Transmission Characteristics of $A_oA_o$ and $A_oS_o$ Modes at a Delamination Interface

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Abstract. During the interaction of the anti-symmetric fundamental Lamb mode ($A_o$) with the front edge of a delamination, it undergoes mode conversion. The mode converted $A_o$ mode ($S_o$) propagates as $A_oS_o$ mode along with the incident $A_o$ mode, which propagates as $A_oA_o$ mode in top and bottom sub-laminates. A detailed study on variation in transmission factors and transmission coefficients revealed that, transmission factor of $A_oA_o$ mode is nearly independent of thickness ratio and excitation frequency. Whereas, the transmission factor of $A_oS_o$ mode depends on the thickness ratio and excitation frequency. Moreover, power transmission coefficient of $A_oA_o$ mode is a strong function of the thickness ratio, but, the power associated with the $A_oS_o$ mode is very less as compared to its $A_oA_o$ counterpart.

1. Introduction

Various techniques have been devised for detecting damages in composite materials. However, Lamb wave methods have recently re-emerged as a reliable way to locate damage in these materials. The Lamb wave has potential to offer an effective method to estimate the location, severity and type of damage. They have also demonstrated suitability for Structural Health Monitoring (SHM) applications having an online sensor and actuator network to assess the state of a structure during operation. A number of researchers have investigated guided Lamb wave method for structural Non-destructive Evaluation (NDE) [1–3].

Yang et al. [4] presented some aspects of numerical simulation of Lamb wave propagation in composite laminates using the FE models with explicit dynamic analysis. Kang et al. [5] showed that generation of Lamb waves using piezoelectric transducers and its propagation using dynamic FEM analysis. Yuan et al. [6] presented wave reflection and transmission in composite beams containing a semi-infinite delamination by using Timoshenko theory for two extreme delaminated surface conditions - open delamination and closed delamination. Mirahmadi and Honarvar [7] improved time resolution and signal-to-noise ratio obtained from inspection of plates by the Symmetric fundamental Lamb mode ($S_o$) in an aluminium plate. Ramadas et al. [8] investigated the propagation of the anti-symmetric fundamental Lamb mode ($A_o$) in a composite laminate containing semi-infinite delamination through numerical simulations and experiments employing the air coupled ultrasonic technique. In addition, turning Lamb modes and a mode converted turning Lamb modes were also observed in numerical simulations and experiments. Hosseini and Gabbert [9] investigated the guided Lamb wave propagation in honeycomb sandwich panels using finite element simulation. A parametric study was used to show the influence of the geometrical properties of honeycomb plates, the material properties of the skin plates and the loading frequency on the group velocity, the wave length and the energy transmission. Each of these properties provided valuable information to design an efficient structural health monitoring system.

To implement delamination damage detection with Lamb waves, it is necessary to understand the mechanism of interaction between Lamb waves and delamination damage. When the fundamental anti-symmetric Lamb mode ($A_o$) is incident at the front edge of delamination, it
interacts and gets transmitted into the sub-laminates. It also undergoes mode conversion and propagate independently in sub-laminate. The mode converted \( A_0 \) mode \((S_0)\) propagates as \( A_0S_0 \) mode along with the incident \( A_0 \) mode, which propagates as \( A_0A_0 \) mode in top and bottom sub-laminates. However, the inherent anisotropy of composites renders a mechanical behaviour that is difficult to predict. Finite element analysis is the most powerful numerical tool used to analyse this mechanism. Hence an investigation has been performed in which the potential of variation in Lamb wave transmission factors and transmission coefficients for composite structures is evaluated by using the Finite Element Method (FEM).

### 2. Modelling of Lamb wave propagation and FEM details

The numerical simulations were carried for the fundamental anti-symmetric Lamb mode \((A_0)\) with the frequencies from 150 kHz to 225 kHz in the steps of 25 kHz. There were five cycles in the excitation pulse modulated using Hanning window function. The ply stacking sequence was unidirectional \((UD)\) and the thickness of each ply was 0.33 mm. There were eight plies \(([0\_8])\) so the total thickness of laminate worked out to be 2.64 mm. The length of model considered was 300 mm with semi-infinite delamination from 150 mm to 300 mm. Since each laminate contains a total number of eight plies, it was possible to have a delamination at any one of the interfaces across the thickness. It was assumed that plane strain conditions prevail. The excitation was given at \( x = 0 \) and receivers were deployed at \( x = 60 \) mm over the top of the main laminate and \( x = 210 \) mm over the top and bottom sub-laminates as shown in Fig. 1.

![Fig. 1: Transmitter and Receivers location](image)

The in-plane and out-of-plane displacement time histories were captured at all the receivers. The material used in numerical modelling was glass/epoxy (GFRP) and its mechanical properties are listed in Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>( E_{11} ) (GPa)</th>
<th>( E_{22} ) (GPa)</th>
<th>( \nu_{13} )</th>
<th>( \nu_{23} )</th>
<th>( G_{13} ) (GPa)</th>
<th>( \rho ) kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass/Epoxy</td>
<td>44.68</td>
<td>6.90</td>
<td>0.280</td>
<td>0.355</td>
<td>2.54</td>
<td>1990</td>
</tr>
</tbody>
</table>

The element selected was 2D plane strain element. The displacement profiles of \( A_0 \) mode at the respective frequencies were applied across the laminate thickness. The Hilbert Transform (HT) was carried out on each signals captured at the respective locations as discussed in the above paragraphs. The peak of HT was taken as the representative time of arrival or Time-of-Flight (ToF) of that wave group. Further, the ToF of the Lamb mode was analytically estimated using group velocity obtained from DISPERSE. There was a good agreement between the arrival times of wave groups from \( A \)-scan and analytically estimated values.
In addition to this, Fourier Transform (FT) of the received signals captured at the respective locations as mentioned in Fig. 1 was carried out. The peak of the amplitude of each signal was at its central frequency of excitation. These two checks ensured the mode of propagation was $A_0$. Further, the transmission factors, based on Fourier Transform (FT), were calculated as the ratio of the amplitude of transmitted / generated signal to that of the incident signal. The transmission factors of the transmitted and generated wave groups were computed for various excitation frequencies and locations of delamination interface across the laminate thickness. The thickness ratio was defined as the ratio of thickness of sub-laminate to that of the main laminate. The variation in transmission factors of Lamb wave groups $A_0A_0$ and $A_0S_0$ with the thickness ratio and central frequency of excitation is shown in Fig. 2 and Fig. 3 respectively.

The power associated with the transmitted and generated wave groups $A_0A_0$ and $A_0S_0$ was estimated through numerical simulations. The estimation was based on the average elemental rate of work over the whole thickness which can be obtained by integrating over the time and thickness. Hence, the power transmission coefficient (ratio of the transmitted power to the incident power) of
wave groups, $A_0S_0$ and $A_0A_0$, were computed using the following equation, which gives the time averaged power flow, $<P>$, across any cross-section [6].

$$<P> = \frac{1}{t_o} \int_{t_o}^{t_o + t_o/2} \int_{-h/2}^{h/2} (\sigma_{xc} u + \tau_{xc} w) dz dt$$  \hspace{1cm} (1)$$

Here, ‘$h$’ is the total thickness of plate. Selection of time integration limits depends on duration of pulse and the thickness of the plate. Since, the number of cycles chosen in the excitation pulse was five, the duration of excitation was limited up to 33.33 $\mu$s to 22.22 $\mu$s in Eq. 1, depending on the central frequency of excitation, which varied from 150 kHz to 225 kHz, respectively. The variation in power transmission coefficients of Lamb wave groups $A_0A_0$ and $A_0S_0$ with the thickness ratio and central frequency of excitation is shown in Fig. 4 and Fig. 5 respectively.

3. Results and discussion

Fig. 2 shows variation in transmission factors of the transmitted wave group, $A_0A_0$ mode, with the thickness ratio for various excitation frequencies. It was found that the influence of excitation
frequency on transmission factors is negligible. For a given thickness ratio, the transmission factor is almost same for all the excitations frequencies in the range 150 kHz to 225 kHz. The following observation was made, when the small changes are also considered. The transmission factor decreases with increase in the thickness ratio, reaches minimum value when the delamination is located symmetrically. With further increase in the thickness ratio, transmission factor increases as shown in Fig 2.

Variation in transmission factors associated with the mode converted $A_o$ modes ($A_oS_o$) mode, with the thickness ratio for various excitation frequencies is shown in Fig 3. Transmission factor of $A_oS_o$ mode keeps on decreasing with increase in the thickness ratio. For a given thickness ratio, transmission factor decreases with increase in the thickness ratio. Transmission factor of $A_oS_o$ mode is more sensitive than $A_oS_o$ mode to the thickness ratio.

Fig. 4 shows variation in power transmission coefficients of the transmitted wave group, $A_oA_o$ mode, and Fig. 5 shows for the mode converted $A_o$ modes, $A_oS_o$ mode with the thickness ratio for various excitation frequencies. The power associated with the wave group $A_oA_o$ and the $A_oS_o$ wave group propagating in the upper sub-laminate and lower sub-laminate is equal for various excitation frequencies in case of symmetrical delamination. However, the power transmission coefficients of the $A_oA_o$ mode shows sharp and continuous increasing trend as the thickness ratio is increased for various excitation frequencies. Whereas, the power transmission coefficients of the $A_oS_o$ mode shows initial rise followed by continuously decreasing trend for various excitation frequencies. The transmitted wave groups, $A_oA_o$, carry the major portion of the incident power compared to the mode converted $A_o$ mode, $A_oS_o$, which carries little portion of the incident power at the structural discontinuity. The detailed numerical study carried out on power distribution at the delamination edge revealed the that the power associated with $A_oA_o$ and $A_oS_o$ modes depends on the location of delamination interface and the central frequency of excitation.

4. Conclusion

A systematic numerical study was carried out on the propagation and transmission characteristics of $A_oA_o$ and $A_oS_o$ modes propagating in the sub-laminates. It is found that transmission factor of $A_oA_o$ mode is nearly independent of thickness ratio and excitation frequency. Whereas, the transmission factor of $A_oS_o$ mode depends on the thickness ratio and excitation frequency. This is because in-plane stiffness is a strong function of direction of plies.

Power transmission coefficient of $A_oA_o$ mode is a strong function of the thickness ratio, but, the power associated with the $A_oA_o$ mode is very less as compared to its $A_oS_o$ counterpart. Further, $A_oS_o$ mode shows strong sensitivity to excitation frequency and the thickness ratio compared to $A_oA_o$ mode. Hence, transmission factor of $A_oS_o$ mode could be a good candidate to predict the depth of delamination along the thickness.

References


