Interpreting results of NDT/SHM methods based on elastic guided waves in composite plate-like structures by a semi-analytic modal simulation tool

Jordan Barras¹, Alain Lhémery¹
¹CEA List, Gif-sur-Yvette, France
*corresponding author, E-mail: alain.lhemery@cea.fr

Abstract
Elastic guided waves (GW) are valuably used for NDT/SHM of metallic or composite plate-like structures. They allow fast scanning of large parts for the detection of all kinds of inner or surface defects and flaws. However, their complex behaviour is quite challenging as far as solving the inverse problem of defect characterization from a set of measured signals is concerned. Consequently, designing optimal NDT/SHM configurations for a given part, accurately interpreting measured signals, make it necessary to grasp the various phenomena that arise. Simulation obviously constitutes an efficient and quantitative way of doing it.

Here, results obtained by means of a newly developed model are presented to demonstrate how the understanding of complex waveforms typically measured in NDT/SHM experiments is greatly simplified as soon as the modal nature of the various wave packets can be predicted, exhibited and understood.

1. Introduction
Elastic guided waves (GW) have long been a promising field of research for non-destructive evaluation and testing (NDT) of thin structures. They can propagate over long ranges and can be used to efficiently scan large parts at their speed. Thus, it is possible to attach a restricted number of transducers to the structure while ensuring full coverage; this is at the origin of the Structural Health Monitoring (SHM), an active research topic supported notably by oil, aeronautic and nuclear industries [1]. Nevertheless, GW have a dispersive and multi-modal behaviour resulting in complex time-dependent signals. Consequently, it is often difficult to design reliable NDT and SHM methods based on GW (this explaining why the industrialization of these methods has been delayed).

Simulation is an effective way to deal with above-mentioned difficulties. Various numerical methods such as the finite element (FE) method have been used to predict GW radiation in thin plates but they can become prohibitively computer intensive for large structures [2]. Simulation tools must account for transducer diffraction effects to ease choosing the most appropriate source and receiver. For aerospace applications, they also must be able to predict wavefields in commonly used composite structures.

In this paper, a semi-analytical model, based on ray technics [3-5] is proposed. It predicts modal wavefields that are more understandable than global ones. It is dedicated to field calculation on a restricted set of points (for example points corresponding to a receiver position). Furthermore, results can be re-used to perform parametric studies on source emission shape, for example. Typical predictions are shown and compared to FE results for validation, demonstrating the great interest of predicting the modal structure of complex wavefields.

2. Modal GW pencil for composite plates
The approach defined below describes the wave spreading of a guided mode generated by a point source emitting a harmonic load on the top surface of a plate. The medium can be a composite laminate made of orthotropic plies. The method, based on geometrical elastodynamics, combine a semi-analytical propagation model and a numerical computation of modes [6]. The propagation model relies on 2D geometry. In this section, the modal GW pencil principle is introduced. The method to deduce particle displacement field from pencils is explained.

2.1. Pencil definition and propagation
The concept of modal GW pencil is easily understandable through geometrical considerations. A pencil is composed of a ray linking a source point \( \mathbf{R}_0(x_0, y_0, d) \) and a calculation point \( \mathbf{r}(x, y, z) \) that follows an energy path, the so-called axial ray. Around it, a bundle of rays originating from the same point as the axial ray but diverging infinitesimally from the latter is considered. These rays are called paraxial rays. Describing the pencil propagation consists of describing the propagation of the axial ray and that of a paraxial ray relatively to the axial one. Thus, a pencil is represented at each of its states by the bi-vector called paraxial ray state vector:

\[
\Psi = \left( \frac{d\zeta}{dp_i} \right),
\]

where the differential paraxial gap between rays \( d\zeta \) and the differential paraxial gap between their slowness \( dp_i \) are represented on Fig. 1. A pencil can be defined for each propagative mode, each energy direction \( \varphi \) and its associated phase direction \( \gamma_m \).
Figure 1: Schematic top view of a pencil axial ray and one of its paraxial rays. Paraxial differential quantities are shown at two pencil states (1 and 2).

It is possible to describe the evolution of $\Psi$ between two states thanks to elementary 2D geometry considerations. Corresponding vectors are linked by the propagation matrix:

$$L_{\text{propa}} = \begin{pmatrix} 1 & \frac{\partial r m}{\partial \phi} \gamma m \cos(\gamma m - \phi) \\ 0 & 1 \end{pmatrix},$$

(2)

2.2. Energy conservation within a pencil

A key point of the pencil method is that the energy is assumed not to leak through the pencil lateral envelop. In other words, energy is conserved within the pencil during propagation. The acoustic intensity at point $r$ depends on the initial acoustic intensity $I_0$ as follows:

$$I_r = \frac{d\Omega_0}{d\Omega_r} I_0,$$

(3)

where surfaces $d\Omega_0$ and $d\Omega_r$ are described on Fig. 2. Furthermore, it is possible to link $I_r$ and the modulus of the displacement field at point $r$ by calculating the mechanical energy.

Figure 2: Schematic view of a pencil envelop of paraxial rays.

Then, $I_0$ is calculated using the modal Green formalism introduced in [7]. Finally, a new expression of the modal Green tensor $g_m$ is obtained. In this expression, the ratio $d\Omega_0/d\Omega_r$ simply corresponds to the upper-right coefficient of the propagation matrix. The displacement field is then expressed as follows:

$$\hat{u}(r, \omega) \approx \sum_m g_m(r; R_0, \omega) \hat{q}(R_0, \omega),$$

(4)

where $\hat{q}$ is the Fourier coefficient of the load imposed as source term.

2.3. Finite-sized sources

In practice, surface loads are imposed over an area of finite size (e.g. PZT transducers). The field radiated by the source is simply obtained by integrating Eq. (4) over its surface:

$$\hat{u}(r, \omega) = \sum_m \left\{ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} g_m(r; R, \omega) \hat{q}(R, \omega) d\rho d\theta \right\},$$

(5)

In Eq. (5), the surface integral has been expressed in polar coordinates $(\rho, \theta)$ centred on the calculation point to simplify its calculation over each incoming energy direction $\theta$.

3. Reflection of a modal pencil in finite-sized plates

The pencil formalism becomes particularly interesting when the plate is of finite size as it can easily deal with an arbitrary number of reflections at plate edges. In this section, the previous model is derived to take them into account.

3.1. Free boundary condition on edges and mode conversions

Let us consider an incident plane GW composed of a single mode $(i)$ reflecting on a plane edge of normal $n$. We want to determine propagative reflected modes, following Snell-Descartes law on wavevectors projections on the interface $k(i) \cdot (e_z \times n) = k(j) \cdot (e_z \times n)$. The distribution of incident energy among reflected modes is driven by the free boundary condition chosen herein, which writes

$$\sigma(i) \cdot n + \sum_j r_{ij} \sigma(j) \cdot n = 0,$$

(6)

where $r_{ij}$ is the amplitude reflection coefficient representing the conversion of the incident mode $(i)$ into the reflected mode $(j)$. Here, the basis of reflected modes is composed not only of propagative modes but also of evanescent ones. Indeed, reflection is a local phenomenon where evanescent modes intervene in the energy balance.

3.2. Geometrical reflection of a pencil

When mode conversions occur, there is generally no more energy conservation in a single pencil. Rather, the energy is distributed among several pencils originating from the same incident mode and the same energy direction. The product of the various energy reflection coefficients $\Pi_{i,f}$ encountered by one pencil satisfies this repartition. Energy reflection coefficients are easily deduced from amplitude reflection coefficients. Equation (3) becomes:

$$I_r = \Pi_{i,f} \frac{d\Omega_0}{d\Omega_r} I_0,$$

(7)
Thus, \( \psi_{i..f} \) is taken into account in the expression of the Green tensor \( g_{i..f} \) which follows the pencil modal sequence denoted by \( i..f \). Fig. 3 shows the path followed by all pencils linking a circular source and a single calculation point after two reflections (on edges A then B) with the modal sequence \{mode 1; mode 2; mode 3\}.

![Diagram of double reflection of pencils originating from a circular source to a single calculation point with two mode conversions.](image)

As previously done for the propagation matrix describing the evolution of the pencil state vector between two positions, it is possible to geometrically build the reflection matrix:

\[
L_{\text{refl}} = \begin{pmatrix}
-\frac{\alpha_2}{\alpha_1} & 0 \\
0 & -\frac{\alpha_3}{\alpha_2}
\end{pmatrix},
\]

where:

\[
\Omega_i = \frac{\cos(\psi_i - \theta_n)}{\cos(\psi_i - \psi_1)},
\]

the angle \( \theta_n \) being the angle between the edge normal and the \( x \)-axis.

By multiplying each elementary matrix (propagation or reflection) by the left following their order of appearance, we obtain the global evolution matrix of a pencil sequence.

### 3.3. Example: reflection in a semi-infinite-sized plate

To illustrate the implementation of the pencil method, the displacement field is computed on the upper surface of a plate for an excitation frequency of 100kHz. The plate is semi-infinite, delimited by an edge located at \( x = 500 \) mm. Plate material is described in section 4, Table 1. At this frequency, wavefields are decomposed on their three fundamental modes \( qA_0 \), \( qSH_0 \), and \( qS_0 \). The norm of particle displacement is plotted on Fig. 4.

![Figure 4: Displacement field at 100 kHz before reflection (top), after edge reflection (2nd line) and their sum (bottom). Results are obtained for the three fundamental modes \( qA_0 \) (left), \( qSH_0 \) (middle) and \( qS_0 \) (right).](image)

On the first line, the field before reflection (pencils directly linking a point source to a calculation point) is given. The \( qA_0 \) mode is preponderant while the radiated amplitude of \( qSH_0 \) is low.

On the second line, modal fields after edge reflection are shown. The mode \( qA_0 \) is not converted into symmetric modes (\( qSH_0 \) and \( qS_0 \)) so it is reflected with an amplitude of the same order as that of the incident field. Conversely, mode conversion occurs from \( qS_0 \) to \( qSH_0 \) with a reflection coefficient that increases when the incident angle increases. This is why the value of reflected \( qSH_0 \) displacement field is higher than that of incident \( qSH_0 \).

The sum of direct and reflected fields is given on the third line showing their interferences, which would be difficult to interpret without the prior knowledge of the separated fields.

### 4. Interpretation with the modal decomposition of time-dependent signals

The composite material modelled in study case is a CFRP composite made 16 transversely isotropic plies of thickness 133.75 \( \mu \)m with the following orientation sequence relatively to the \( x \)-direction: [45°, 0°, −45°, 90°, 45°, 0°, −45°, 90°]_S. Table 1 gives its stiffness coefficients (only the independent values) and density:

<table>
<thead>
<tr>
<th>( c_{11} ) (GPa)</th>
<th>( c_{22} ) (GPa)</th>
<th>( c_{66} ) (GPa)</th>
<th>( c_{12} ) (GPa)</th>
<th>( c_{23} ) (GPa)</th>
<th>( \rho ) (g.cm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>145.5</td>
<td>11.5</td>
<td>5.2</td>
<td>5.0</td>
<td>5.258</td>
<td>1.59</td>
</tr>
</tbody>
</table>

![Table 1: Stiffness coefficients for a ply of 0° orientation](image)
In this section, a comparison of time-dependent signals with results provided by a Finite Element (FE) code is performed as a validation mean. Then, the modal decomposition of these signals is given in order to understand the underlying physics.

4.1. Validation with a FE code
Results are computed using an in-house FE code called ONDO [8] solving problems in acoustics or elastodynamics. The studied plate is a square one (2000 mm on each side). The source, located at the centre of the plate, is a uniform load of normal stress with a time-dependency given by a 10 cycle-Hann windowed cosine at 100 kHz.

Fig. 5 presents superimposed signals obtained at point (1400, 1200, 1.07) for each component of the particle displacement. The semi-analytical model has been stopped after the second reflection. This is why several fast propagating third-reflected wave packets are not predicted. Predicted packets fit very well with those computed by the FE model, this partially validating the pencil method. Nevertheless, an unpredicted packet around 1550 µs comes from the diffraction of the $qA_0$ mode by the nearest corner. Indeed, no diffraction effect can be taken into account by the pencil model alone.

Figure 5: x (left), y (middle) and z (right) components of the particle displacement at point (1400, 1200, 1.07) predicted by FE ONDO (top) and the present method (bottom).

4.2. Modal decomposition
The pencil method was derived to provide a modal solution to a problem involving complex wave phenomena. Its interest is illustrated in Fig. 6, by taking as an example the x-component signal shown on Fig. 5. As the source does not generate $qSH_0$ mode, sequences starting with this mode do not exist and corresponding signals are not shown. For antisymmetrical modes, the interpretation is straightforward: one gets a single direct wave packet, 4 packets involving one reflection (one per plate edge) and then several packets involving two reflections. For symmetrical modes, wave packet are much more difficult to interpret (this would have been impossible without modal interpretation). Indeed, reflected wave trains due to a single or two reflections are overlap because of multiple mode conversions between $qSH_0$ and $qS_0$ modes.

Figure 6: Modal decomposition of the x-component of particle displacement at (1400, 1200, 1.07). Left: symmetric modes. Right: antisymmetrical modes.. Top: direct contribution. Middle: contribution involving one reflection. Bottom: contributions involving two reflections.

4.3. Discussion
Although signals presented in this section are relatively simple (only three modes are concerned; the plate being large, wave packets are rather well separated; the excitation bandwidth is narrow), it is already difficult to interpret them without the help provided by their modal decomposition. In more complicated cases, it soon becomes impossible. The knowledge of modal decomposition is also interesting as it is associated to accurate knowledge of mode shapes and wavelengths. Each mode will have a particular behaviour when interacting with a particular type of flaw and it is easier of course to fully understand these interactions with a thorough modal analysis.

5. Conclusion
A newly developed method to predict GW fields radiated by finite-sized sources in finite-sized composite plates has been described. It is inherently modal so that predictions are associated to a full understanding of the complex wave phenomena involved. This is very helpful to rationally design NDT/SHM methods and setups.
As far as computing performances are concerned, the implementation of the method is fully parallelizable: each calculation (for a single calculation point, frequency and modal sequence) is independent from the others. Its extension to deal with radiation in slowly continuously inhomogeneous media is under investigation. Its hybridization with a FE code (harmonic) to deal with GW scattering by a localized defect is also under investigation. This will allow one to predict modal coefficients for the scattering by defects and will give access to a new basis of interpretation of complex waveforms typically used to form flaw images in NDT/SHM practices.
References


