INVERSE ANALYSES AND THEIR APPLICATIONS TO NONDESTRUCTIVE EVALUATIONS

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Abstract

Inverse problems and inverse analyses, which deal with estimation of inputs or source from outputs or results, have been receiving increasing attention in various fields of science and engineering. A reasonable classification of the inverse problems is described. The nature of inverse problems and difficulties encountered in the inverse analyses are discussed. Typical inversion analysis schemes for solving the inverse problems are summarized. Nondestructive evaluations can be regarded as inverse problems. As an example of nondestructive evaluations the identification of cracks and defects from electric potential distributions observed on cracked bodies is discussed. The author and his coworkers proposed the active electric potential CT (computed tomography) method, in which the measurements of electric potential under various electric current application conditions are compared with numerical analyses of electric potential distributions for crack identification. The uniqueness and the stability of the identification are discussed. Examples of the applications of the active electric potential CT method to identification of two-dimensional cracks and three-dimensional cracks are given. By using the piezoelectric film, the present author and his coworkers proposed the passive electric potential CT method, which did not require electric current application. The applicability of the passive electric potential method to identification of two-dimensional and three-dimensional cracks and delamination in composite materials are demonstrated. The enclosure method and the probe method proposed recently by Ikehata for direct reconstruction of voids and inclusions are introduced. Inverse analysis schemes related to the residual stress and strain estimation and the defect identification using thermography are also discussed.

1. Introduction

Inverse problems and inverse analyses have been receiving increasing attention in various fields of science and engineering. Increasing number of books and review papers have been published on inverse problems [1-8]. Inverse problems can be defined as problems concerning the determination of input or source from output or response. This is contrary to the direct problems, in which output or response are determined using information about input or source. It was found that there are many inverse problems of different types for every research area of science and engineering. The X-ray computed tomography in radiography, identification of heat source in thermodynamics, three-dimensional optical microscope imaging in optics, acoustic inverse scattering for identifying defects in structural mechanics, estimation of the structure of the earth from data on wave propagation in geophysics, estimation of distribution of elastic constants in solid mechanics, and even induction of laws governing not well-established phenomenon are typical examples of the inverse problems. Nondestructive evaluations can be regarded as inverse problems.

In this paper definition and the nature of inverse problems, and typical inverse analysis schemes are described. As an example of inverse problems the detection of cracks and defects in solids and structures by the active and passive electric potential CT (computed tomography) [9-18] proposed by the present author and his co-workers is described. The enclosure method, the probe method and some other inverse analyses schemes for nondestructive evaluations are introduced.

2. Categorization of inverse problems

The meaning of inverse problems sometimes depends on research area. The inverse problems can be defined as the problems, which cannot be categorized into direct problems. Therefore a rational definition of inverse problems can be given by referring to the definition of direct problems [6, 7].

To make possible a direct analysis of distribution of a physical quantity u, the following information is indispensable.

(a) Domain Ω with boundary Γ where u is defined.

(b) Equation governing the variation of the physical quantity u,

$$L(\kappa)u = f, \tag{1}$$

where L, κ and f denote an operator, material properties and force/source term, respectively.

(c) Boundary conditions on boundary Γ, and initial conditions, if necessary.

(d) Force or source term f defined in domain Ω.

(e) Distribution of material properties κ.

When all information of these items is available, we can determine output or response using conventional analytical or numerical schemes, such as the finite element method, the boundary element method, and the finite difference method.

If one of requisites (a) to (e) is lost, we cannot conduct the direct analysis to obtain the distribution of the physical quantity u. As was described in the foregoing, those problems not classified as the direct problems can be classified into inverse problems.

Then, for the problem dealing with the distribution of u, there are the following categories of inverse problems corresponding to the lack of requisites (a) to (e) for the direct analyses.

(A) Estimation of the shape of domain Ω, its boundary Γ or unknown inner boundary (domain/boundary inverse problems).

(B) Inference of the governing equation (governing equation inverse problems).

(C) Estimation of the boundary conditions on the entire or partial boundary and/or estimation of the initial conditions in Ω (boundary value/initial value inverse problems).

(D) Estimation of force or source f applying in Ω (force/source inverse problems).

(E) Estimation of material properties κ defined in Ω and involved in the governing equations (material properties inverse problems).

Any combination of these inverse problems can be another inverse problem.

We will be faced with these kinds of inverse problems when we start to understand a new phenomenon. Inverse problems are then natural and important problems we find in science and engineering.

3. Information used for inverse analyses and the nature of inverse problems

The inverse problems are inherently lacking in information as compared with direct problems. Additional information is necessary to conduct inverse analyses for inverse problems. Output or response can be used as primary information to conduct inverse analyses. This is contrary to the direct problem, in which output or response is determined from input or source. The information concerning output or response can be obtained by measurements, for example. A priori information, such as physical constraints or knowledge based on experience about the input, can be effectively used in the inverse analyses. This kind of information can be called secondary information or subsidiary information.

It is well-known that the existence, the uniqueness and the stability of the solution are usually assured for the direct problems. On the contrary the solutions of the inverse problems lack in the existence, the uniqueness or the stability. Then inverse problems are called ill-posed. If a solution for a posed problem does not have uniqueness, it is not possible to obtain a reliable solution without additional information, since the solution can be a fake. Even when the existence and the uniqueness are guaranteed, most inverse problems suffer from the lack of stability of solution: the solution is very sensitive to the primary information used in the inverse analysis. To overcome the difficulty of the loss of the stability, inverse analysis schemes incorporating regularization are applied.

4. Typical schemes for inverse analysis and regularization

Among various inverse analysis schemes, the selection method simply uses a comparison between the observed response $u^{(m)}$ with the calculated one $u^{(c)}$ for assumed input or unknown parameters p to be estimated. As a measure of the comparison the following residual R_s is evaluated on measurement boundary Γ_m.

$$R_s = \int_{\Gamma_m} (u^{(c)}(p) - u^{(m)})^2 d\Gamma \tag{2}$$

The combination of parameters giving the smallest value of R_s is employed as a quasi-solution.

In the Tikhonov regularization, penalty term or regularization term $\Lambda(p)$ is added to regularize the solution:

$$\Pi = \int_{\Gamma_m} (u^{(c)}(p) - u^{(m)})^2 d\Gamma + \alpha\Lambda(p) \qquad (3)$$

Here α is the regularization parameter. As the function of $\Lambda(p)$, norm of p is often employed.

To compare models involving different number of parameters, the AIC (Akaike information criterion) defined by the following equation is used.

$$\mathrm{AIC} = N \log_e (R_s / N) + 2P \qquad (4)$$

Here N denotes the total number of measurements, and P is the number of parameters.

In many cases a matrix equation for unknowns can be deduced from observation equation:

$$[A]\{p\} = \{B\}. \qquad (5)$$

Here $\{p\}$ is a vector consisting of unknown parameters, $\{B\}$ is a vector calculated from observations, and $[A]$ denotes a matrix relating these vectors. Matrix $[A]$ is severely ill-conditioned due to the ill-posed nature of the inverse problems. The singular value decomposition for $[A]$ gives:

$$[A] = [U][S][V]^T \qquad (6)$$

Here $[S]$ is a diagonal matrix whose components are given by singular values of matrix $[A]$. $[U]$ and $[V]$ are unitary matrices. By ignoring small singular values in $[S]$, $[S*]$ is constructed. Then the solution is given by,

$$\{p*\} = [V][S]^{-1}[U]^T\{B\} \qquad (7)$$

The use of the subsidiary information is important in the solution of inverse problems in the existence of noise in observation. Some inverse analysis schemes incorporate the subsidiary information. Combination of fundamental solutions satisfying the subsidiary information can be applied.

In the regularization schemes, the regularization parameter in the Tikhonov regularization, the number of singular values to be considered or the number of fundamental solutions used should be determined. For determining these regularization parameters the discrepancy principle and the AIC minimum criterion are widely applied. The L-curve method, reference problem method, and admissible condition number method can be also used.

5. Nondestructive evaluations as inverse problems

In evaluation of structures and components nondestructive inspection of cracks and defects is very important. The inspection can be regarded as one of the domain/boundary inverse problems, since cracks or defects corresponding to unknown inner boundaries are estimated from certain observations. Ultrasonic inspection, eddy current method, A.C. electric potential method, D.C. potential method, radiation method, elastodynamic response and strain measurement have been used for the inspection.

Mechanical properties of fracture process region developing in the vicinity of a crack tip cannot be measured directly. The estimation of tension-softening characteristics in fracture process region can be recognized as one of the boundary value inverse problems. The characteristic can be estimated by using load-deflection diagrams.

It is well-known that fatigue strength and crack propagation rates of structures and their components are strongly affected by residual stresses introduced in construction or forming processes. The estimation of residual stresses and strains are therefore important in strength evaluation. The estimation of the distribution of residual stresses and strains in a body can be regarded as one of the force/source inverse problems.

Some nondestructive evaluations and applications of inverse analyses for them are introduced in the following.

6. Active electric potential CT method for crack identification

The present authors proposed the active electric potential CT (computed tomography) method for the detection and quantitative identification of cracks [9-14]. In this method the electric potential distributions observed on the surface of cracked body under electric current application is used to identify the crack.

6.1 Crack identification using electric potential distribution as one of inverse problems

For monitoring crack propagation and for measuring crack length, electric potential method has been used which is based on the fact that the existence of cracks gives rise to disturbance in electric potential readings under D.C. electric current application. The location, size and shape of a two- or three-dimensional crack can be also estimated from the electric potential distribution.

Under the application of D.C. electric current the spatial variation of electric potential u in an electric conductive homogeneous body is expressed by the governing equation of electrostatics. The potential u obeys the following equation.

$$\nabla \cdot \gamma \nabla u = 0, \tag{8}$$

where γ denotes the conductivity. When γ is constant Eqn. (8) is reduced to the Laplace equation:

$$\nabla^2 u = 0 \tag{9}$$

For a usual direct boundary value problem of electrostatics, domain Ω with boundary Γ is given. Boundary Γ consists of Dirichlet boundary Γ_1 where the value of electric potential u is prescribed, and Neumann boundary Γ_2 where the value of normal derivative or flux $\partial u / \partial \nu = q$ is prescribed. The boundary condition involved in the inverse problem of crack identification is different from those for the direct problems. Since the location, size and shape of the cracks are unknown in advance and cracks constitute themselves other flux free Neumann boundary, the boundary conditions are incompletely given. Hypothetical boundary Γ_0 can be introduced, which contains the cracks to be identified and on which neither potential u nor flux q is prescribed. In some cases even Γ_0 is not known. Then Γ_0 can be called incompletely-prescribed boundary.

In this inverse problem the lack of information about boundary conditions can be compensated by introducing over-prescribed boundary Γ_3, where both u and q are given. This over-prescribed boundary can be introduced by measuring potential u on some parts of Neumann boundary Γ_2. The use of the boundary values on the over-prescribed boundary together with other boundary values makes it possible to solve the inverse problem of crack identification.

6.2 Formulation of inverse analysis schemes

Two inverse analysis schemes based on the boundary element formulation are proposed: the inverse boundary integral equation method and the least residual method. Outlines of these two schemes are described in the following.

The boundary integral equation method gives an equation for the value of potential u at point A on boundary Γ expressed by a boundary integral using the fundamental solution u^* and its normal derivative q^* for Eqn. (9):

$$c(A)u(A) + \int_{\Gamma} [q^*(A,B)u(B) - u^*(A,B)q(B)]d\Gamma(B) = 0 \tag{10}$$

Coefficient $c(A)$ is dependent on the geometry of boundary in the vicinity of point A. By discretizing boundary Γ, Eqn. (10) is reduced to a matrix equation interrelating u and q on boundary Γ.

In the inverse boundary integral equation method, this reduced matrix equation is solved for unknown boundary values from prescribed boundary values on Γ_1, Γ_2 and Γ_3. The cracked portions on in plane Γ_0 are identified as its flux-free portions. Thus in the inverse boundary integral equation method the problem of crack identification is reduced to one of the boundary value inverse problems.

The least residual method searches a quasi-solution of a crack giving the minimum norm between the observed and computed potential readings among admissible cracks. Cracks expressed by various combinations of plane Γ_0 containing cracks, crack location, size and shape are assumed. This makes it possible to separate boundary Γ_0 into cracked portions and remaining uncracked portions. When boundary values of q only are used on over-prescribed boundary Γ_3 a direct analysis can be made, which gives computed potential distribution $u^{(c)}$ on over-prescribed boundary Γ_3. To obtain an estimate of the crack, the square sum R_s of residuals is evaluated between $u^{(c)}$ and the measured potential distribution $u^{(m)}$ on Γ_3 using a weighting factor w.

$$R_s = \int_{\Gamma_3} w(u^{(c)} - u^{(m)})^2 d\Gamma \tag{11}$$

The quasi-solution is determined as the crack giving the smallest value of R_s.

6.3 Uniqueness of crack identification

The present author discussed the condition for the uniqueness of the inverse solution in crack identification from electric potential distribution on over-prescribed boundary [12]. It was shown that the electric potential distribution is uniquely determined, if there exists a continuous over-prescribed boundary and if the solution exists.

It is shown that the cracks can be uniquely identified from the electric potential distribution, when plane Γ_0, which contains cracks, is known in advance. When Γ_0 is not known, the electric potential distributions under two current application conditions are necessary to determine a single two-dimensional crack embedded in a body. To determine a single three-dimensional crack in an unknown plane, electric potential distributions under three independent current application conditions are necessary.

6.4 Simulations and experiments of crack identification

Numerical simulations were made for identifying two- and three-dimensional cracks by the inverse boundary integral equation method [10,11]. It is found that the inverse boundary equation method can be applied for identifying a single crack and plural cracks in a two- and three-dimensional bodies.

The inverse boundary integral equation method is however very sensitive to the errors involved in the potential data used in the inverse analyses. Constraints or regularizations are needed to obtain a reasonable estimate by this method.

Numerical simulations and experiments were conducted to examine the applicability of the least residual method to estimation of the crack location, size and shape of two- and three-dimensional cracks.

Identification of a two-dimensional inclined crack embedded in a stainless steel strip was made [13]. The crack location, angle and size were unknown in advance and to be determined from the potential distributions measured on flux free side faces.

As was described in the foregoing, if the plane containing the crack is not known the crack cannot be uniquely identified from electric potential distribution under only one current application condition. To ensure the uniqueness of the crack identification, the multiple current application method was proposed, in which potential data measured under several current application conditions were processed simultaneously. The crack was identified by the least residual method. As a criterion for identifying the most plausible combination of crack location, angle and size from several sets of electric potential distributions, sum R_s of residual for the *i*-th current application condition $R^{(i)}$ over all current application conditions was employed:

$$R_s = \sum_i R^{(i)} \tag{12}$$

Experiments were made for identifying a three-dimensional surface crack in a steel plate. In the identification of a three-dimensional crack by the least residual method number of assumed cracks increases dramatically as detailed assumption is made to obtain an accurate estimate. For efficient identification of the three-dimensional surface crack by the least residual method, a hierarchical inverse analysis scheme was proposed, in which two-dimensional scanning inverse analyses were combined with three-dimensional inverse analyses.

This least residual inverse analysis scheme incorporating the hierarchical schemes was successfully applied to the identification of three-dimensional internal cracks introduced by diffusion bonding technique in a steel bar.

Since singular fields develop near crack tip, fracture mechanics parameters expressing the intensity of the singularity can be applied to the crack identification. For two-dimensional elastic body, the following *J*- and *M*-integrals are path-independent for a path C enclosing a crack tip in a two-dimensional body.

$$J_k = \int_C (W\nu_k - \sum_i T_i \frac{\partial u_i}{\partial x_k}) d\Gamma \tag{13}$$

$$M = \int_C (W\sum_i x_i \nu_i - \sum_{i,j} T_j \frac{\partial u_j}{\partial x_i} x_i) d\Gamma \tag{14}$$

Here W denotes the strain energy density, x_i is the Cartesian coordinates, u_i is displacement, ν_i is outward unit normal to C and T_i is traction vector. The intensity of the singularity is given by J_k, and the location of crack tip x_k can be evaluated by the ratio of M the J_k [19].

Using the analogy between elasticity and electrostatics, the following path-independent integrals can be used for estimation of crack location and the intensity from boundary measurement.

$$j_k = \int_C \sum_i \left(\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_i}\frac{\nu_k}{2} - \frac{\partial u}{\partial x_i}\nu_i\frac{\partial u}{\partial x_k}\right)d\Gamma \quad (15)$$

$$m = \int_C \sum_{i,j} \frac{\partial u}{\partial x_i}\left(\frac{\partial u}{\partial x_i}\frac{\nu_j}{2} - \frac{\partial u}{\partial x_j}\nu_i\right)x_j d\Gamma \quad (16)$$

7. Passive electric potential CT method for crack and defect identification

On piezoelectric material electrical charge proportional to a change in mechanical strain is incurred. When the piezoelectric film is glued on cracked body, which undergoes mechanical load, electric potential distribution is incurred due to the piezoelectric effect without applying the electric current on the cracked body. The passive electric potential CT method uses this incurred electric potential [15-18].

In piezoelectric material mechanical and electrical effects are coupled as can be seen in the following equations:

$$\{\sigma\} = [C]\{\varepsilon\} - [e]^T\{E\} \quad (17)$$

$$\{D\} = [e]\{\varepsilon\} + [g]\{E\} \quad (18)$$

where $\{s\}$ and $\{e\}$ are stress and strain vectors, $[C]$, $[e]$ and $[g]$ are stiffness matrix, piezoelectric coefficient matrix and dielectric constant matrix, respectively. $\{E\}$ is electric field vector. $\{D\}$ is electric displacement vector.

Finite element method can be applied to calculate the electric field on the piezoelectric film as well as the deformation field.

The passive electric potential CT method was applied to the identification of two-dimensional crack. It is found that the electric potential distribution has peaks around crack location. The location of local minimum of potential coincides with location of the crack. It is also found that the peak value of electric potential increases with increase in crack length and decrease in crack depth.

As the inverse analysis method for identification of cracks, the least residual method was applied. In this method, the residual R_s is evaluated between the computed electric potential distribution and the measured distribution. The combination of crack location and size, which minimized R_s, was employed as the most plausible one among all the assumed combinations of the crack location and size. For effective inverse analysis, a hierarchical calculation scheme was introduced, in which rough estimation was followed by detailed estimation using an optimization scheme.

The passive electric potential CT method was applied to the identification of a two-dimensional plural cracks, a three-dimensional crack, and a delamination in fiber-reinforced composites also. Numerical simulations and experiments have shown the applicability of the method for the identification.

8. Enclosure method and probe method for direct reconstruction of cavities and inclusions

Ikehata [20-25] proposed the enclosure method for direct reconstruction of a convex hull of polygonal cavities or inclusions from boundary measurement.

As an example, consider a conductivity problem, whose governing equation is given by Eqn. (8). In domain Ω there is an unknown domain D, which corresponds to cavities or inclusions. In domain Ω, the value of γ is constant except in D, where γ takes another constant value. The determination of the location and shape of D is made from Cauchy data, i.e. the non-constant distribution of $u = f$ and its normal derivative $\partial u/\partial \nu = g$ on the boundary Γ.

The region of D can be expressed by the so-called support function h_D of D.

$$h_D(\omega) = \sup_{x \in D} x \cdot \omega, \quad (19)$$

Here ω is a unit direction vector $\omega = (\omega_1, \omega_2)$. If we know h_D the convex hull of D can be obtained by enclosing regions:

$$\mathbf{I}_{\omega}\{x \in \mathrm{R}^2 | x \cdot \omega < h_D(\omega)\} \quad (20)$$

For a unit direction vector $\omega^{\perp}$ perpendicular ω and satisfying $\det(\omega, \omega^{\perp}) > 0$, the following special harmonic function v^* is defined for $\tau > 0$ and $i = \sqrt{-1}$.

$$v^* = \exp(\tau x \cdot (\omega + i\omega^{\perp})) \quad (21)$$

The indicator function $I_{\omega}(\tau, t)$ is defined by

$$I_{\omega}(\tau, t) = e^{-\tau t} \int_{\Gamma} (g v^* - \frac{\partial v^*}{\partial \nu} f) d\Gamma \quad (22)$$

Then the construction of function h_D is given by the following equation.

$$\lim_{\tau \to \infty} \frac{\log |I_{\omega}(\tau, t)|}{\tau} = h_D(\omega) - t \quad (23)$$

For $t > h_D(\omega)$ the left hand side of Eqn. (23) goes to 0, while $t < h_D(\omega)$ it blows up.

The enclosure method has an advantage that the voids or inclusions can be reconstructed without requiring information about the value of γ.

Ikehata [26, 27] proposed the probe method also for direct reconstruction of an obstacle from boundary measurement.

As an example, consider a problem of determination of a sound-soft or a sound-hard obstacle in a medium, where the governing wave equation is given by the Helmholtz equation:

$$\nabla \cdot \nabla u + k^2 u = 0, \quad (24)$$

where k denotes the frequency.

Consider a straight needle whose tip is given by x. Let v_1, v_2, $\ldots$ denote a sequence of solutions converging to the fundamental solution of the Helmholtz equation with a source at point x. The boundary value of v_n is denoted by f_n. The Λ_D denotes the so-called Dirichlet-to-Neumann map:

$$\Lambda_D f = \left. \frac{\partial u}{\partial \nu} \right|_{\Gamma} \quad (25)$$

Λ_0 denotes Λ_D in the case of $D = 0$. Function $I_n(x)$ is defined by

$$I_n(x) = \int_{\Gamma} \{(\Lambda_0 - \Lambda_D) \overline{f}_n\} f_n d\Gamma \quad (26)$$

Then the indicator function I is given by the following equation.

$$I(x) = \lim_{n \to \infty} I_n(x) \quad (27)$$

If x is outside D the indicator function I is finite:

$$\sup_{dist(x,D) > \varepsilon} I_n(x) < \infty \quad (28)$$

The value of $I(x)$ blows up when x reaches D. By moving x one can estimate the location and shape of D.

9. Thermography for identifying defects and cracks

The techniques in thermographic imaging have been remarkable improved in the last decade and thermography was successfully applied to the detection and sizing of cracks and defects in various kind of structures [28-30]. The combination of the measurements with analyses is promising for understanding the measured data and attaining reasonable identification of cracks and defects.

10. Estimation of residual stresses

Since eigen strains induce residual stresses, the estimation of residual stress can be reduced to the estimation of eigen strain distribution, which is one of force/source inverse problems [31, 32]. The nature of eigen strains, such as isotropy, incompressibility can be effectively introduced in the inverse analyses. The decomposition of eigen strains into compatible and incompatible components was also applied in the inverse analyses.

The distribution of eigen strains was estimated from limited number of measurements concerning deformations and residual stresses available at selected points [33]. To conduct this severely under-determined estimation, the singular value decomposition together with constraints such as isotropy and the non-positiveness of eigen strains was successfully applied.

The estimation of initial residual stress fields from residual stresses redistributed due to crack initiation and propagation can be regarded as one of the force/source inverse problems. The present author proposed an inversion scheme based on the fundamental function expansion and inverse sensitivity matrix [34]. The estimated residual stress distribution was used for estimation of a remaining fatigue crack propagation life.

11. Concluding remarks

As can be seen in the foregoing, the inverse analyses can give rational understanding of the data and reasonable identification. Promotion of the combination of measurements and inverse analyses are strongly recommended. Because of the lack of space, details of nondestructive evaluations will be given in the presentation. The readers are referred to the original papers for further details.

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