A damage localization procedure based on sparse reconstruction for Lamb waves

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Abstract

Lamb wave is a promising tool for structural health monitoring (SHM) in composite plate because of its attractive features, such as the ability to propagate a relatively long distance with less attenuation, high sensitivity to damage. Generally, a typical configuration is a sparse array of fixed or embedded piezoelectric disc sensors, which can both excite and receive guided waves. A damage localization procedure is presented for Lamb waves in this work. A scattering wavefield dictionary, which its atoms are the signal Hilbert envelopes of the corresponding scattering signals, is constructed for the sparse representation of the damage scattering Lamb waves. Using a priori knowledge that damage spot sizes are sparse compared with the area of interest, a sparse reconstruction model is built for damage localization. The sparse representation coefficients, representing the probability of damage, can be obtained by solving the $l_1$-minimization problem. Then the adaptive weighting factors, which are the cross-correlation coefficients of the scattering signal and the wavefield dictionary, are used to be the weights of the sparse representation coefficients. The presented damage localization procedure is validated using experimental data for a 2-mm carbon fiber composite.

Keywords: Lamb waves, Damage localization, Sparse representation, Carbon fiber composite

1. Introduction

The widely used composites in aerospace structures may suffer extreme loading and unknown impact, which may lead to damage, such as crack and delamination [1]. The accurate detection of damage is important to perceive the integrity information of structures. Among different approaches that are used in structural health monitoring (SHM) and non-destructive evaluation (NDT), Lamb wave based approaches have got a lot of attention [2]. Lamb waves, which are ultrasonic guided waves propagating in thin-wall plate structures, are a promising tool for SHM and NDT as its features such as the sensitivity to multiple defects, and the ability to inspect a relatively large structure with sparse sensing array [3]. In general, Lamb wave based damage detection approaches use a spatially distributed array of sensors in the structure to excite and receive Lamb waves, which is known as active-passive networks. If there is damage in the structure, the interaction of Lamb waves and damage will result in reflection and scattering, which will be recorded in the received signals.

There are several approaches have been developed to achieve the aim of damage detection and localization. Some algorithms utilize the time of flight of echoes of several sensor pairs and the wave propagation velocity to locate damage, such as delay-and-sum (DAS) imaging [4] and minimum variance distortionless response (MVDR) imaging [5]. In some cases of that the accurate wave propagation velocity is difficult to obtain, the reconstruction algorithm for the probabilistic inspection of damage (RAPID) [6] can be used to locate damage. But this algorithm needs a high density sensor networks in order to cover the area of interest. Recently, sparse reconstruction based imaging algorithms have been proposed [7,8,9]. Those algorithms utilize the wave propagation model to generate a scattering dictionary and obtain the value of damage probability via sparse reconstruction. The imaging quality mainly depends on the dictionary constructed.
This work presents a damage localization procedure based on sparse reconstruction for carbon fiber composite. A sparse signal reconstruction based imaging algorithm is used to locate damage, and adaptive weights based on correlation are used to suppress noise and improve the imaging quality. The effectiveness of the presented procedure is valid by experimental study.

2. Damage localization procedure

2.1 Lamb wave scattering waveform

In an isotropic or quasi-isotropic thin-wall plate, sensors attached on a single surface of the plate can measure a Lamb wave’s two displacement components, which contain the vibration in the direction of wave propagation and perpendicular to the plate’s surface. Under the excitation with function \( S(\omega) \) in frequency domain, the measured direct Lamb waveform at received sensor can be represent as

\[
X(r, \omega) = \sum_m \frac{1}{\sqrt{r k_m(\omega)}} S(\omega) G_m(\omega) e^{-jk_m(\omega)r},
\]

where \( X(r, \omega) \) is the measured Lamb waveform in frequency domain, \( r \) is the distance between excitation and received sensor, \( \omega \) is the angular frequency, \( G_m(\omega) \) and \( k_m(\omega) \) represent the amplitude response and wavenumber function of each mode \( m \), respectively. In this work, the amplitude response \( G_m(\omega) \) is assumed to be independent of frequency as narrowband excitation is used. If there are damage-free in the plate, the received Lamb waveform, \( y_{\text{fre}}(t) \), can be expressed as

\[
y_{\text{fre}}(t) = y_{\text{dir}}(t) + n(t),
\]

where \( y_{\text{dir}}(t) = F^{-1}\{X(r, \omega)\} \) is the direct waveform without any reflections, and \( n(t) \) is noise. It is important to note that, the scattering waveforms from the boundaries of the plate are included in \( n(t) \). Those additional contributions in composite plate are relatively weak in general compared with the direct waveform from the excitation to the received sensor.

Now consider to model the received Lamb waveforms when there is a scatterer in the plate. The received waveform contains the component measured in damage-free state and the component scattering from the scatterer. Suppose that the distances from the excitation location to the scatterer, and the scatterer to the received sensor location are \( r_1 \) and \( r_2 \), respectively. The scattering waveform, which propagates from the excitation location to the scatterer, and then to the received sensor location, can be expressed as follows without considering mode conversion

\[
y_{\text{sca}}(t) = \sum_m \frac{1}{\sqrt{(r_1 + r_2)k_m(\omega)}} S(\omega) G_m(\omega)e^{-jk_m(\omega)(r_1 + r_2)}. \]

Thus the received waveforms, \( y_{\text{rec}}(t) \), can be represented as

\[
y_{\text{rec}}(t) = y_{\text{fre}}(t) + y_{\text{sca}}(t).
\]

In Eq. (4), the received waveforms contains two main components: one is the baseline, which is free of the scatterer; the other is the scattering waveform, which is closely related to the scatterer and its location. In general, most Lamb wave based detection method need to get the baseline of the measured structure. The baseline is usually measured when the measured structure is damage-free. Owning the baseline, the scattering waveform only related to the scatterer and its location can be separated from the received waveform by subtracting the baseline.
\[ y_{\text{sc}}(t) = y_{\text{rec}}(t) - y_{\text{Ref}}(t). \]  

2.2 Damage imaging based on sparse reconstruction

In order to utilize sparse reconstruction to achieve the aim of damage imaging, one needs to transform the imaging problem into a sparse representation form. The detected area is discretized into \( M \) pixels and each pixel value represents the probability of damage. The whole \( M \) pixel values can be represented as a \( M \times 1 \) vector \( \mathbf{a} \). For a sensor network containing \( L \) sensors, if a round-robin fashion is used, which one of the sensors excites at a time and the other record at the same time, then a total of \( L(L - 1)/2 \) exciter-receiver pairs can be used to measure Lamb waveforms (the signal from sensor \#i to sensor \#j is considered to be the same with the one from sensor \#j to sensor \#i). For a specific scatterer located at the \( i \)th pixel of the plate, the scattering waveform for the \( i \)th exciter-receiver pair can be obtained using Eq. (3), as follows

\[ \mathbf{d}_i = \sum_{m} \frac{1}{\sqrt{(\kappa_1^2 + \kappa_2^2)}} S(\omega) G_m(\omega) e^{-j \kappa_m(\eta_i + \tau_i)}. \]  

Therefore a total of \( L(L - 1)/2 \) scattering waveforms can be obtained for the scatterer located at the \( i \)th pixel of the plate and it can be formed a vector as

\[ \mathbf{d}_i = [\mathbf{d}_{i,1}, \mathbf{d}_{i,2}, \ldots, \mathbf{d}_{i,L(L-1)/2}]^T. \]  

Then a dictionary can be built using all the scattering waveforms for different scatterer locations from the first pixel to the \( M \)th pixel. The dictionary is formed as

\[ \mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_M]. \]  

The actual measured scattering waveforms are obtained by Eq. (5) for each exciter-receiver pair. We form all the scattering waveforms together into a vector as

\[ \mathbf{y} = [y_{\text{sc,1}}(t), y_{\text{sc,2}}(t), \ldots, y_{\text{sc,L(L-1)/2}}(t)]^T. \]  

It should note that the length of waveform \( y_{\text{sc},i}(t) \) must be the same with \( \mathbf{d}_i \) in order to form the linear model

\[ \mathbf{y} = \mathbf{D} \mathbf{a} + \mathbf{e}, \]  

where \( \mathbf{e} \) is noise term. One can obtain the pixel values by solving the Eq. (10). The problem of Eq. (10) can be transformed into a basis pursuit denoising (BPD) problem

\[ \hat{\mathbf{a}} = \arg \min_{\mathbf{a}} \| \mathbf{a} \|_1 \text{ subject to } \| \mathbf{y} - \mathbf{D} \mathbf{a} \|_2 \leq \sigma, \]  

where \( \sigma \) is the standard deviation of noise. In this work, \( \sigma \) is specified to be 0.2 \( \| \mathbf{y} \|_2 \).

2.3 Adaptive weights based on correlation

The adaptive weight for each pixel value are defined as

\[ w_j = \frac{\text{Cov}(\mathbf{y}, \mathbf{d}_j)}{\sigma_y \sigma_{\mathbf{d}_j}}, \]  

where the covariance, \( \text{Cov} \), is defined as

\[ \text{Cov}(\mathbf{y}, \mathbf{d}_j) = \sum_{n=1}^{N} (y_n - \mu_y)(d_{n,j} - \mu_{\mathbf{d}_j}), \]  

and the standard deviations, \( \sigma_y \) and \( \sigma_{\mathbf{d}_j} \), are defined as

\[ \sigma_y = \sqrt{\sum_{n=1}^{N} (y_n - \mu_y)^2}, \sigma_{\mathbf{d}_j} = \sqrt{\sum_{n=1}^{N} (d_{n,j} - \mu_{\mathbf{d}_j})^2}, \]
where $y_n$ is the $n$th element of vector $y$, and $d_n^i$ is the $n$th element of vector $d_i$. $N$ is the recording length of vector $y$, $\mu_y$ and $\mu_d$ are the mean values of vector $y$ and $d_i$, respectively. By multiplying the adaptive weight with the corresponding pixel value in vector $\alpha$, the weighted pixel value can be represented as

$$\alpha'_i = \alpha_i \mu_y \mu_d,$$

where $\alpha'_i$ is the $i$th weighted pixel value, and $\alpha_i$ is the corresponding $i$th original pixel value obtained from Eq.(11).

In the above procedure, the Lamb waveforms used for imaging contain both amplitude and phase information. While in Lamb wave application, the phase information may lead to poor performance as the scattering behaviour is closely related to the phase information and their relationship is complicated and hard to access in many practical situations. Therefore all the waveforms above are replaced by their Hilbert envelopes to keep the amplitude information and discard the phase information.

3. Experimental study

Lamb waves are recorded by permanently attaching PZTs to the surface of a carbon fiber composite of 16-layers (angles of fiber layers are $[0/\pm 45/90]_2$) as shown in Fig. 1(a). Lamb waves are generated and recorded using eight PZTs that are glued to the same surface of the plate as per locations shown in Table 1. The origin of coordinate is the geometric center of the plate. The plate is 2 mm thick and its dimensions are $400 \times 400$ mm. The damage is simulated by two $\varnothing 10$ mm $\times$ 10 mm cylindrical neodymium magnets adsorbing on both surfaces of the plate, as shown in Fig. 1(b). Lamb waves are generated by exciting one of the eight sensors in turn with a 60 volt 5-cycle Hanning windowed tone burst excitation centered at 80 kHz, and the response signals received by each the other seven sensors are recorded. The A0 mode Lamb waves are dominant with such excitation according to the wavelength tuning effect [10]. Thus 56 signals are recorded in total for each measurement but only 28 of those 56 signals are used for further analysis as for reciprocity between each sensor pair. Two such measurements are made, one (baseline) without the simulated damage and one with the simulated damage. The scattering signals are obtained by subtracting the signals with the simulated damage from the baseline.

Fig. 1. (a) Specimen with sensors. (b) Diagram of the plate showing locations of the eight sensors (numbered circles) and the two masses within the $400 \times 400$ mm area of the plate.
In this work, the anisotropic nature of the plate and its dispersion effect is not considered, or more specifically, the term $k_x(\omega)$ in Eq. (6) is linearized and it can be approximated by $\omega / c_g$, where $c_g$ is the average measured group velocity of the recorded Lamb waves. Only the A0 mode Lamb waves are considered and included in the dictionary, and the S0 mode Lamb waves are treated as noise. All the 28 scattering signals are replaced by their corresponding Hilbert envelopes. The weights of pixels are shown in Fig. 2(a). It can be seen that the values of the weights of pixels around the damage locations are larger than those far away from the damage. However, it cannot effectively identify the two simulated damage from Fig. 2(a). The imaging result of the proposed procedure is shown in Fig. 2(b) with a 20 dB scale. From this imaging result, one can clearly distinguish the two damage locations. The two simulated damage is successfully detected and located by the proposed damage localization procedure.

### Table 1. Coordinates of sensors and masses.

<table>
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<tr>
<th>Description</th>
<th>x (mm)</th>
<th>y (mm)</th>
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<tbody>
<tr>
<td>Sensor #1</td>
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<td>-100</td>
</tr>
<tr>
<td>Sensor #2</td>
<td>100</td>
<td>-100</td>
</tr>
<tr>
<td>Sensor #3</td>
<td>100</td>
<td>0</td>
</tr>
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<td>Sensor #4</td>
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<td>100</td>
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</tr>
<tr>
<td>Mass #2</td>
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<td>-75</td>
</tr>
</tbody>
</table>

![Fig. 2. (a) The weights of pixels. (b) The weighted imaging result showed in 0~20 dB scale. The white dots represent the locations of the sensors, and the “+” are the two locations of the simulated damage.](image)

### 4. Conclusion

In this work, a damage localization procedure is presented for Lamb wave SHM. Some conclusions can be obtained as follows.

1) The correlation based adaptive weights can roughly highlight the location of damage but may not fine enough to distinguish the number of defects.
2) The sparse signal reconstruction based imaging method combined with the adaptive weights can effectively locate damage and suppress noise.
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References