12th A-PCNDT 2006 - Asia-Pacific Conference on NDT, 5th - 10th Nov 2006, Auckland, New Zealand

Ultrasonic Non-destructive evaluation of stress around the tip of a crack

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Abstract
More recently, interest has focused on the stress-dependent effect of the velocity of sound as well as on non-destructive stress measurement methods based on what is known as acoustoelasticity. The propagation velocities in deformed isotropic elastic material are determined as functions of the strain or the stress. In this paper from measurement of the wave propagation time within the material it is possible to calculate directly the stress intensity factors k_I and k_{II} , even in the case of mixed mode.

1. Introduction
the application of fracture mechanics principles bears largely upon the stress intensity factor. An essential part of the solution of a fracture problem in linear elastic fracture mechanics is the establishment of the stress intensity factor for the crack problem under consideration. In cases of relatively simple geometry use can be made of analytical methods, but in view of the complexity of the boundary conditions, numerical solution of the equations is soon necessary. In engineering problems of complex geometry and with complicated stress systems, finite element methods can be used. An experimental determination of the stress intensity factor is sometime useful to obtain an approximate value.

The stress intensity factor cannot be measured directly in an experiment, but it can be found through the relations between K and a measurable quantity, such as strain, displacement, etc.

Some methods are applicable only in laboratory experiment, but a few may have a limited use under service conditions, provided the load on the structure can be measured also. A typical laboratory technique is the use of Acoustoelasticity.

The basic relations of the acoustoelasticity are deduced by means of the infinitesimal wave propagation in a deformed isotropic elastic material.

The main purpose of this paper is a Non-destructive evaluation method which is useful and effective in determination of the stress intensity factors K_I and K_{II} , and thus to evaluate the danger level of the crack. This method does not require knowledge of the geometric dimensions of the crack.

2. General theory

2.1 Acoustoelastic theory
Hughes and Kelly derived expressions for the speeds of elastic waves in a stressed solid using Murnaghan's theory of finite deformations and third order terms in the strain energy expression. They showed that the speeds of plane waves propagating as shown in Fig.1 in the 2 direction, and having particle displacements in the 1,2, or 3 directions in a initially isotropic body subjected to a homogeneous triaxial strain field, are given by[1]

$$r^0 V_{22}^2 = l + 2m + (2l + l)q + (4m + 4l + 10m)a_1 \quad (1a)$$

$$r^0 V_{21}^2 = m + (l + m)q + 4ma_1 + 2ma_2 - \frac{1}{2}na_3 \quad (1b)$$

$$r^0 V_{23}^2 = m + (l + m)q + 4ma_1 + 2ma_3 - \frac{1}{2}na_2 \quad (1c)$$

Where

r^0 = initial density

V_{22}, V_{21}, V_{23} = speeds of waves propagating in the 2 direction with particle displacements in the 2,1,3 directions respectively

l, m = Lame constants

l, m, n = Murnaghan's constants

$q = a_1 + a_2 + a_3$

Where a_i indicate the principal strain. V_{22} is the velocity of a longitudinal wave propagating in the 2-direction ; V_{21}, V_{23} are the velocities of two shear waves polarized perpendicular to each other.

In order to develop equation describing the influence of stress state on the ultrasonic velocity it is assumed that the principal axes of strain coincide with the principal axes of stress. Using the generalized Hooke's law, the strains are replaced by the stresses and the elastic moduli.

Replacing $l + 2m$ in equation (1a) by $r^0 (V_{22}^0)^2$ and using the approximation $V_{22} + V_{22}^0 = 2V_{22}^0$, simple treatments result in :

$$\frac{V_{22} - V_{22}^0}{V_{22}^0} = \frac{A}{C}.s_2 + \frac{B}{C}.(s_1 + s_3) \quad (2)$$

Where V_{22}^0 is longitudinal wave speed at zero axial strain. A, B, C are combinations of the elastic constants of the material:

$$A = 2(l+m)(4m+5l+10m+2l) - 2l(2l+l) \quad (3a)$$

$$B = 2(2l+l)(l+m) - l(2l+l) - l(4m+5l+10m+2l) \quad (3b)$$

$$C = 4m(l+2m)(3l+2m) \quad (3c)$$

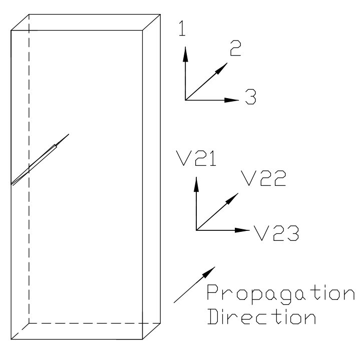


Figure 1.Speed of plane waves on orthogonal coordinate

The experimental work covered in this paper falls into the area of plane stress with the acoustic waves propagating normal to the plane of stress. Thus , $s_2 = 0$ and we obtain

$$\frac{V_{22} - V_{22}^0}{V_{22}^0} = \frac{B}{C}.(s_1 + s_3) \quad (4)$$

To examine the acoustoelastic response of a material, an accurate measurement of the wave speed is necessary. In this study, the double-pulse overlap technique described in detail by Ilic al. (1982)[2],is used to measure the phase change of a wave propagating through the specimen. The velocity is determined by combining this phase shift with the change in path length .

A longitudinal acoustic pulse from the transducer traverses the water path to be reflected by the front and back faces of the specimen. The transit time of the pulse through the double thickness of the specimen is indicated by time delay in the return of the back echo after the front echo. Variations in transit time are measures of changing thickness and pulse velocity.

The first back-face echo has a transit time through the sample of [3]

$$T = 2L / V_{22} \quad (5)$$

Where L is the specimen thickness and V the wave velocity.

Therefore, the total phase shift between the front- and back-face echoes is

$$f = wT = 2pf\left(\frac{2L}{V_{22}}\right) \tag{6}$$

Where w is the radian frequency.
By logarithmically differentiating equation (6) , it will be seen that

$$\Delta f / f = \Delta f / f^{0} + \Delta L / L^{0} - \Delta V_{22} / V_{22}^{0} \tag{7}$$

If zero phase change ($\Delta f = 0$) is maintained during the measurement by adjustment of frequency w, then

$$\frac{\Delta V_{22}}{V_{22}^{0}} = \frac{\Delta f}{f^{0}} + \frac{\Delta L}{L^{0}} \tag{8}$$

Where the superscript zero indicates evaluation at zero load. The term $\Delta L / L^{0}$ is strain in the direction of wave propagation and in plane stress,

$$\frac{\Delta L}{L^{0}} = e_{Z} = -\frac{n}{E}\left(s_{x} + s_{y}\right) = -\frac{n}{E}\left(s_{1} + s_{3}\right) \tag{9}$$

Where s_{1} , s_{3} are the principal stress components in x-y plane. Then

$$\frac{dV_{22}}{V_{22}^{0}} = \frac{\Delta f}{f^{0}} - \frac{n}{E}\left(s_{1} + s_{3}\right) \tag{10}$$

So, equation (4) can be rewritten as

$$\frac{\Delta f}{f^{0}} = \left(\frac{B}{C} + \frac{n}{E}\right)\left(s_{1} + s_{3}\right) \tag{11}$$

Or,

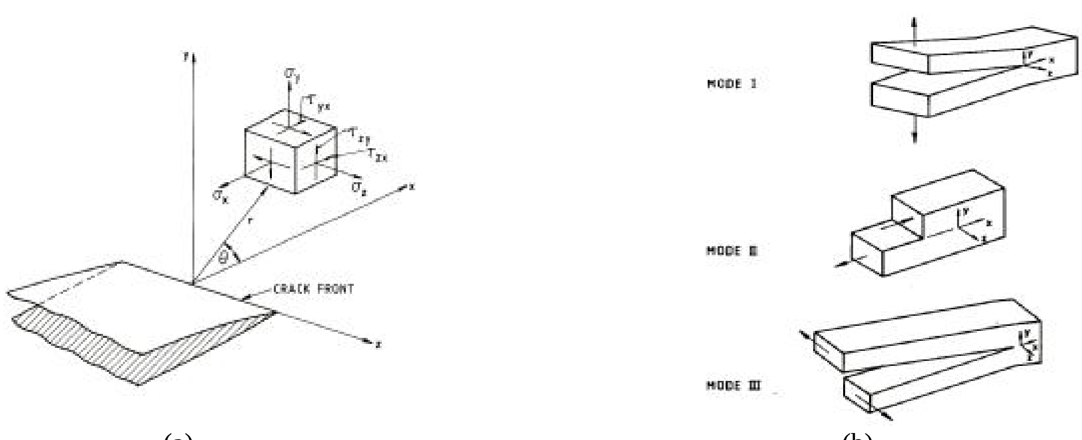
$$\frac{\Delta f}{f^{0}} = N\left(s_{1} + s_{3}\right) \tag{12}$$

Where material constant, N, is

$$N = \left(B/C + n/E\right) \tag{13}$$

2.2 Stress fields in the vicinity of a crack tip

A crack in a solid can be stressed in three different modes, as illustrated in figure 2 . Normal stresses denoted as mode I. The displacements of the crack surfaces are perpendicular to the plane of the crack. In-plane shear results in mode II. The displacement of the crack surfaces is in the plane of the crack and perpendicular to the leading edge of the crack. Mode III is caused by out-of-plane shear. Crack surface displacements are in the plane of the crack and parallel to the leading edge of the crack.[4]



(a) (b)

Figure 2.(a)-Crack-tip coordinates and stress state,(b)- Basic modes of loading and associated crack surface displacements.

Stress field equation equations for mode I case, are given by :

$$s_x = \frac{K_I}{\sqrt{2pr}} \cos\frac{q}{2}\left(1 - \sin\frac{q}{2}\sin\frac{3q}{2}\right) \quad (14a)$$

$$s_y = \frac{K_I}{\sqrt{2pr}} \cos\frac{q}{2}\left(1 + \sin\frac{q}{2}\sin\frac{3q}{2}\right) \quad (14b)$$

For the second mode (II) of deformation :

$$s_x = -\frac{K_{II}}{\sqrt{2pr}} \sin\frac{q}{2}\left(2 + \cos\frac{q}{2}\cos\frac{3q}{2}\right) \quad (15a)$$

$$s_y = \frac{K_{II}}{\sqrt{2pr}} \cos\frac{q}{2}\sin\frac{q}{2}\cos\frac{3q}{2} \quad (15b)$$

Where K_I and K_{II} are the stress intensity factors for mode I and mode II , respectively, and r and q the distance and the angle in polar coordinates of the point considered in respect to the tip of the crack. (see Fig . 2)

Practical structures are not only subjected to tension but they also experience shear and torsional loading. Cracks may therefore be exposed to tension and shear, which leads to mixed mode cracking. The combination of tension and shear gives a mixture of modes I and II . For mixed mode, stress field at a crack determined by adding $s_x = s_{xI} + s_{xII}$, and $s_y = s_{yI} + s_{yII}$,etc., where s_{xI}, s_{yI} are the stresses due to mode I and s_{xII} , s_{yII} are the stresses due to mode II. We will now write expressions(1) to (10) for distribution of stresses s_{ij} in the vicinity of a crack tip in some different form

$$s_{ij} = \frac{K_I}{\sqrt{2pr}} f_{Iij}(q) + \frac{K_{II}}{\sqrt{2pr}} f_{IIij}(q) \quad (16)$$

Here r, q are the polar coordinate of a point ; f_{Iij} and f_{IIij} are combinations of trigonometric functions. Therefore, by adding s_x and s_y in mixed modes I and II, we have

$$s_x + s_y = \frac{2}{\sqrt{2pr}}(K_I \cos\frac{q}{2} - K_{II}\sin\frac{q}{2}) \quad (17)$$

By combining equation (12) and equation (16) ,($s_x + s_y = s_1 + s_3$)

$$\frac{1}{N}\left(\frac{\Delta f}{f^0}\right) = \frac{2}{\sqrt{2pr}}(K_I \cos\frac{q}{2} - K_{II}\sin\frac{q}{2}) \quad (18)$$

3. Experimental techniques

To demonstrate the accuracy, stability of the method test under more complicated loading conditions, an experiment was conducted in which a specimen was subjected to both biaxial tension and shear deformation. The specimen was made of steel with Young's modulus E=208.7GPa, Poisson's ratio $n = 0.27$ and yield stress s_{Yield}=220 MPa, elastic constants $l = 110GPa$, $m = 82GPa$, $l = -290GPa$, $m = -666GPa$, $n = -720GPa$.

An immersion technique based on a pulse-echo method was used to design an ultrasonic apparatus for scanning measurements with longitudinal waves on plane specimen. Immersion of the transducer and specimen in water provided an acoustic path and permitted easy scanning of the transducer. The two dimensional motion and location of the transducer at measurement positions is accomplished by a perpendicular pair of digital stepping motor driven translatory slidesThe machine has provisions for precisely maintaining the relative position and alignment of the transducer and the specimen. A water-filled tank for immersion of the specimen and transducer is an integral part of the assembly[5]. A loading

machine with tensile load capability of 30 Ton was developed for these experiments. Loading is provided by a hydraulic cylinder supplied with oil at an accurately controlled pressure. (Fig. 3)
A crack slanted at 45^0 to the boundary of a rectangular plate was selected for the present study. The plate loaded by a uniform end tension. The geometry of the plate is shown in Fig.4.

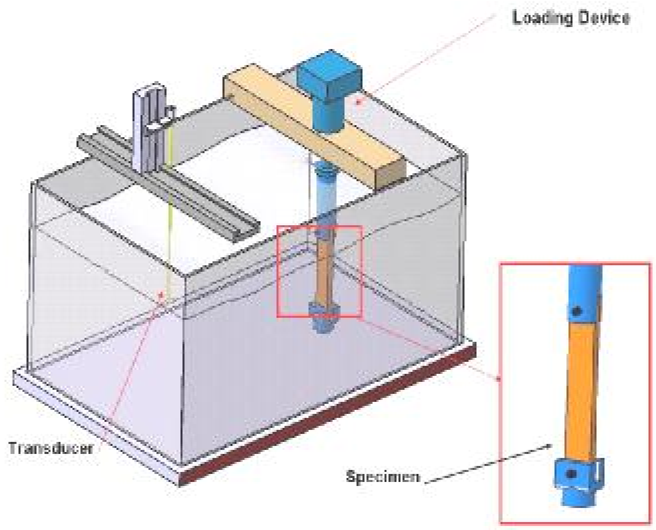
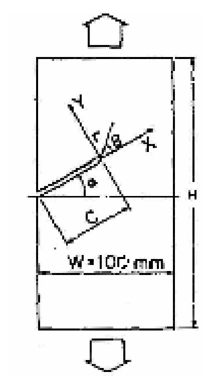


Figure 3 . Experimental system

Expressed in coordinates normal and tangential to the crack, this leads to the biaxial tension and shear tractions shown in Fig. 4 .



H=400mm
$a = 45^0$
$C/W = 0.5$
$s = 40MPa$

Figure 4. Geometry and loading for oblique edge cracked plate[6].

4. Results and discussion

In this section, the finite element results obtained for crack tip parameters in the specimen for mixed mode I/II are presented and compared with experimental results.
Experimentally measured sum total of main stresses have been employed in order to calculate the stress intensity factors, according to equation (18).For determination of sum total of main stresses, the time of flight measurement were effected at 119 points around the crack tip. These measurement grid, are shown in fig. 5.[7]
Subjecting the specimen to various load level up to a maximum nominal stress of 40 MPa.

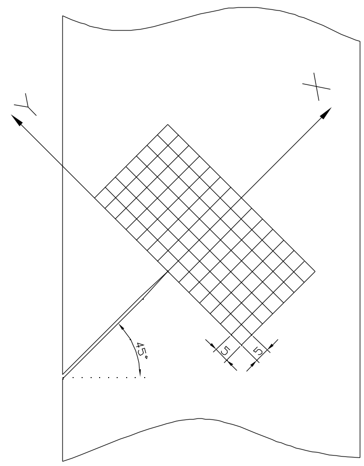


Figure 5. Inspection grid for scanning measurements with longitudinal wave in (mm)

Utilizing all of the measurements affected, considering as known the position of the crack tip and thus the variable r , 119 equations of type eq.(18) were obtained, which, when resolved by least squares, furnished the values : $K_I = 640\,MPa.(mm)^{1/2}$ and $K_{II} = 285\,MPa.(mm)^{1/2}$.

A theoretical check can be used directly to determine the stress intensity factors [8]

$$K_I = 1.27s\sqrt{pC} \quad , \quad K_{II} = 0.58s\sqrt{pC} \tag{18}$$

The theoretical values are

$$K_I = 636.68\,MPa.(mm)^{1/2} \quad , \quad K_{II} = 290.76\,MPa.(mm)^{1/2}$$

The agreement between theoretical and experimental data confirms the validity of the approach.

A numerical analysis was performed using a two-dimensional elastic-plastic finite element method, employing the incremental theory of plasticity under the assumption of plane stress. The VonMises effective stress-strain relationship was used. (see Fig.6)

Figure 7 and 8 shows the behavior of sum total of main stresses as actually measured and theoretically evaluated around the tip of the crack. A good agreement is seen between both results.

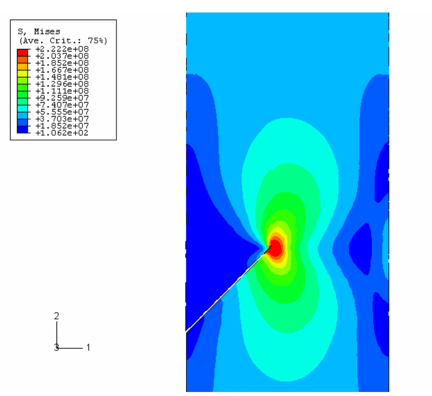


Figure 6. Finite element model

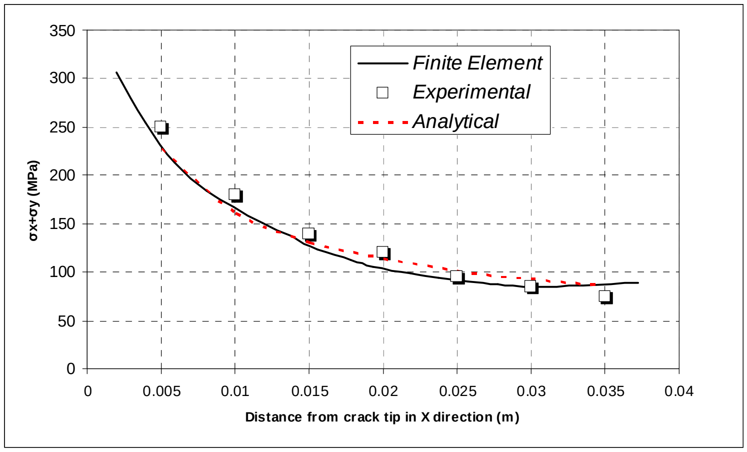


Figure 7. Comparison of the calculated and experimental results of the sum total of main stresses around the tip of the crack in X direction.

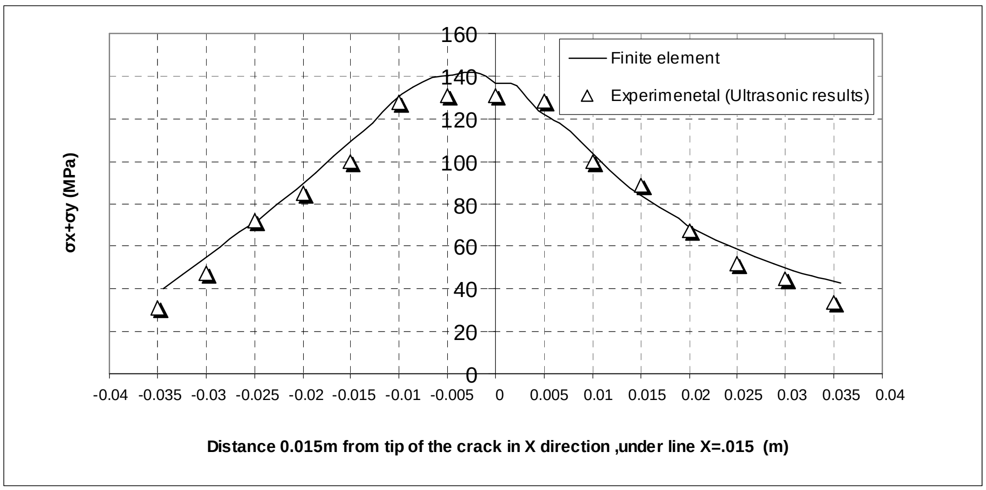


Figure 8. Comparison of the calculated and experimental results of the sum total of main stresses under line X=0.015.

5. Conclusions

A technique for Nondestructive evaluation of stress in around the tip of the crack has been presented. Test results have shown that the automatically scanned double pulse echo technique for ultrasonic imaging can be applied to quantitative measurement of stress-field profiles. Also, the test results have demonstrated that the double pulse echo method of measuring ultrasonic velocities can be effectively adopted for determining the stress intensity factor in mixed mode loading. It is also interesting to note that this method dose not require knowledge of the geometric dimensions of the crack.

Acknowledgments

We gratefully acknowledge the partly financial support of P.O.G.C of Iran.

References

1. Egle D.M and Bray D.E , 'Measurement of acoustoelastic and third-order elastic constants for rail steel' , Journal of the Acoustical society of America, Vol 60, No 3 , pp 741-744 , September 1976.
2. Ilic D.B and Kino G.S , 'Computer-Controlled system for measuring Two-dimensional acoustic velocity fields' , Review of Scientific Instruments, Vol 50 ,No 12, pp 1527-1531, 1979

3. Kino G.S and Hunter J.B and Johnson G.C , ‘Acoustoelastic imaging of stress fields’ ,Journal of applied physics , Vol 50, No 4 , pp 2607-2613 , April 1979.
4. Meguid S.A , ‘Engineering fracture mechanics’ , Elsevier applied science, 1989.
5. King R.B and Herrmann G ., ‘Acoustoelastic Determination of forces on a Crack in Mixed – Mode Loading’, Journal of Applied Mechanics , Vol 50, pp 379 – 382 ,1983.
6. Itoh Y.Z and Murakami T , ‘Proportional extrapolation techniques for determining stress intensity factors’ , Engineering Fracture Mechanics, Vol 31, No 2, pp 297-308,1988.
7. Vangi D, ‘Ultrasonic technique for experimental evaluation of structural integrity’ , Journal of Testing and Evaluation, Vol 29, No 3, pp 301-305, May 2001.
8. Tada H , ‘The stress analysis of cracks Handbook’ , Del Research Corp, Hellertown , PA, 1973 .