

# AN OVERVIEW OF TESTING APPLICATIONS OF WAVELET IN GUIDED WAVES\*

Fangcheng He<sup>1,2</sup>, Yifei Gao<sup>1</sup>, Zhenggan Zhou<sup>1</sup>, Jicheng Bai<sup>3</sup>

<sup>1</sup>School of Mechanical Engineering and Automation, Beihang University, Beijing 100083, P. R. China

<sup>2</sup>Department of NDT, Beijing Institute of Aeronautical Materials

<sup>3</sup>School of Mechatronics Engineering, Harbin Institute of Technology, Harbin 150001, P. R. China

## Abstract

Guided Waves Testing is a new technology of ultrasonic testing of NDT bloomed in recent years especially for pipes and boards inspection in industry. Different methods used in guided waves signal processing are analyzed. The contrast between Fourier and Wavelet analysis is presented to support the outstanding features of wavelet in the processing of signals with sudden changes, such as guided waves. The state - of - the - art of the research in the application of wavelet to guided waves is described in detail. It is concluded that more abundant and clear information can be get and several certain bandwidth signals can be extracted with appropriate wavelet in guided waves in order to find the defects and their status. Wavelet is a potential analysis approach which would be used widely in guided waves inspection.

**Key Words:** Ultrasonic, Guided Waves , Wavelet Analysis

## 1. Introduction

Compare with traditional ultrasonic, guided waves are more efficient and sensitive in nondestructive testing (NDT) of large scale plate or long range pipe. Due to travel along with the object surface, guided waves scan in a line instead of a point, which makes high performance in testing. By using multiple mode, guided waves can detect different types of defects especially holes and flaws with small size [1]. Although this technology has already been applied in many cases of NDT, but there are still a lot problems to be solved. Signal processing of guided waves is an important topic among these problems because of the complexity of its signals. The information of interest can be separated from a composite signal includes noise, scatter and some modes which are not care about by using this powerful tool.

## 2. Characteristics of guided waves signal [1]

The guided waves have much different to bulk waves because they propagate in finite media. The interactive between the guided waves and media

boundaries induce mode conversions between longitudinal waves and shear waves. Therefore the signals of guided waves include much more information of the media than the common bulk waves. The dispersion and multiple modes are two main characteristics.

The velocities of longitudinal waves and transverse waves are all same in different frequency, whereas the velocities of guided waves vary with frequency, this is called disperse phenomenon. For the example of Aluminum, the phase velocity disperse curves are illustrated in Fig 1.

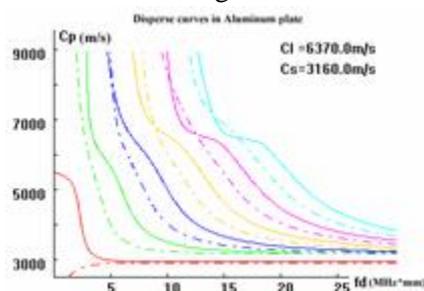


Figure 1: *Disperse curves in Aluminum*

Known from this graph, in a certain product of frequency and thickness have at least two modes, therefore the signal obtained from receiver is the mixture of different modes of signal. In fact, the sample with different material or different structure will have different kinds of propagation modes,

which makes prediction and analysis of the guided waves become very difficult.

The guided waves can provide abundant information about the sample in the scan line, because of the characteristics of them. This information contains the signals of different modes which travel in different velocity, and certain mode is more sensitive than the others to some kinds of defects, so the signals should be processed for the different purpose.

### 3. Signal processing of guided waves

The methods of time series analysis, such as averaging filter, median filter and etc, are earliest applied in signal processing, they can improve the signals obviously. This situation was last until the early of eighteen century, before Fourier Transforms appeared.

In 1807, J.B.J Fourier asserted that any periodic function can be expressed by linear combination of sine and cosine functions with different frequency and amplitude [2], expressed as

$$x(t) = \sum_{-\infty}^{+\infty} a_k e^{jk(2p/T)t} \quad (1)$$

The coefficients  $a_k$  can be given by

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2p/T)t} dt \quad (2)$$

The Fourier transform, a powerful tool of signal analyze, holds the important status in the signal processing. It has shown that any complex signal in the time domain can be split into some simple harmonic waves. With its help the signal can be converted from time domain to frequency domain and these two kinds of conformations from different analysis domain compose the Fourier transform pair which can help to study the characters of the signal more exact and refined. At the same time, Fourier analyses also has defects, whatever in the time or frequency domain, the Fourier transform is defined in the scope of real number which makes it can not fit for the local time series and spectrum signature. For this reason, Fourier transform is few applied in sudden change signals. Windowed Fourier

Transform comes into being to meet the requirements of local analysis for sudden change signals. WFT can be written as

$$(Gf)(w, b) = \int_R f(t) w(t-b) e^{-iwt} dt \quad (3)$$

The window function  $w(t)$  is one function of rectangular, spline, gaussian or the others.

WFT makes a great promotion based on Fourier transform, whereas it is still unfit for precise local signals analyses because of the fixed window size. The low frequency signal changes slowly in a long time scope whereas the high frequency changes very fast and consequently need a window function which can adjust the window size with the signal frequency automatically. WFT is not competent for the task which promotes the development of wavelet, a multi-scale representation of signals [3].

### 4. Wavelet and Time-Frequency analysis [2] [3] [4] [5]

In the local area analyses of time frequency domain, an adaptive window with variable size in different frequencies is demanded. The window about interest regions has long time intervals for the signals on low frequencies and shorter for high frequencies, as shown in fig. Wavelet transform is such an ideal method meets the requirements of multi-scale.

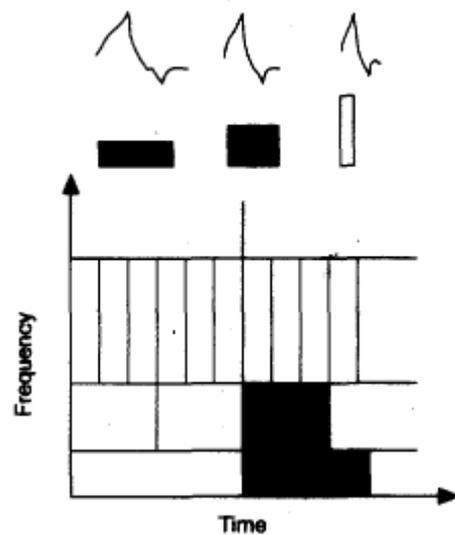


Figure 2: Time-frequency plane with Daubechies wavelet basis functions [6]

If the function  $y(t) \in L^2(R)$ , and satisfied permissibility condition:

$$\int_{-\infty}^{+\infty} \frac{|\hat{y}(w)|^2}{|w|} dw < \infty \quad (4)$$

Where  $\hat{y}(w)$  is Fourier transform of  $y(t)$ . The function  $y(t)$  is called mother wavelet.

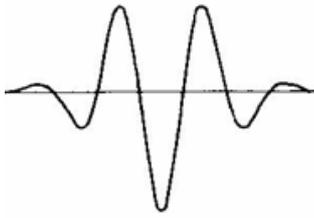


Figure 3: A mother wavelet.

The mother wavelet  $y(t)$  should attenuate very quickly in the practice in order to have compactly support. Because of the permissibility condition,  $\hat{y}(w)$  is a continuous function, so that  $\hat{y}(0) = \int_{-\infty}^{\infty} y(t) dt = 0$ . Only when  $y(t)$  have positive and negative values can make  $\hat{y}(0) = 0$ . For these reasons, the mother wavelet attenuates quickly and oscillates.

Stretching and shifting the mother wavelet  $y(t)$  with stretching factor  $a$  (also be called scale factor) and shifting factor  $b$  construct the wavelet function:

$$y_{a,b}(t) = |a|^{-\frac{1}{2}} y\left(\frac{t-b}{a}\right), \quad a, b \in R, a \neq 0 \quad (5)$$

If the width of the mother wavelet window is  $\Delta t$ , and the center is  $t_0$ , then the window of wavelet function  $y_{a,b}(t)$  which is stretched and shifted can be illustrated by these parameters using:

$$t_{a,b} = at_0 + b \quad (6)$$

$$\Delta t_{a,b} = a\Delta t \quad (7)$$

Where  $t_{a,b}$  is the center of the window and  $\Delta t_{a,b}$  is the width of the window.

If  $f(t) \in L^2(R)$ , its continuous wavelet transform can be defined as:

$$W f(b, a) = \langle f(t), y_{a,b}(t) \rangle = |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t) \overline{y\left(\frac{t-b}{a}\right)} dt \quad (8)$$

The most important different between Wavelet transform and Fourier transform is that wavelet functions are localized in the space where as sin and cosine functions are continuous. Wavelet transform has the parameters of scale factor  $a$  and shifting factor  $b$ , which make Wavelet transform have a variable size of analyses window. The function  $f(t)$  can be projected to two dimensions time scale plane by decomposed with wavelet basis. In this plane, the frequency increases with the scale decreases, and vice versa. This localization feature is fit for sudden change signals, and makes Wavelet transform overcomes some limitations of Fourier transform.

### 5. Some applications of wavelet in guided waves

Wavelet is a powerful tool of time-frequency analyses for sudden-change signal, therefore it is used in a lot of applications in guided waves testing [7], such as the examples following.

#### 5.1 The Research of Characters of Guided Waves Propagation [8] [9]

Because of disperse and multi-mode of guided waves, the research of guided waves propagation is very difficult. In traditional, an inefficient approach is applied in order to get the disperse curve of the sample. In the first, the incident angle is fixed, the frequency of excitation signal is changed little by little, which is called frequency scan, a horizontal line in the dispersion curves which describes incidence angle versus frequency can be obtained. In the second, change the incident angle from 0 to 360 degree with a small increasing, so that every

horizontal line in the curves will be acquired. Also by using the same method, the dispersion curves about the relationship of frequency and group velocity or frequency and phase velocity can be drawn.

If using a broad band signal to excite the probe will send out guided waves with different frequencies, accordingly the received signals contain the information of different modes in a range of frequency varying continuously, which represent a horizontal line in the dispersion graph.

In reference [8] [9], Gabor wavelet was applied to investigate the dispersions of guided waves propagated in a thin plate. The guided waves' signal was produced by breaking a lead, received by an amplifier in the acoustic emission experiments. The strength of each mode is related to the breaking angle, the A0 and S0 mode with high power can be emitted in a certain angle.

Gabor wavelet was used to decompose the signal in order to get the signals in each frequency. A conclusion can be drawn after wavelet transform that the signal contains many modes, and A0 mode and S0 mode are two main components. The signal peaks of A0 and S0 mode in different frequencies was shown in the contour map, the propagation time was measured from the beginning to the each mode's peak time. By this means, several dispersion curves can be calculated with only one broad band echo signal, which makes the research of the propagation characters of guided waves become more efficient and quick.

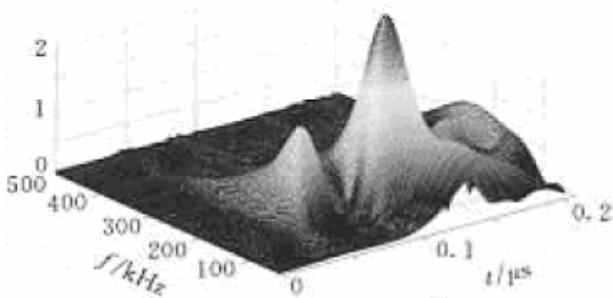


Figure 4: Gabor Wavelet transform decomposition of guided wave signal in time-frequency analysis [9].

## 5.2 Pipe Inspection [10]

Pipe inspection is a kind of technology which is already put into practice, and also becomes one of the main applications of guided waves. Furthermore, the approach is a potential method to the inspection of pipe in service whereas the traditional ultrasonic is not suitable for these special situations. In the applications, the system parameters are controlled strictly so as to generate the waves with selected modes. Comb transducer, band pass filters and other facilities are used to acquire specific propagation modes. Wavelet is another choice to achieve this difficult work. With mother wavelets, the signal can be decomposed into components in several levels. These levels represent frequency from low to high, and the original signals can be recovered by add them up. Reconstruct the signal by reserving some levels and discarding the others can take the same effect as band-pass filters. Wavelet transform can retain even improve signal/noise ratio of signal whereas band-pass filters would decrease it in most of the cases.

The mother wavelet Daubechies3 was applied in the result analyses. It is one of Daubechies wavelet function, which is orthogonal and has an irregular shape as shown in Fig 2. The signal was break down approximations by Discrete Wavelet Transform and removed the parts of high frequencies by abandoning the details in some scales.

The result shows that compared with band pass filter, Wavelet transform can not only carry out denoising, but also improve signal/noise ratio up to around 12DB, so that the echo signals of defects become clarify and distinct.

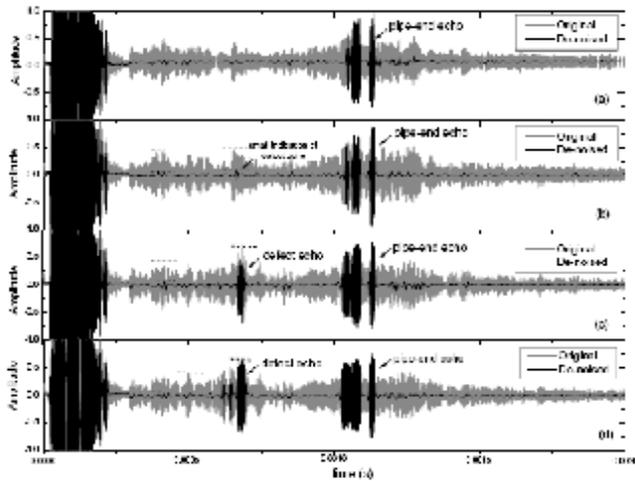


Figure 5: Original and denoised signals: (a) no defect; (b) defect 4 mm; (c) defect 7mm and (d) through defect [10].

### 5.3 Long-range Defect Detection in Railroad Tracks [11] [12]

Multimode and dispersive caused by complex structure of rail tracks induce complications of testing especially in mode identification.

In some applications, Wavelet transformation coefficient was used to detect defects directly. From the discontinuous coefficient, defects can be identified [12].

In another case [11], Lamb waves were excited to detect defects in 115-lb AREMA tee rail. A commercial finite element package, ABAQUS EXPLICIT, was used to model the propagation of lamb waves at frequencies below 100 KHz in rail track structure. The simulation results showed guidelines to the experiments and presented reference dispersion curves of lamb waves propagated in such a complex structure.

Gabor wavelet applied with the continuous Wavelet transform performed the joint time frequency analysis of the signals in order to simplify the operation. By using broad band guided waves, Gabor wavelet transform (GWT) analysis can calculate the group velocity dispersion curves only with one received signal in a single detection point. The amplitude of the signal in the joint was described in the GWT graph in the time frequency domain, as shown in Fig. 6.

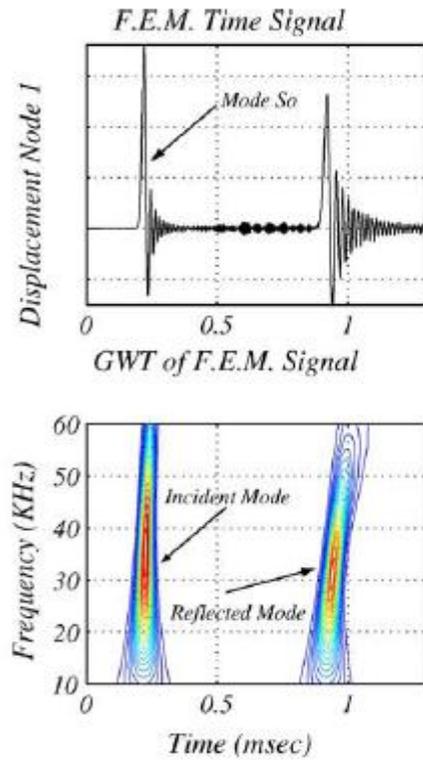


Figure 6: ABAQUS simulations of Lamb waves of S0 mode and GWT time-frequency of signal [11].

Two nodes were placed in the sample with 750mm distance between them, and the group velocity of guided waves can be calculated from the arrival times of the signal peaks get from the GWT graph for the two nodes following:

$$C_g(f) = \frac{L}{t_2(f) - t_1(f)} \quad (9)$$

Where  $c_g$  is group velocity,  $L$  is the distance between the two nodes,  $t_1$  and  $t_2$  are the arrival times of the signal peaks.

The result of GWT was employed to compute the energy reflection coefficient which was needed in following analysis in order to find the defect. The reflection coefficient can be given by:

$$R(f) = \frac{\int_{\Delta t_2}^{GWT_2(t, f) dt}{\int_{\Delta t_1}^{GWT_1(t, f) dt}} \quad (10)$$

Where  $\Delta t$  is the duration of the signal at frequency  $f$ , and the subscripts 1 and 2 refer to the first arrival and the edge reflection. The result shows that

defects with different size can be judged from the reflection coefficient.

## 6. Challenges and summary

The application of Wavelet transform is still in the primary stage although it has already performed in many cases to overcome the limitations of traditional signal processing methods. At first, selecting the mother wavelet fit for the different purposes is difficult. At present, considerable work have been done in researching of the theory and approach about the mother wavelet construction. The mother wavelet with different types and characters, such as orthogonality, symmetry and slickness, meets the different requirements in guided waves processing and the proper mother wavelet can improve precision of analyses in the future [3] [5].

At second, the propagation characters in the complex structure components of guided waves are not fully grasped, the dispersion curves are calculated mainly by experiment datum. So that the signification of the result of Wavelet transform is un-clarity. So much information provided by Wavelet transform is neglected because of the lack of understanding of its meaning. It has great potential in data mining of this information so that to expand the functions of guided waves.

At last, with popularization of high performance computer and Wavelet transform software package, Wavelet transform will be more practical and become a kind of general approach in sudden change signals analysis.

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