INTEGRATED APPROACH TO DEMONSTRATE OPTIMUM SENSOR POSITIONS IN A GUIDED WAVE BASED SHM SYSTEM USING NUMERICAL SIMULATION

Ramanan SRIDARAN VENKAT ¹, Christian BOLLER ¹, Lei QIU ³
Nitin BALAJEE RAVI ², Debiprosad ROY MAHAPATRA ², Nibir CHAKRABORTY ²

¹ Universität des Saarlandes, 66125 Saarbrücken, Germany
Ramanan.sridaran@uni-saarland.de, c.boller@mx.uni-saarland.de

² Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560012, India
nitinb@aero.iisc.ernet.in, droymahapatra@aero.iisc.ernet.in, nibir.chakraborty@aero.iisc.ernet.in

³ Key State Lab of Mech. & Control of Mech. Struct., Nanjing University of Aeronautics & Astronautics (NUAA), 210016 Nanjing, P.R. China
lei.qiu@nuaa.edu.cn

Abstract
In aeronautical applications a guided wave based structural health monitoring (SHM) system has the advantage of covering comparatively larger distances with high sensitivity for monitoring structural damages specifically when the structure has plate like shapes being common in various aeronautical structures. According to the damage tolerant principle, the structure has been designed for an allowable damage and the condition of the damage has been continuously monitored using piezoelectric wafer acoustic sensors (PWAS) that record signal having been generated by a piezoelectric actuator. The location of such actuators and sensors around the tolerable damage to be monitored has been widely regarded as the important prerequisite for an efficient SHM system design. Numerical simulations provide thorough understanding of wave mechanics within the structure. FEM based COMSOL Multiphysics has significant advantages of coupling piezoelectric and structural mechanics modules to model the guided wave propagation in a more efficient way. The advantage of viewing wave patterns at various time intervals for pristine and damaged conditions would allow one to compute differential signals/ differential images in order to identify the hot spot areas where the actuators and sensors could be placed to reliably detect the tolerable damage. However, FEM based simulation is time consuming when the structure to be simulated is large. In a modular approach, ray tracing has been combined with FEM for simulating those large structures in a less computation time. A plate with three holes and a crack on the center has been identified as the demonstrator for this paper. The simulated results have been validated with the experiments. Besides the optimal sensor placement, this paper attempts to introduce the coated piezoelectric material onto the plate based on the differential wave pattern from the FEM simulation.

1. Introduction

An active structural health monitoring (SHM) involves integrated actuators and sensors usually based on PWAS (Piezoelectric Wafer Active Sensors) attached onto the structure to continuously monitor the integrity of the structure considered [3]. Numerical simulation of guided waves brings thorough understanding of governing mechanisms of the wave propagation when it interacts with structural features and has been used as a major tool in scientific community. One of the prime requirements of the guided wave SHM approach is to deploy actuators/sensors for determining the known damage size. For a given appropriate loading
condition, the probable location of the damage can be obtained from a load (stress/strain) distribution simulation analysis. Based on the inputs of location and size of the tolerable damage, the number of actuators/sensors required to be placed on the structure for reliably detecting the respective damage can be determined. There are numerous ways in which one could achieve the optimum actuator-sensor pattern for the given allowable damages to be present in the structure for a general SHM system and are reported [12]. Theoretically, a dense sensor network would be more certain to reliably detect the damages. However, this may be highly impractical to implement since the volume of data to be processed should be big for a large structure. A sparse network with a high probability of detecting the tolerable damage has to be opted for a SHM applications. Qui et al [8] has reported a quantitative multi-damage monitoring method for large-scale aircraft component made of composites. In general, the sensors and actuators should be positioned not too close to the boundaries to avoid the influence of mutual interference between the waves from the damage and the waves from the boundaries [14]. Early work on optimum sensor positions for damage detection was performed by Hemez et al. [5]. In his work, he used strain energy distribution as a criterion to decide the optimum sensor placement. Lee [1] describes in his paper that the best possible solution is to place the sensors close to the damage while a first wave packet in a signal has to be considered. Stepinski et al [14] mentioned that at least three sensors are required to triangulate a damage instance with more accuracy. In recent study, Wandowski et al [15] used laser vibrometry along with PWAS actuator for non-contact sensing that allows to speed up the measurement process for circular sensing network in order to verify if a sensing network is valid candidate for damage detection or not.

There are numerous simulation methods listed for carrying out damage detection using guided waves [1, 10]. Among all those methods, FEM is well-established method and could be accomplished using commercial tools. In this paper, FEM approach and raytracing method are opted to simulate the guided wave propagation on a plate with the crack at the centre and the results from the simulations are used to optimize the sensor positions based on differential imaging and damage index measures.

2. FEM model

2.1 Governing equations

Three-dimensional FEM model is developed by means of COMSOL-Multiphysics that couples piezoelectric devices and the structure mechanics modules. The material behavior due to applied force is studied with in structure mechanics module and the piezoelectric effect is studied with in the electrostatics domain. This is shown in fig.1 where piezoelectric material is being applied as a common interface between the two coupling physics. 2D-FEM model was studied Nieuwenhuis et al [6] for the generation and detection of guided waves using PZT wafer transducers. The governing equations for piezoelectric effect i.e., the generated electric field for the applied stress, electrical displacement and converse piezoelectric effect i.e., total strain generated for the given electric field, stress can be written as follows:

\[ E = -g \cdot T + \beta^T \cdot D \]  
\[ S = s_E \cdot T + d^T \cdot E \]

Where,
\[ \mathbf{S}, \mathbf{T} \] - strain and stress
\[ \mathbf{E}, \mathbf{D} \] - electric field and electric displacement(charge)
d, $d^T$ - coupling between electrical and mechanical variables
$S_E$ - Strain per unit stress
$\varepsilon_T$ – Electric permittivity
$g, \beta^T$ - Piezoelectric coefficients and Impermittivity

The eq. (2) is used for modelling actuator and eq. (1) for modelling the sensor behaviour in the absence of temperature influence. The geometry and material parameters are shown in fig.2 and Table.1 respectively.

![Fig.1. FEM Multiphysics model](#)

<table>
<thead>
<tr>
<th>Material type</th>
<th>Properties</th>
<th>Dimensions</th>
<th>Crack geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium Plate</td>
<td>Young’s modulus-70 GPa Density- 2700 kg/m³ Poisson’s ratio-0.33</td>
<td>600mm x 112mm Thickness-1mm Ø6mm -3 holes at the centre.</td>
<td>Crack length 25 mm Crack height- 1 mm</td>
</tr>
<tr>
<td>PZT -5A</td>
<td>Ref- COMSOL material library</td>
<td>Ø8mm and 0.2mm thickness</td>
<td>Actuator at (x-56, y-250)</td>
</tr>
</tbody>
</table>

**Table 1. Model properties**

### 2.2 Model geometry

![Fig.2. Model geometry](#)

### 2.3 Excitation function

Since guided waves are dispersive waves, tone burst excitation was chosen in order to excite coherent single-frequency waves to minimize the dispersion effects and also to understand the characteristics of elastic wave propagation. But the raw tone burst excitation has side frequencies or side lobes associated with the sharp transition at the start and the end of the signal so, Hanning window function of the form (eq.3) is used to smooth the raw tone burst excitation to apparently reduce the side lobes [16]. The windowed signal and frequency spectrum of the smoothed tone burst signal is shown in fig.3 which does not show any side lobes.
Fig. 3. Hanning window tone burst excitation

\[ V(t) = \frac{1}{2} \left[ 1 - \cos \left( \frac{\omega_0 t}{N_C} \right) \right] \sin(\omega_0 t), t \in \left[ 0, \frac{N_C}{f} \right] \]  

(3)

Where,
- \( \omega_0 \) – excitation voltage
- \( \omega_0 \) – angular frequency
- \( N_C \) – number of counts that matches the length of the Hanning window

2.3 Mesh and critical time step

It is to be noted that the mesh element size used for the simulation in FEM models is critical for the accuracy of the solution. The definition of the maximum length element (\( \Delta x_{\text{max}} \)) is presented in eq.4

\[ \Delta x_{\text{max}} = \frac{\lambda}{R} \]  

(4)

where \( \lambda \) is the wavelength of the guided wave mode propagated in the direction of incidence and \( R \) is the number between 5 to 10. The use of a value of \( R \geq 8 \) was recommended by Ghose et al [2]. For the evaluation of critical time step, CFL (Courant-Friedrichs-Lewy) condition should be met to achieve the stable solution [9]. It is defined by the following relationship eq. (5).

\[ \Delta t_{\text{critical}} < \frac{\Delta x_{\text{max}}}{C} \]  

(5)

where \( C \) is the longitudinal ultrasonic wave of the material. In our model, \( \Delta x_{\text{max}} \) of 1 mm is used in the structure domain and \( \Delta x_{\text{max}} \) of 0.2mm is used for piezoelectric domain. The time step of 0.1\( \mu \)s satisfies the CFL criteria. Boundary conditions play very important role in the simulation, free boundary conditions have been adapted for this current model. However, in reality the results from the simulation and the experiment could vary due to the external factors such as temperature, actual loading etc., L. Qui et al [7] has introduced time varying boundary conditions as a Gaussian mixture model in order to account the external factors and that could even be considered for the numerical simulation.

3. Differential imaging

When the excitation voltage is applied on the piezoelectric material, the strain obtained in the aluminum plate is used for simulating the wave propagation problem in COMSOL Multiphysics. The relationship between the strain and the displacement is obtained by equation of motion (eq.6)
\[ \sigma_{ij} + \rho f_i = \rho \ddot{u}_i \]  

(6)

For the known elastic constants and by means of Hooke’s law \( \sigma_{ij} = C_{ijkl} \varepsilon_{kl} \), following constitutive relation is established between strain and the displacements and is implemented in COMSOL structural mechanic’s module.

\[ \varepsilon_{ij} = \frac{1}{2} \left( \dot{u}_{i,j} + \dot{u}_{j,i} \right) \]  

(7)

\[ \varepsilon_{zz} = \frac{\partial u_z}{\partial z} \]  

(8)

\[ \frac{\partial \Delta u}{\partial z \partial t} = \left( \frac{\partial u_{\text{crack}}}{\partial z \partial t} - \frac{\partial u_{\text{pristine}}}{\partial z \partial t} \right) \]

(a) Time at 30 \( \mu s \)  

(b) Time at 40 \( \mu s \)

Fig. 4. Guided wave propagation at various time steps

Where, \( \sigma \) and \( \varepsilon \) are the Cauchy’s stress and strain tensors and \( u \) is the displacement. One can write the differential displacements in the form eq. (8). Differential imaging method has been studied by the author and the method is used to obtain optimum sensor location for large aircraft panel [4]. The results of the FEM simulation are wave propagation at all the time steps and the recorded voltage signals at the sensor locations. The wave propagation at various time intervals is shown in fig.4 in 2D and is shown in fig.5 in 3D.
In guided wave SHM, the sensor signals at the present state of the structure are often compared with the signals that were stored during the pristine condition. This process is done only after placing the sensors which are supposed to be an optimum location for measuring the damage but in order to find the optimum position of the sensors, differential imaging is applied which is very similar to differential signals [16]. In this method, the wave propagation at particular time interval for the given damage is subtracted from the wave propagation of the pristine condition. The resultant wave propagation shown in the fig.6 shows the points of maximum and minimum residual displacements as the wave propagates over time and is given in eq. (8). The differential FEM model is built by implementing parametric sweep for an undamaged and damaged condition using COMSOL Multiphysics 5.2 or COMSOL allows to extract 2D plot data of the displacements and one can achieve differential imaging via Matlab tool. In this paper, crack of length 25mm is placed at the center of the plate is simulated.

![Fig.5 Wave propagation at different time intervals](image)

![Fig.6 Differential images of undamaged and damaged (25mm crack)](image)

**4. Evaluation of sensor signals based on damage index**

Damage detection in SHM of guided wave signals has been reported in many literatures [16, 17]. The simplest form of identifying the presence of damage is by signal differentials where the current state of the signal is subtracted from the signal in pristine condition. Higher the differential signals amplitude, more severe is the damage. The damage index or damage metric value is used to measure the amount of damage present in the structure through the statistical processing of the resultant signal with the pristine state signals. If any damage is present in the structure, the spectral content of the signal would be affected, the resultant scalar value is called as damage index. The simplest of the damage index could one calculate is RMSD (Root Mean Square Deviation) and the correlation coefficient deviation (CCD). In this paper, damage index based on RMSD will be discussed. It is expressed as in eq. (9)
Fig. 6 Differential images of undamaged and damaged (25mm crack)

\[
\sqrt{\frac{\sum_{i=1}^{N} (S_i - S_i^0)^2}{\sum_{i=1}^{N} (S_i^0)^2}} \tag{9}
\]

Where,
- \( S_i^0 \) - frequency spectrum of the sensor signal in pristine condition.
- \( S_i \) - frequency spectrum of the sensor signal in damaged condition (crack 25mm length).

In order to correlate the differential image with the damage index, RMSDs have been evaluated for various sensor positions and is shown in fig.6. Data is obtained by placing 8 sensors above the crack and 2 sensors (sensors 9 and 10) below the crack. Another feature that could be extracted from the sensor is its sum of energy of the whole signal. It is given by the eq.(10).

\[
\sum_{i=1}^{N} x_i(t) \tag{10}
\]

Where, \( x_i(t) \) - Amplitude of the signal at particular time. Energy plot is obtained for different sensor positions for the crack length of 25mm as shown in fig.9.

RMSD and energy plots shown above obviously agree with the differential image for the time at 60 µs. The RMSD (eq. 8) measures the deviation due to the presence of the damage in the signal whereas, eq.(9) measures the reflected energy. The sensor-6 and sensor-10 are at equal distance from the crack. Sensor-10 records high energy due direct reflection from the crack whereas the sensor-6 records the maximum differential energy. The intensity of differential image fig.6 for the sensors-6 and 10 are not the same because, the actuator is placed close to the sensor-10. Damage index scheme described here is highly recommended for numerical analysis and they vary for the experimental data due to time varying parameters.
5. Experimental validation

Experimental setup is shown in fig. 8 a) and b), where the actuator is excited at $\pm 70$V using the frequency generator and the power amplifier. The oscilloscope reads the actuator and sensor signals which are digitized at the sampling rate of 10 M samples per second. In fig 8.c), noise due to electromagnetic field is seen at the beginning and this noise is mixed with the transmitted signal in the fig.8 d). The experimental results of the sensor 6 and sensor 10 match closely with the simulated response and the difference in amplitudes are negligible.

6. Piezoelectric coating based on differential pattern

Generation and detection of guided waves using piezoelectric polymers is an alternative method to the conventional PWAS which are attached to the structure through an adhesive bonding layer. Piezoelectric-polymer composites were first developed as transducers to achieve mechanical and electrical properties, which cannot often be obtained with single phase materials. Recent advancements see that thin piezoelectric polymer composites are developed in order to be used as flexible transducers [11, 13] in SHM. The electrical, magnetic and mechanical properties of polymer composite materials can be tuned adjusting the
filler concentration. One of the main features of these materials include their ease of production, which makes it possible to obtain different sizes and shapes [11]. During the manufacturing of such composites, the most important requirement is to obtain a flexible and soft material with adequate thickness range, while maintaining good piezoelectric properties. Piezoelectric polymers, such as PVDF and its copolymers, represent an interesting choice as a matrix for piezoelectric composites due to its good electromechanical coefficients and high conductivity. However, it requires high electric field to pole and has poor thermal stability. Another suitable candidate for the polymeric matrix in piezoelectric composites is the epoxy resin, which is used in many applications due to its good mechanical and electrical properties, ease of manufacturing and low cost. The development of the fabrication process and characterization of a BaTiO3/epoxy resin piezoelectric composite is described in detail [10]. The BaTiO3 used in this study is a commercially available powder (Inframat advanced materials, USA). It was characterized as a material having an average particle size of 700 nm (reported by the manufacturer), $\varepsilon_r$ of 771 and a specific surface area of 1.8 m$^2$/g. The polymer used is an epoxy resin (Araldite MY750) with low viscosity (Hunstman, Belgium) hardened with sufficient amounts of isophorone diamine (Fluka, Germany). The development and characterization of coated piezoelectric composite is carried out in a joint collaboration with INM (Leibniz Institute for New Materials), Saarland, Germany. In this first attempt, a circular mould has been prepared to coat the piezoelectric composite onto the structure.

7. Conclusion

Advantages of differential images are as follows;
- It shows direction of maximum differential energy traveling on a specimen for a given damage condition.
- The coordinates of the maximum differential energy could be used to position the sensors for damage detection.

In guided wave SHM it is obvious that damage detection and damage sizing play key role in damage evaluation. The former is based on the comparison of two states; undamaged and the damaged. According to the differential image, the sensors at the coordinates of maximum differential path i.e., above and below the crack show higher probability of damage detection than the coordinates outside this region. Damage index based on RMSD reveal that the sensors above and below the crack regions record different transmitted and reflected differential signals. The sensors located close to the actuators and below the crack regions record high reflected energy from the crack. For the given crack configuration, it is necessary to position one sensor above the crack to have high POD and one sensor below the crack to get the maximum information of the crack which could be useful for damage sizing.

Another promising aspect investigated in this paper was the application of differential imaging to piezoelectric coating. New coating methodology based on BaTiO3/Epoxy resin is proposed as coating material which acts as a sensor for recording maximum differential displacements. Ability to fabricate and coat the required size and shape of such coated sensors play a significant role and they could be determined from the wave patterns of the differential image. Although, the research on the coated sensors is ongoing, the initial developments show that in future, large structures could be coated in order to efficiently monitor the given tolerable damage. Numerical simulations using FEM are time consuming when we have to simulate large aircraft wings etc., Raytracing combined with FEM is a hybrid approach which is highlighted in this paper shows lot of advantages when compared to FEM approach. The scheme described in this paper is adapted for finding optimal position for sensors provided the actuator positions are known. However, a more similar like scheme could be used for optimizing the actuator positions for a given tolerable damage.
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References
[9] Leonardo Zepeda-Nunez et al, ”Time-stepping beyond CFL: a locally one-dimensional scheme for acoustic wave propagation”, article referred from Dept. of Mathematics and Earth source lab, Massachusetts Institute of Technology