ELECTRICAL LOADING EFFECTS ON LEAKY LAMB WAVES FOR PIEZOELECTRIC PLATE BORDERED WITH A FLUID: ANALYSIS AND MEASUREMENTS

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Abstract

Leaky Lamb waves are important research topics in the area of NDE and acoustic wave sensing technology. For waves propagating in a piezoelectric plate immersed in a fluid (dielectric or conductive), both mechanical and electrical loading effects present and result in phase velocity variation and wave attenuation. To quantitatively characterize these loading effects play an important role in various applications of these plate waves. In this work, we apply the partial wave theory for analyzing the fluid's dielectric and conductive loading effects on an *XZ*-cut LiNbO3 plate. Both phase velocity variation and wave attenuation as a function of the fluid's permittivity and conductivity are numerically determined. Furthermore, a new differential type measurement system is developed to accurately and sensitively measure the loading effects on the leaky Lamb waves. Good agreements between experimental data and numerical results are observed, which verifies the analytic model of the leaky Lamb wave and can be useful for further development of dielectric or conductivity sensors based on leaky Lamb waves.

1. Introduction

Lamb waves are guided elastic waves propagating in a plate placed in vacuum or air, so that both plate surfaces are traction-free. It is also known as the free or pure mode of Lamb wave. If one or both of the plate surfaces are in contact with a fluid, part of the energy will leaks into the adjacent fluid. This attenuated guide plat wave can be easily found in many applications such as biosensors or conductivity sensors [1~3] and is called leaky Lamb wave in order to distinguish it from the free mode Lamb wave. In above applications, the wave guides are usually the piezoelectric plates subjected to fluid loadings. The guided plate waves leak energy into adjacent fluids from mechanical and electrical loadings through the mechanical and electrical coupled boundary conditions respectively at the plate/fluid interfaces.

The mechanical loading, also known as mass loading, is from the mass of the fluid molecules loading on the plate surface. The electrical loadings are induced from the electrical fields inside the piezoelectric plate coupled into the electrical fields in the adjacent fluids, which can be distinguish in dielectric and conductivity loadings. In practice, all kinds of these loadings will combine together and affect the wave propagation behaviors in two ways, phase velocity change and wave attenuation.

Since 1980, Adler [4] and Nayfeh [5] have been analyzed this problem with an approach known as partial wave theory. However, only the fluid mechanical loading effects are considered in Nayfeh's research. Sequentially, Yang and Chimenti [6-8] analysis leaky Lamb wave problems theoretically and experimentally. Both the mechanical and electrical (dielectric) loading effects are considers in their investigations. Further, the same theory has been also adopted by Yang et al [9-11] for analyzing similar problems involving fluid conductivity loading effects.

In reviewing all previous works, we find that the fluid electrical loading effects can strongly dominate the leaky Lamb wave on piezoelectric plate in theoretical analysis [10, 11]. The conspicuous case is the fundamental symmetry mode (S0 mode) Lamb wave propagating along 40^o azimuthal angle of XZ-LiNbO3 wafer [11]. Under certain conditions, the S0 mode Lamb wave velocity can be changed about 900m/s [11] fluid loading under fluid loadings. However, in previous calculations [6-11], the leaky Lamb wave problems of piezoelectric plates are only partially solved since the wave attenuation is missing. It is because in previous works [4-11], an incident acoustic wave is always involved, which will vanish the wave attenuations of leaky Lamb wave and force the wave number as a real number[12]. Therefore, the exact solutions of partial wave theory are not completely given yet.

Besides, a variety of methods have been developed for leaky Lamb wave measurement in previous works such as immersion-type dual-transducer wave reflection measurements [8, 9], pulsed laser ultrasound technique with a 2D FFT waveform processing method [10, 11], and directly mounted inter-digital transducer (IDT) on the piezoelectric plate for wave generating and receiving [3]. However, there are no convincing experimental measurements for the leaky Lamb waves of piezoelectric plates to support the theoretical results. This is because the fluid-loading effects can be very small and the experimental methods in previous works may be inadequate to detect them. Therefore, a more precise and accurate measurement method is required to detect these small variations in the propagating characteristics of leaky Lamb waves.

Hence, in this paper, we will analysis the leaky Lamb wave of piezoelectric plate subjected to fluid electrical loadings theoretically and experimentally. A 500μm thickness XZ-LiNbO3 wafer will be applied as a sample plate and we will focus on the S0-mode leaky Lamb wave propagates along 40^o azimuthal angle. The sample plate will be immersed in the fluid with different dielectric and conductivity loadings. In theoretical works, we will re-visit the partial wave theory with the goal to establish an accurate way of calculating the characteristics of leaky Lamb wave. Both the leaky Lamb wave velocity and wave attenuation influenced by dielectric and conductivity fluid loadings can be determined quantitatively. In experimental works, a novel measurement system is developed to determine the fluid loading effects with a differential measurement concept. This new concept allows this system to measure the fluid loading effects with high accuracy. With this novel system, this paper tries to offer some convincing experimental measurements to support the partial wave theory or not.

2. Theoretical Analysis

To analyze the leaky Lamb waves, the partial wave theory is applied. The basic idea of partial wave theory is first to reconstruct the formula based on superposition of homogeneous and inhomogeneous waves in the plate and the fluid, then the boundary conditions at the plate/fluid surfaces are applied to solving the wave velocity and attenuation of the leaky Lame wave. This standard derivation can be found in Refs [5-11] as well as in a monograph by Nayfeh [13], however, there is no incident wave involved in our approach.

2.1. Wave Fields in Piezoelectric Plate

Consider an XZ-$LiNbO_3$ plate having a thickness *2h* and oriented along the Cartesian coordinate system as shown in Fig.1. In Fig.1, the (X, Y, Z) coordinates are the crystolograpical system and the wave propagation coordinates is (x_1-x_3-x_2). The (x_1-x_3) plane is the sagittal plane of Lamb wave propagation and the coordinate x_1 indicates the wave propagating direction which is at an azimuthal angle q measured from Y-axis direction..

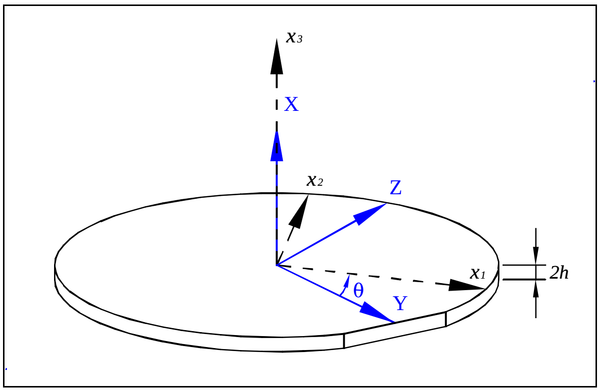


Figure 1: *The coordinate systems of XZ-$LiNbO_3$ plate.*

With the coordinate system shown in Fig.1, the constitutive equations for a piezoelectric material are,

$$T_{ij} = C_{ijkl} S_{kl} - e_{kij} E_k \quad (1)$$

$$D_k = e_{kij} S_{ij} + e_{ki} E_i \quad (2)$$

where T_{ij} is the stress tensor, C_{ijkl} is the elastic constants, S_{kl} is the strain tensor, e_{ijk} is the piezoelectric constants, E_k is the electric field vector, D_i is the electric displacement vector and e_{ij} is the permittivity. For a dielectric material in quasi-static electrical field assumption, the government equation of the harmonic wave can be derived from Eq.(1)and (2),

$$C_{ijkl} u_{k,lj} + e_{kij} f_{,kj} = r \ddot{u}_i , \quad (3)$$

$$e_{ikl} u_{k,li} - e_{ik} f_{,ki} = 0 , \quad (4)$$

where u_k is the displacement, f is the electrical potential. Then assuming the formal solutions of the displacement and electrical potential, One can obtain the wave equation of the f and u_k by solving the Christoffel's equation derived from Eq.(3) and (4),

$$(u_1,u_2,u_3,f)=\sum_{q=1}^{8}(U_q,V_q,W_q,\Phi_q)\cdot A_q\cdot e^{ix(x_1+a_q x_3)}\cdot e^{-iwt}. \quad (5)$$

where a_q and (U_q,V_q,W_q,Φ_q) (q=1~8) are the eigenvalues and eigenvectors of the Christoffel's matrix. x is the complex wave number for the leaky Lamb wave which can be expressed as,

$$x=\frac{w}{c}(1+i\cdot g)=k(1+i\cdot g). \quad (6)$$

where c and k are the phase velocity and the real part of wave number along x_1 direction, respectively, and g is a non-dimensional attenuation coefficient. With Eq.(5), one can derive the stress and electrical displacement components as,

$$(T_{33},T_{13},T_{23},D_3)=\sum_{q=1}^{8}(D_{1q},D_{2q},D_{3q},D_{4q})\cdot A_q\cdot e^{ix(x_1+a_q x_3)}\cdot e^{-iwt}. \quad (7)$$

2.2. Mechanical Wave Fields in Inviscid Fluid

The wave equations of an inviscid fluid can be modeled as a kind of isotropic material from the government equations,

$$T_{ij}=C_{ijkl}S_{kl}. \quad (8)$$

Neglecting the shear modulus, the wave equations of the displacement and stress for an inviscid fluid can be presented as,

$$\left(u_1^f,u_2^f,u_3^f,T_{33}^f\right)^u=(1,\ 0,\ a_f^u,\ ir_f w^2/x)\cdot F^u\cdot e^{ix(x_1+a_f^u\cdot x_3)}\cdot e^{-iwt},\ (x_3>+h) \quad (9)$$

and

$$\left(u_1^f,u_2^f,u_3^f,T_{33}^f\right)^l=(1,\ 0,\ a_f^l,\ ir_f w^2/x)\cdot F^l\cdot e^{ix(x_1+a_f^l\cdot x_3)}\cdot e^{-iwt},\ (x_3<-h) \quad (10)$$

where r_f is the mass density of the fluid and the superscripts u and l descript the upper and lower plate surface respectively. The a_f^u and a_f^l are the constant and have to satisfy,

$$a_f^u,a_f^l=\pm\sqrt{(\frac{k_f}{x})^2-1}. \quad (11)$$

where $k_f=w/c_f$ is the wave number in the fluid.

2.3. Electrical Wave Fields in Inviscid Fluid

For a piezoelectric plate, the electrical potential in the plate can induce the electrical field disturbance in the adjacent fluid. This electrical field of the fluid E_k^f can be presented as,

$$E_k^f=-f_{f,k}. \quad (12)$$

where f_f is the electrical potential of the fluid. Apparently the electrical potential f_f is a harmonic wave having the same angular frequency w as the mechanical wave in the plate. Therefore, the electrical potential and the electric displacement of the upper and lower surface fluids can be represented as,

$$\left(f_f,D_3^f\right)^u=(1,-x\,e_f)\,\Theta_f^u\cdot e^{-x\cdot x_3}\cdot e^{ix\,x_1}\cdot e^{-iwt} \quad (13)$$

and

$$\left(f_f,D_3^f\right)^l=(1,x\,e_f)\,\Theta_f^l\cdot e^{x\cdot x_3}\cdot e^{ix\,x_1}\cdot e^{-iwt} \quad (14)$$

respectively. where D_3^f is the electric displacement in x_3 direction and e_f is the permittivity of the fluid. If the fluid is conductive, the permittivity e_f can be replaced by a complex number e_f^*,

$$e_f^*=e_f+i\cdot(s_f/w) \quad (15)$$

where s_f is the conductance of the fluid.

2.4. Boundary conditions

When the piezoelectric plate is immersed in a fluid, there are three kinds of boundary condition cases under consideration, namely the "dielectric", "shorted" and "conductive loading" surface boundary conditions [12, 14, 15]. In the "dielectric" case, the electrical conductivity of the adjacent fluid is very small and can be neglected so that the effective fluid permittivity in Eq.(15) is a real number. Therefore, the only electrical loading from the fluid is it's permittivity.

The opposite case of the "dielectric" type is the "shorted" case. Owing to the fluid's high conductivity and its shielding effect, the electrical boundary conditions at the plate/fluid interface are effectively shorted. Between these two extreme types is the "conductive loading" case. The conductivity is in a medium level so both the conductivity and dielectric loading can influence the plate. Therefore, the effective fluid permittivity in Eq.(15) is a complex number. With the mechanical loading in inviscid fluid, the boundary conditions of “dielectric”, “shorted” and “conductive loading” cases are presented as follows.

First, for the “dielectric” inviscid fluid case, the boundary conditions are $u_3 = u_3^f$, $T_{33} = T_{33}^f$, $T_{13} = 0$, $T_{23} = 0$, $f = f_f$ and $D_3 = D_3^f$, which yields a 12×12 linear algebra system in matrix form

$$\left\{ \begin{matrix} \begin{pmatrix} u_3 - u_3^f \\ T_{33} - T_{33}^f \\ T_{13} \\ T_{23} \\ f - f_f \\ D_3 - D_3^f \end{pmatrix}_{x_3 = +h} \\ \begin{pmatrix} u_3 - u_3^f \\ T_{33} - T_{33}^f \\ T_{13} \\ T_{23} \\ f - f_f \\ D_3 - D_3^f \end{pmatrix}_{x_3 = -h} \end{matrix} \right\} = \left\{ 0 \right\}. \tag{16}$$

For the “shorted” case, the boundary conditions are $u_3 = u_3^f$, $T_{33} = T_{33}^f$, $T_{13} = 0$, $T_{23} = 0$ and $f = 0$ which yields a 10×10 linear algebra system,

$$\left\{ \begin{matrix} \begin{pmatrix} u_3 - u_3^f \\ T_{33} - T_{33}^f \\ T_{13} \\ T_{23} \\ f \end{pmatrix}_{x_3 = +h} \\ \begin{pmatrix} u_3 - u_3^f \\ T_{33} - T_{33}^f \\ T_{13} \\ T_{23} \\ f \end{pmatrix}_{x_3 = -h} \end{matrix} \right\} = \left\{ 0 \right\}. \tag{17}$$

For “conductive loading” case, the boundary conditions are $u_3 = u_3^f$, $T_{33} = T_{33}^f$, $T_{13} = 0$, $T_{23} = 0$, $f = f_f$ and $D_3 = D_3^f$, which yields a12×12 matrix form,

$$\left\{ \begin{matrix} \begin{pmatrix} u_3 - u_3^f \\ T_{33} - T_{33}^f \\ T_{13} \\ T_{23} \\ f - f_f \\ D_3 - D_3^f \end{pmatrix}_{x_3 = +h} \\ \begin{pmatrix} u_3 - u_3^f \\ T_{33} - T_{33}^f \\ T_{13} \\ T_{23} \\ f - f_f \\ D_3 - D_3^f \end{pmatrix}_{x_3 = -h} \end{matrix} \right\} = \left\{ 0 \right\}. \tag{18}$$

For non-trivial solutions, the matrixes discussed in Eq.(16), (17) and (18) can be further yield into the characteristic equations of the leaky Lamb wave dispersion relations. Given all material properties of the plate and fluid and a propagation direction q in Fig.1, one can first choose a frequency w and then search for the wave number x which will satisfy the characteristic equation. Owing that the wave number x in Eq.(6) is a complex number, both the real and image parts need to be determined simultaneously. In this paper, a computer program in Matlab® (The MathWorks Inc., MA, USA) programming language is developed to carry out the 2D search of complex x for solving the characteristic equation of leaky Lamb wave.

3. Measurement Method

For measuring the leak Lamb wave effect, a differential measurement system aiming at accurately determining the leaky Lamb wave velocity change at chosen frequency is developed. The experiment Setup is shown in Fig.2

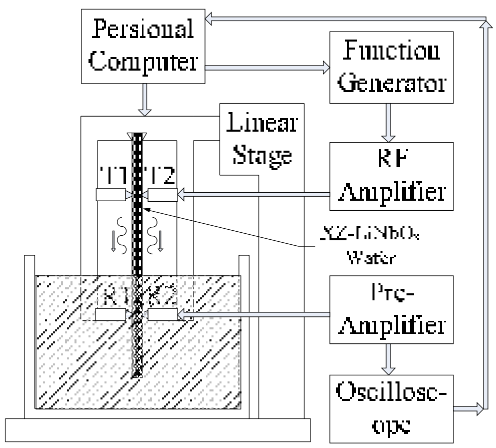


Figure 2*: Schematic diagram of experiment setup.*

In Fig.2, a loading fixture is designed and constructed to mount both the XZ-$LiNbO_3$ plate under measurement and the four laboratory-made line PZT transducers. These PZT transducers are the key elements, which consists of a long slender PZT bar with trapezoid cross-section. The length, height, widths in the bottom and top surfaces are 12.7, 0.5, 1.0, and 0.2 mm, respectively. The PZT bar is packaged with a copper block, tungsten/epoxy mixture, and a metal case, as well as electrically connected and shielded. It can act as an acoustic wave transmitter when applying electrical voltage to the PZT element, or an acoustic wave receiver reversely. In this system, these four PZT transducers have the same configurations. Two of them (T1 and T2) are for wave generation and the other two (R1 and R2) are for wave receiving. Fig.3 and Fig.4 show the explosive and composite diagram of one of these PZT transducer.

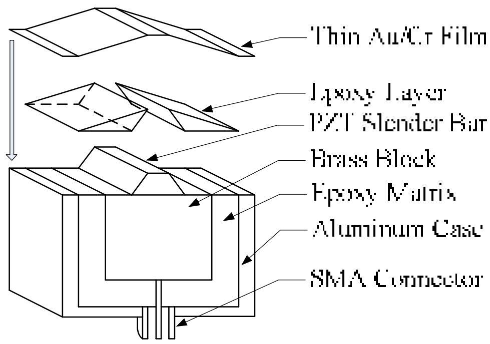


Figure 3*: Explosive diagram of line PZT transducer.*

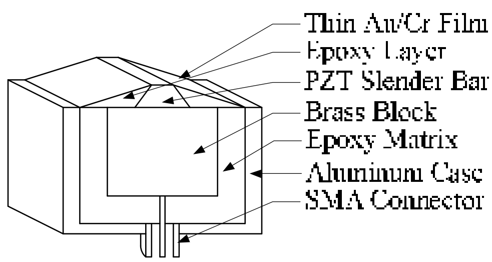


Figure 4*: Composite diagram of line PZT transducer.*

During the measurement, the T1 and T2 transducers are carefully mounted at the symmetrical positions on opposite sides of the XZ-LiNbO$_3$ plate for generating the symmetrical modes of Lamb waves, therefore, the generated acoustic waves can propagate in the plate along the direction perpendicular to the long axes of T1 and T2 transducers. The other two PZT transducers, R1 and R2, are aligned parallel to those of T1 and T3 transducer so that the plate waves can be most effectively received.

In order to generate Lamb waves at a chosen frequency, a tone-burst electrical system is constructed. The original tone-burst signal is generated by a function generator and amplified by a RF amplifier. Then the tone-burst electrical signal is divided by a 0^o power splitter into two signals which are in-phased and of equal amplitude for exciting the transducers T1 and T2 simultaneously. After traveling for a certain distance, the plate waves are receiving by R1, R2 and amplified by a preamplifier. This signal is recorded by a digital oscilloscope. A personal computer controls and synchronizes the generation of the exciting tone-burst signal as well as the acquisition of the received signal waveforms from the digital oscilloscope. This PC also connects to a linear-stage for controlling the vertical position of the loading fixture mounted with sample plate and transducers.

3.1. Velocity Change

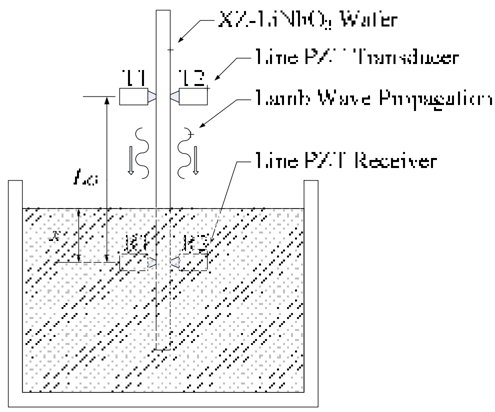


Figure 5*: The configurations of experimental measurement method (I).*

In the beginning of the measurement as shown in Fig.5, the sample plate with R1, R2 receivers are partially immersed in the fluid. The distance between the transmitters and the receivers are L_o which is fixed during the measurements. The distance between the receivers to the air/fluid interface is x. Let t_1 be the time-of-flight of a plate wave traveling form the transmitters to the receivers, one has,

$$t_1 = \frac{L_o - x}{c_o} + \frac{x}{c} \tag{19}$$

where c_o and c are the wave velocities of the free Lamb wave and the leaky Lamb wave, respectively.

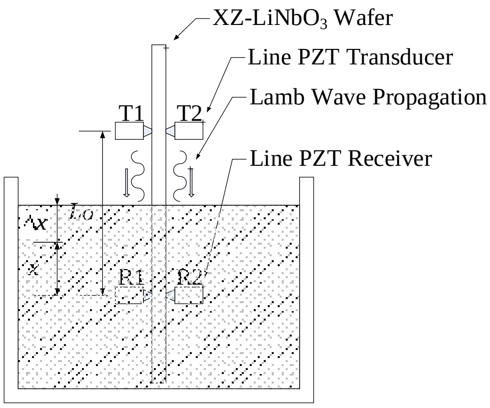


Figure 6*: The configurations of experimental measurement method (II).*

Subsequently, moving the whole sample plate and transducers assembly into the fluid by a distance of Δx as shown in Fig.6, one can has the time-of-flight becomes,

$$t_2 = \frac{L_o - x - \Delta x}{c_o} + \frac{x + \Delta x}{c} \tag{20}$$

From Eq.(19) and Eq. (20), the difference in time-of-flight between Fig.5 and Fig.6 is,

$$\Delta t = t_2 - t_1 = \frac{-\Delta x}{c_o} + \frac{\Delta x}{c}. \tag{21}$$

Re-writing Eq.(20), one has,

$$c = \left(\frac{\Delta t}{\Delta x} + \frac{1}{c_o} \right)^{-1}. \tag{22}$$

Therefore, with given wave velocity c_o of free Lamb wave, the leaky Lamb wave velocity can be directly determined by measuring the linear correlation between the change in time-of-flight (Dt) and the change in distance (Dx). This differential measurement type is very sensitive to the wave velocity change and is less vulnerable to measurement errors and noises.

3.2. Amplitude Attenuation

The measurement of wave attenuation of leaky Lamb wave shares the same principle as in the measurement of velocity change, and therefore follows a similar measurement approach. The two basic assumptions needed to be made here are: (a) the wave attenuation of free Lamb wave is much smaller than that of a leaky Lamb wave and hence can be negligible; (b). the output signals of transducer R1 are linearly proportional to the input acoustic waves in the plate. Based on these two assumptions, the measurement of wave attenuation factor can be easily obtained by,

$$g = -\frac{\ln\left(|A_2|/|A_1|\right)}{\Delta x} \cdot \frac{1}{k}. \tag{23}$$

where A_1 and A_2 can be considered as the wave amplitude measured in Fig.5 and Fig.6 respectively. g is a non-dimensional attenuation factor in Eq.(6). Again, Eq.(23) is a differential type of measurement focusing on the amplitude variation of received signal waveforms caused by different length of liquid-covered path. To be even more accurate in the measurement, one can take a number of waveforms continuously when moving the sample plate into the fluid over a wide range of Δx.

4. Conductivity Loading Experiment

In analysis the leaky Lamb wave subjected to conductivity fluid loadings, the water adding with different amount of salt is used to apply the fluid conductivity loadings. The conductivity of the salt solution is continuously varying from 0.0001 S/m to 0.5 S/m and monitored by a conductivity sensing meter. Fig.7 (a) and (b) show the calculation result of the velocity changed and wave attenuation of S0 mode leaky Lamb waves subjected to conductivity loadings.

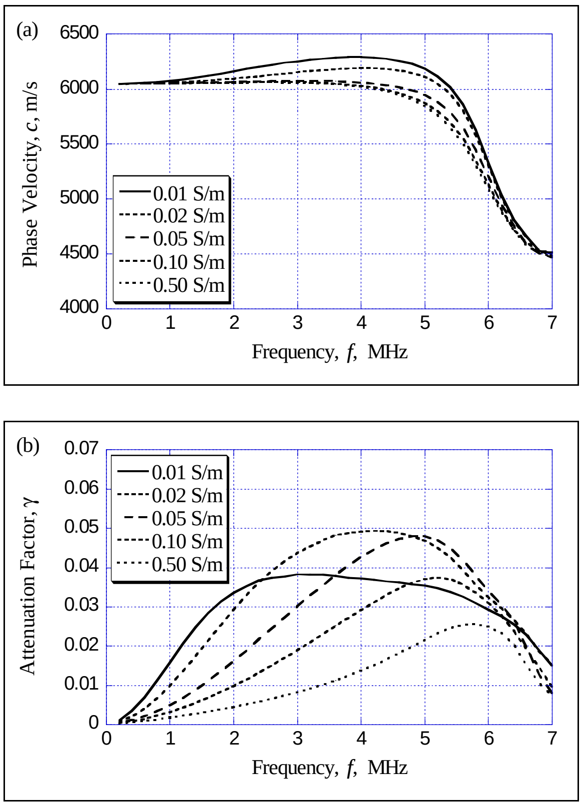


Figure 7*: The loading effects of fluid conductivity on the S_0 mode of Leaky Lamb*

waves (a) the wave velocity and (b) the wave attenuation factor

In experiment, the condition settings are following the theoretical analysis. Fig.8 and Fig.9 displays the measured velocities and attenuation factor as functions of water's conductivity at 2, 3, 4, and 5 MHz.

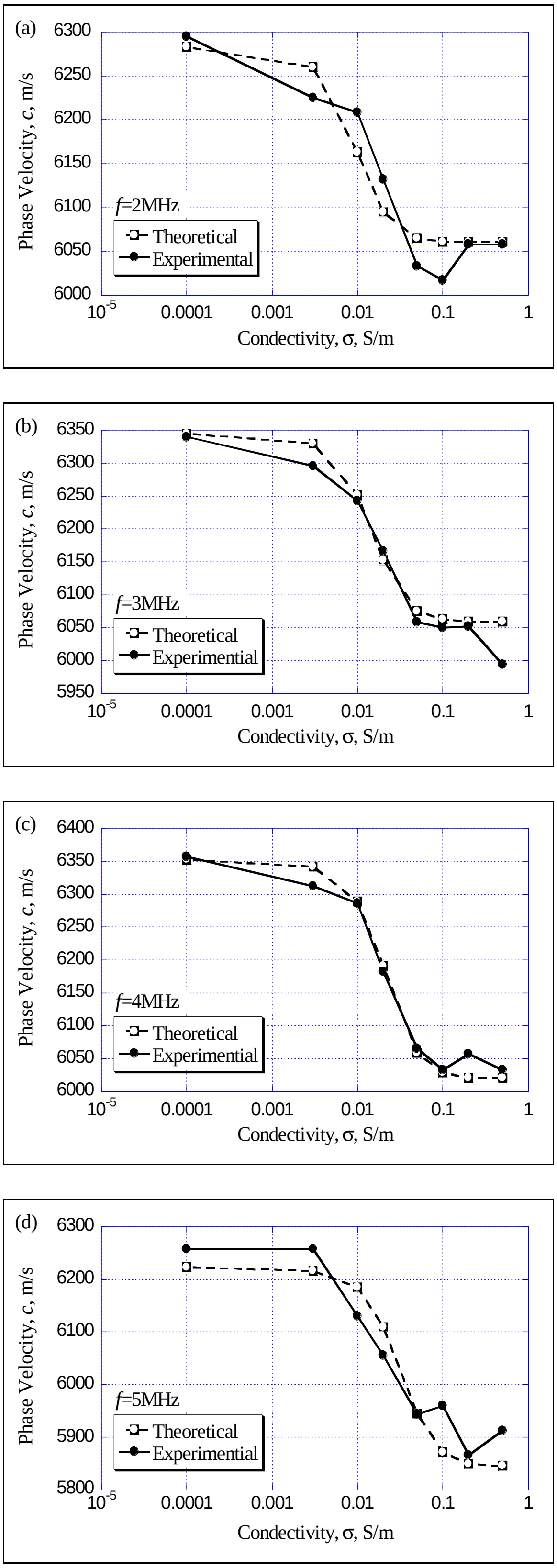


Figure 8*: Conductive loading effects of the leaky Lamb wave velocity, both theoretical and experimental data at (a) 2 MHz, (b) 3 MHz, (c) 4 MHz, and (d) 5MHz.*

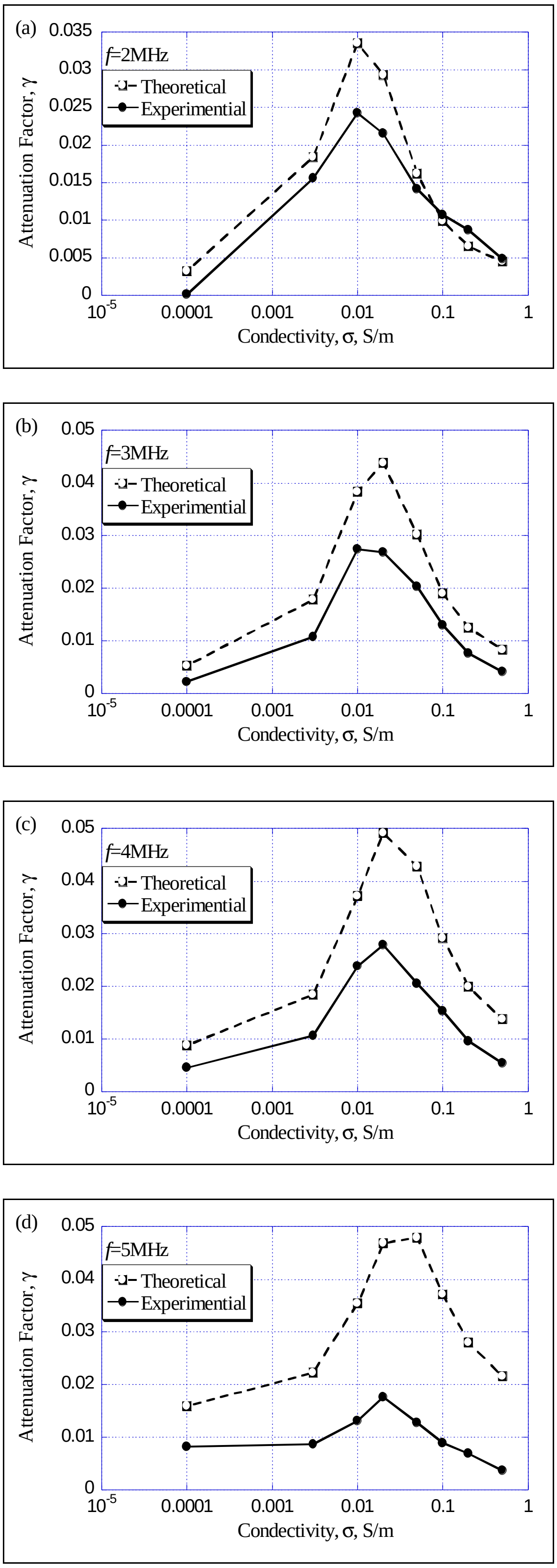
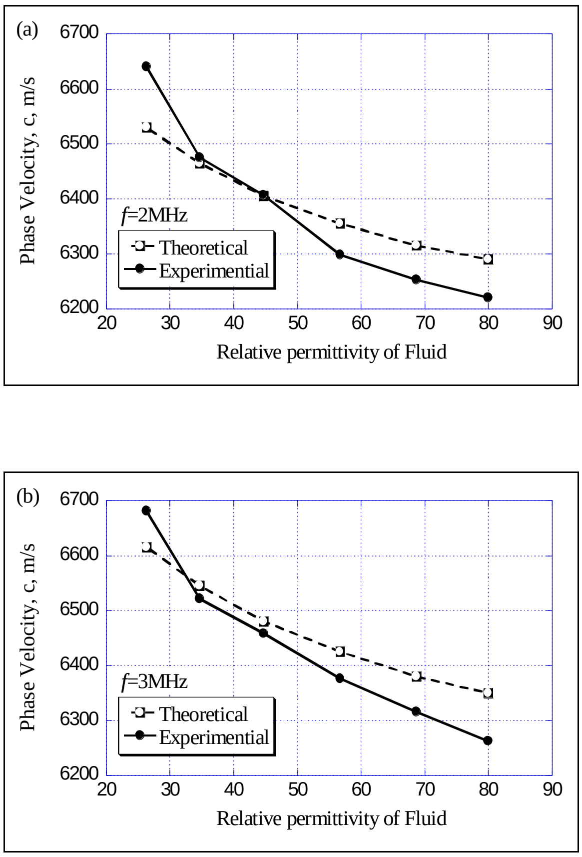


Figure 9*: Conductive loading effects of the leaky Lamb wave attenuation, both theoretical and experimental data at (a) 2 MHz, (b) 3 MHz, (c) 4 MHz, and (d) 5MHz.*

For the wave velocities, good agreements between the experimental and theoretical data are observed in Fig.8. When the conductivity is varied from 0.0001 S/m to 1 S/m, the wave velocity continuously drops by about 4 % (2MHz) to 6 % (5MHz). For the wave attenuation, the value of the attenuation factor are first increasing rapidly to a peak value around 0.012 S/m and then drop to a constant level. This phenomenon is due to electrically shielding effect. Though there are some deviations in magnitude between the measured and theoretical result, this electrically shielding phenomenon still has been caught in our experiments.

5. Dielectric Loading Experiment

In dielectric loading analysis, the ethanol/water solutions are used to provide the varying dielectric loading effects. By varying the concentration of ethanol in water, the permittivity of the solution can substantially be changed from 80 (pure water) to 26 (pure ethanol) with varying other material properties (density, wave velocity) in an acceptable level. The theoretical calculations show that the fluid's permittivity can have significant and dominate influence on the wave velocities of leaky Lamb waves [16]. Therefore, the experiments about dielectric loadings will focus on the phase velocity change only and Fig.10 shows the experiment result.



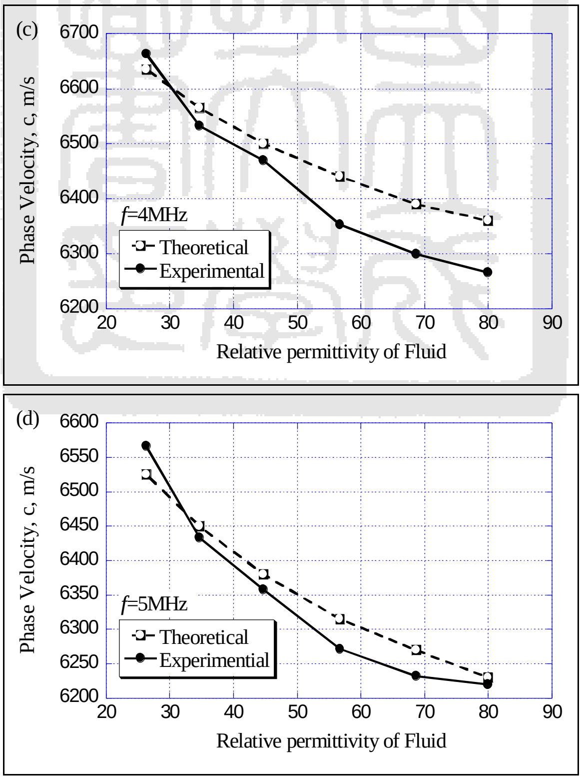


Figure 10*: Comparison between experimental and theoretical leaky Lamb wave velocities at, (a) 2 MHz, (b) 3 MHz, (c) 4 MHz, and (d) 5 MHz.*

The measured velocities of leaky Lamb wave gradually decay when the relative permittivity of the fluids is increasing from 26 to 80. This experiment results quantitatively conform the simulation results. Therefore, the partial wave theory shows good agreements between experimental and theoretical data of dielectric loading effects.

6. Conclusion

In this paper, the wave velocity changes and the wave attenuation factors of leaky Lamb waves induced by the fluid dielectric and conductivity loading effects are successfully determined both theoretically and experimentally. Using a *XZ*-$LiNbO_3$ piezoelectric plate as a sample plate, the partial wave theory with 2D search method provides a good prediction about leaky Lamb wave velocity changes in theoretical analysis. For the wave attenuations, although the experimental results do not perfectly match the theoretical predictions, but the discrepancies are still in a small and acceptable ranges and the important features have been predicted successfully.

Besides, the experiment system introduced in this paper has demonstrated its outstanding performance in terms of good sensitivity and accuracy. That is because the measurement method is designed and operated under exactly the same mechanical and electrical conditions except for the relative traveling distances between free and leaky waves. Therefore, this system can avoid possible errors or uncertainties during the measurements. Comparing with other experimental methods, this system shows its uniqueness and strength in measuring the characteristic of the leaky Lamb wave. With this paper, the numerical approaches and the measurement data could be very useful in designing or analyzing new types of liquid acoustic wave sensors based on leaky Lamb waves.

7. References

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