission, thermo-graphic, X-ray scan, tomographic testing etc. Among all those method guided wave based damaged detection method becomes popular [1] because it can propagate longer distance without much attenuation and can bring damage induced signature from hidden location. Using guided wave based damage detection method damage location is determined form time of flight which can be calculated from the responses whereas size of the damage estimated from the wave packet characteristics. Information of delamination depth can be predicted from the damage induced scattering and local wavelength analysis [2,3]. Both $A_0$ and $S_0$ are widely used in damage detection structure. $S_0$ wave is non-dispersive in nature and have low attenuation compared to $A_0$ wave [4].

There are several mathematical techniques available to model wave propagation in structure including finite difference, boundary element, finite element method etc. All those methods have their unique advantage depending upon the complexity of the problem. Simulation of ultrasonic wave propagation in structure using conventional method requires
fine spatial as well as temporal discretization [5, 6] which increase the computational size enormously. Spectral methods are applied successfully in modelling wave propagation structures to reduce the computation cost. There are two types of spectral method available in literature, one is frequency domain spectral method [7] and other one is time domain spectral method [8]. Time domain spectral method has the advantages over frequency domain counterpart when dealing with complex 2-D or 3-D structure [9]. Time domain spectral finite element method has been used in modelling wave propagation in rod, beam and shell structure [9, 10] with superior accuracy and convergence.

In this study effect of $S_0$ wave in cross-ply laminate with large delamination is studied using analytical model based on Euler-Bernoulli beam approximation as well as simulation using time domain spectral finite element method (TSFEM). Wave interaction with delamination causes reflection and transmission. Reflection happens from the leading delamination tip as well as trailing delamination tip. If the delamination length is small reflection from both the delamination tip cannot be identified separately as responses overlaps. To study the delamination tip effect on the reflection and mode conversion, very large delamination is considered that is wave length is very small compared to delamination length. Reflection characteristic as well as mode conversion is studied for various cases with different interface positions of delamination for various frequencies.

2. Mathematical modelling for semi-infinite delamination

In this section, analytical model of wave propagation in cross ply laminate with semi-infinite delamination is derived based on the Euler-Bernoulli beam approximation to study the delamination tip effect on the wave propagation. When incident wave reaches the delamination tip, some of the energy is reflected and rest is split into two sub-laminates. In order to identify the effect of the tip on wave interaction, semi-infinite delamination is considered. In reflected wave content there can be symmetric as well as anti-symmetric wave mode due to the mode conversion. Similarly in sub-laminates also both the wave modes can be present. It can be considered that wave propagation takes place in three regions. In base-laminate (region-1) forward moving incident $S_0$ wave, backward moving reflected $S_0$ and $A_0$ waves are present. In upper sub-laminate (region-2) and in lower sub-laminate (region-3) forward moving $S_0$ and $A_0$ waves are present.

Figure-1: Schematic diagram of tip of the delamination

First, displacement fields for a particular frequency are assumed, and all the displacement fields are considered with respect to neutral axis of corresponding regions. Notations are as follows, $\hat{u}_i^{(i)}$ is axial wave corresponding to $i^{th}$ region, $\hat{u}_i^{(i)}$ and $\hat{w}_i^{(i)}$ are amplitudes of forward and backward moving axial wave in region-i. Similarly, $\hat{w}_i^{(i)}$ denotes the transverse wave in region-i, $\hat{w}_i^{(i)}$ and $\hat{w}_i^{(i)}$ are the amplitudes of propagating part and evanescent part of
transverse wave respectively in region-i. \( k_{ai} \) and \( k_{bi} \) are the respective wave number for axial and transverse wave in \( i^{th} \) region. Displacement field due to axial and transverse waves at neutral axis in base laminate are given respectively

\[
\hat{u}^{(1)}(x, \omega) = \hat{u}_i^{(1)} e^{-ik_{ai}x} + \hat{u}_i^{(1)} e^{-ik_{bi}(L_1-x)}
\]

\[
\hat{w}^{(1)}(x, \omega) = \hat{w}_i^{(1)} e^{-ik_{ai}x} + \hat{w}_i^{(1)} e^{-ik_{bi}(L_1-x)}
\]

Displacement field of upper sub-laminate (region-2) is given by

\[
\hat{u}^{(2)}(x, \omega) = \hat{u}_i^{(2)} e^{-ik_{ai}x}
\]

\[
\hat{w}^{(2)}(x, \omega) = \hat{w}_i^{(2)} e^{-ik_{ai}(x-L_1)} + \hat{w}_i^{(2)} e^{-ik_{bi}(x-L_1)}
\]

Displacement field in lower sub-laminate (region-3) is given as

\[
\hat{u}^{(3)}(x, \omega) = \hat{u}_i^{(3)} e^{-ik_{ai}(x-L_1)}
\]

\[
\hat{w}^{(3)}(x, \omega) = \hat{w}_i^{(3)} e^{-ik_{ai}(x-L_1)} + \hat{w}_i^{(3)} e^{-ik_{bi}(x-L_1)}
\]

Following boundary conditions can be imposed to determine the wave amplitude in different region in terms of amplitude of incident \( S_0 \) wave. Continuity in axial displacements at the tip of the delamination gives

\[
\hat{u}^{(1)}|_{x=L_1} = \hat{u}^{(2)}|_{x=L_1} = \hat{u}^{(3)}|_{x=L_1}
\]

Continuity of transverse displacements at delamination tip provides

\[
\hat{w}^{(1)}|_{x=L_1} = \hat{w}^{(2)}|_{x=L_1} = \hat{w}^{(3)}|_{x=L_1}
\]

Continuity of bending slopes at delamination tip

\[
\frac{d\hat{u}^{(1)}}{dx} \bigg|_{x=L_1} = \frac{d\hat{u}^{(2)}}{dx} \bigg|_{x=L_1} = \frac{d\hat{u}^{(3)}}{dx} \bigg|_{x=L_1}
\]

Continuity in axial force at the tip of the delamination

\[
E_{eff}^{(i)} A_{eff}^{(i)} \frac{d\hat{u}^{(i)}}{dx} \bigg|_{x=L_1} = E_{eff}^{(2)} A_{eff}^{(2)} \frac{d\hat{u}^{(2)}}{dx} \bigg|_{x=L_1} + E_{eff}^{(3)} A_{eff}^{(3)} \frac{d\hat{u}^{(3)}}{dx} \bigg|_{x=L_1}
\]

where, \( E_{eff}^{(i)} \) and \( A_{eff}^{(i)} \) are the effective modulus of elasticity and cross section area of \( i^{th} \) region respectively. Effective modulus of elasticity is calculated from

\[
E_{eff} = \sum E_i h_i / \sum h_i
\]

where, \( E_i \) and \( h_i \) are the modulus of elasticity in axial direction and thickness of \( i^{th} \) lamina respectively. Equilibrium of shear force gives

\[
V_i|_{x=L_1} = V_2|_{x=L_1} + V_3|_{x=L_1}
\]

where, \( D_{11}^{eff} \) is flexural stiffness of beam corresponding to \( i^{th} \) region. Equilibrium in bending moment about neutral axis at \( x = L_1 \)

\[
M_i|_{x=L_1} = M_2|_{x=L_1} + M_3|_{x=L_1} + P_2 z_{n2} - P_3 z_{n3}
\]

where, \( P_2 \) and \( P_3 \) is the axial force exerted to the base laminate by sub-laminates (region-2 and region-3 respectively). \( z_{n2} \) and \( z_{n3} \) are the distance of neutral axis of upper and lower sub-laminates respectively from neutral axis of base laminate. From the boundary conditions we get a system of equations as given in equation (7-12) to solve for the amplitude of waves in different region in terms of amplitude of incident \( S_0 \) wave in base laminate.
3. Wave scattering from tip of a large delamination

An eight layer laminated composite beam is considered for analysis. Material properties of the laminate are as follows $E_{11} = 144.8$ GPa, $E_{22} = 9.65$ GPa, $G_{12} = 4.14$ GPa, $G_{23} = 3.45$ GPa, $\nu_{12} = 0.3$, $\nu_{23} = 0.49$, density $\rho = 1389$ Kg/m$^3$. Cross ply laminate has the stacking sequence of [(0/90)$_2$]. And a through thickness semi-infinite delamination is considered, tip of which lies at $L_1$ distance from free end. Wave amplitude in different region is calculated for various frequencies and result is compared with TSFEM simulation. For TSFEM simulation beam of 1200 mm length is considered and a long delamination with tip at 600 mm from the free end extended till the fixed end as shown in the figure-2. Actuators are mounted on both top and bottom surface of the beam at the tip as shown in the figure-2. Each lamina is modelled using one element in thickness direction and there are 2400 elements in axial direction. 3rd order polynomial is used to approximate the solution. And input voltage is applied to both the actuators in phase which creates the $S_0$ wave. Although axial wave is not dispersive but due to wave interaction with delamination may create $A_0$ wave which is dispersive in nature. So the single frequency wave is preferred for interrogation. Therefore five cycle tone-burst signal is considered to generate narrow-band wave. Responses are captured at the point A and point B to capture responses which are located at 500 mm and 700 mm away from the free end respectively. Point A captures the incident wave and reflected wave from the delamination using transducer $A_1$ and $A_2$, and responses for transmitted wave are captured at point B using transducers $B_1$ and $B_2$ for region-2&3 respectively. Wave generated at the tip propagates through the beam and interacts with the delamination. Wave scattering from delamination depends on its interfacial position.

![Figure-2: Composite beam with delamination modelled with actuators and stacking sequences given at right side](image)

Analysis is performed for cases with delamination located at mid-plane (between 4th and 5th lamina) and away from mid-plane (3rd interface that is delamination located between 3rd and 4th lamina). Wave amplitude in terms incident $S_0$ wave amplitude in different region for various frequencies are plotted in figure-3to5 for the delamination located at 3rd interface. From both the approaches it is observed that transverse wave also present in sub-laminates and base laminate, which indicate the mode conversion happens due to the incident $S_0$ wave. Wave field in different time is visualised in terms strain distributions in figure-6. As we can see that as wave reaches to delamination some of the part of wave energy is reflected rest transmitted. In case of delamination is in mid plane there is no reflection as well as no mode conversion and wave in both the sub-laminate propagates in same velocity as shown in figure-6(a). But when delamination is in 3rd interface there is mismatch in stiffness properties of both the sub-laminates, reflections and mode conversion happens. Waves in both the sub-laminates propagate in different velocities. Reflected and transmitted wave contains both $S_0$ as well as $A_0$ wave. As $S_0$ wave has higher velocity therefore $S_0$ wave propagates faster than the $A_0$ wave as observed in figure-6(b).
Figure-3: (a) Reflection coefficient ($\tilde{u}_2^{(1)}/\tilde{u}_1^{(1)}$) of axial wave and (b) reflection coefficient ($\tilde{w}_1^{(1)}/\tilde{u}_1^{(1)}$) of $A_0$ wave in region-1.

Figure-4: (a) Transmission coefficient ($\tilde{u}_1^{(2)}/\tilde{u}_1^{(1)}$) of axial wave and (b) transmission coefficient ($\tilde{w}_1^{(2)}/\tilde{u}_1^{(1)}$) of transverse wave in region-2.

Figure-5: (a) Transmission coefficient ($\tilde{u}_1^{(3)}/\tilde{u}_1^{(1)}$) of axial wave and (b) transmission coefficient ($\tilde{w}_1^{(3)}/\tilde{u}_1^{(1)}$) of transverse wave in region-3.
4. Conclusions

$S_0$ mode Lamb wave propagation in cross-ply with large delamination is modelled using analytical model as well as time domain spectral finite element method. $S_0$ wave does not create any reflection in case of delamination is in mid-plane hence is not sensitive to mid-plane delamination for symmetric cross-ply laminate. Off axis delamination cause the reflection as well as the mode conversion. Strength of the reflected wave and mode conversions depends on the difference in effective modulus of elasticity in axial direction between sub-laminates. If there is no difference in effective modulus of elasticity reflection is very week.
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References


