High Resolution Damage Imaging Based on Linearly-dispersive Signal Construction with Measured Relative Wavenumber Curves

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ABSTRACT

In Lamb wave-based structural health monitoring (SHM), because of dispersion, the signal wavepackets can be easily elongated and deformed. This can make the signal interpretation and flaw information extraction become hard. The signal resolution is then degraded and the final damage detection results could be badly affected. The dispersion effect can be compensated with traditional Fourier-domain signal processing methods, which usually require the theoretical dispersion relations derived with structural material parameters. However, the parameters could be probably unavailable to make the theoretical wavenumber relations hard to be attained. To solve the problem, linearly-dispersive signal construction (LDSC) is presented for dispersion compensation with in-situ measured relative wavenumber curves. Associated with delay-and-sum algorithm, LDSC is then applied for high resolution damage imaging. Finally, an experimental study is arranged on a glass fiber reinforced composite plate with unknown material parameters.

1. Introduction

Lamb waves are a kind of ultrasonic waves that remain constrained between two free surfaces of a plate or shell. With the ability of long distance travelling and high sensitivity to both the surface and internal defects, Lamb waves have been widely used as a promising tool in structural health monitoring (SHM) for plate-like structures[1-5]. Due to the dispersion and multi-mode characteristics, the propagation of Lamb waves is very complicated. Even for the single mode signals of narrowband, the dispersive wavepackets will spread out in time and space with their waveforms distorted as they propagate[6]. This can affect the results of damage identification, because the wave resolution is grievously degraded and the sensor signal interpretation becomes less straightforward. Consequently, there is a critical need to effectively resolve the dispersion problem in Lamb wave inspection or monitoring.

Time reversal process (TRP)[6-7] can automatically compensate the dispersion effect on Lamb waves. Unfortunately, the time of flight (TOF) of the compensated signals is also eliminated at the same time. Another strategy for dispersion removal is the Fourier-domain signal processing using the wave dispersion relations. Sicard et al[8] proposed a numerical reconstruction method for time compaction of S₀ mode signals in steel plates. Wilcox solved the dispersion problem by mapping Lamb wave signals from the time to distance domains[9]. Considering signal waveform correction, an improved time-distance domain transform (TDDT) method was introduced[10]. Marchi et al[11] and Fu et al[12] carried out dispersion compensation of Lamb waves with warped frequency transform (WFT). Liu and Yuan[13] proposed the linear mapping (LM) technique to effectively recompress the dispersive A₀
mode signals in aluminum plates. From LM, the signal construction-based dispersion compensation approaches were developed.\cite{14}

The above signal processing methods can efficiently eliminate the dispersion effect, but usually require the wavenumber relations of Lamb waves in the tested structures. For the common structures, the wavenumber relations can be theoretically derived using the well-documented geometric and material parameters based on Rayleigh-Lamb equation. However, this could be inapplicable for the structures with their material parameters unavailable. To solve the practical problem, a dispersion compensation method of linearly dispersive signal construction (LDSC) just with measured relative wavenumber curves is presented and applied for high resolution damage imaging. The remaining content is organized as follows: section 2 discusses LDSC based on the Lamb wave sensing model. The relative wavenumber curve measurement method is proposed in section 3. In section 4, a LDSC-based high resolution damage imaging method is developed. The experimental validation is arranged in section 5. Conclusions are made in the last section.

2. LDSC for Dispersion Compensation

2.1 Lamb Wave Sensing Model

With piezoelectric (PZT) wafers applied as actuators and sensors, a Lamb wave signal assumed of single wavepacket can be represented in frequency domain as \cite{6,10}

\[
V_0(\omega) = V_a(\omega)H(\omega) \tag{1}
\]

where \(\omega\), \(V_a(\omega)\) and \(V_s(\omega)\) are the angular frequency, frequency-domain excitation signal and sensor signal, respectively. \(H(\omega)\), regarded as the transfer function of the whole procedure including Lamb wave exciting, propagating and sensing, can be expressed as

\[
H(\omega) = A(r, \omega)e^{-ik_s(\omega)r} \tag{2}
\]

where \(A(r, \omega)\) is the amplitude spectrum of \(H(\omega)\), \(r\) is the travelling distance, \(K_s(\omega)\) is the wavenumber of the Lamb wave mode and

\[
c_p(\omega) = \omega / K_s(\omega), \quad c_g(\omega) = d\omega / d[K_s(\omega)] \tag{3}
\]

where \(c_p(\omega)\) and \(c_g(\omega)\) are the phase and group velocities of the mode, respectively.

For the narrowband excitation signal \(V_a(\omega)\), \(A(r, \omega)\) varying slightly within the limited frequency range can be simplified as “1” to facilitate the following analysis. A simplified sensing model of Lamb waves can be then deduced as

\[
V_s(\omega) = V_s(\omega)e^{-ik_s(\omega)r} \tag{4}
\]

The phased delay factor \(e^{-ik_s(\omega)r}\) in the model indicates that, compared with \(V_s(\omega)\), the phase of \(V_s(\omega)\) is reduced by \(K_s(\omega)r = \omega r / c_p(\omega)\), which corresponds to the delay \(r / c_p(\omega)\) in time domain. For the dispersive Lamb wave mode, \(K_s(\omega)\) is nonlinear with \(\omega\) and \(c_p(\omega)\) is frequency-dependent. Different frequency components of \(V_s(\omega)\) will have different time delays to make the signal wavepacket spread out temporally and spatially.

2.2 LDSC method

As discussed above, it is the nonlinear wavenumber relation \(K_s(\omega)\) that results in the dispersion effect
In LDSC, a linearly dispersive version $V_{in}(\omega)$ of $V_{s}(\omega)$ is constructed for dispersion compensation. The wavenumber relation $K_{in}(\omega)$ of $V_{in}(\omega)$ can be linearized as the first order Taylor series of $K_{in}(\omega)$ around the central angular frequency $\omega_c^{[13-14]}$

$$K_{in}(\omega) = k_0 + k_1(\omega - \omega_c)$$

where $k_0 = K_{in}(\omega_c)$, $k_1 = dK_{in}(\omega)/d\omega|_{\omega=\omega_c} = 1/c_e(\omega_c)$.

Based on the signal construction principle$^{[14]}$, $V_{in}(\omega)$ with the wavenumber relation $K_{in}(\omega)$ can be numerically computed in time domain as

$$v_{in}(t) = \text{IFFT}\left[V_{o}\left[\Omega_{in}(\omega)\right]C(\omega)\right]$$

where $v_{in}(t)$ is the constructed linearly dispersive signal in time domain, $\text{IFFT}[ ]$ denotes inverse Fast Fourier Transform (IFFT) operation, $\Omega_{in}(\omega)$ and $C(\omega)$ are the interpolation mapping sequence in LDSC and the waveform correction factor, respectively.

$$\Omega_{in}(\omega) = K_{in}^{-1}\left[K_{in}(\omega)\right]$$

$$C(\omega) = V_s(\omega)/V_s[\Omega_{in}(\omega)]$$

where $K_{in}^{-1}(k)$ is the inverse function of $K_{in}(\omega)$.

### 3. Relative Wavenumber Curve Measurement for LDSC

From equation (6), $K_{in}(\omega)$ is requisite in LDSC. While in Lamb wave detection, $K_{in}(\omega)$ could not be theoretically derived when the structure property parameters are unavailable. For the practical situation, a relative wavenumber curve measurement method is proposed in this section. In theory, $K_{in}(\omega)$ is defined as

$$K_{in}(\omega) = \Phi(\omega)/L$$

where $\Phi(\omega)$ is the phase difference between the excitation signal $v_{s}(t)$ and the sensing wavepacket of the chosen Lamb wave mode in a Lamb wave sensor signal $v_{s}(t)$, $L$ is the travelling distance of the sensing wavepacket, which can be extracted from $v_{s}(t)$ with a rectangular time window. To inhibit the spectral leakage influence, the raising and trailing edges of the window are designed as the left and right parts of Hanning window, respectively.

The optimized window function is

$$f_w = \begin{cases} 
0 & (0 \leq t \leq t_1 - t_s) \\
\frac{1}{2} - \cos \frac{\pi (t - t_1)}{2t_u} & (t_1 - t_u \leq t < t_l) \\
1 & (t_l \leq t \leq t_2) \\
\frac{1}{2} - \cos \frac{\pi (t - t_2)}{2t_u} & (t_2 < t \leq t_2 + t_s) \\
0 & (t_2 + t_s < t < T)
\end{cases}$$

where $T$, $t_u$, $t_1$ and $t_2$ are the duration of $v_{s}(t)$, half-width of Hanning window, the starting and terminal points of the sensing wavepacket in $v_{s}(t)$, respectively.
Take $V_1(\omega) = \text{FFT}[v(t)]$, $V_{ss}(\omega) = V_1(\omega)/V_s(\omega)$. $\Phi(\omega)$ can be computed from the arctangent function as

$$\Phi(\omega) = \begin{cases} \text{Arctan}[\text{Re} / \text{Im}] & (\text{Re} \geq 0) \\ \text{Arctan}[\text{Re} / \text{Im}]+\pi & (\text{Re} < 0, \text{Im} \geq 0) \\ \text{Arctan}[\text{Re} / \text{Im}]-\pi & (\text{Re} < 0, \text{Im} \leq 0) \end{cases}$$

where $\Phi(\omega)$ is the computation result of $\Phi(\omega)$, Arctan$[ ]$ denotes arctangent operation, Re and Im are the real and imaginary parts of $V_{ss}(\omega)$, respectively. Due to the arctangent algorithm, the range of $\Phi(\omega)$ is constrained to $-\pi$–$\pi$. The discontinuities can be found in $\Phi(\omega)$ whenever $\Phi(\omega)$ exceeds the restricted range$^{[15]}$.

To exempt the sophisticated absolute phase determination, $\Phi(\omega)$ is immediately unwrapped to get the relative phase difference $\Phi'(\omega) = \Phi(\omega) + 2n\pi$, where the term $2n\pi$ accounts for the $2\pi$ ambiguity in phase-unwrapping process. The relative wavenumber curve $K'_0(\omega)$ is finally estimated with equation (9) as

$$K'_0(\omega) = \Phi'(\omega) / L$$

4. LDSC-based High resolution Damage Imaging

Combined with the typical delay-and-sum imaging algorithm, LDSC is applied to enhance the spatial resolution of damage imaging. Consider a sparse array of $N$ PZTs integrated in the monitored structure. For a PZT pair $P_{ij}(i \neq j; i=1,2,\cdots,\Omega; j=1,2,\cdots,\Omega)$ composed of $P_i$ at $(x_i, y_i)$ and $P_j$ at $(x_j, y_j)$, the scattering propagation path with respect to an arbitrary point $O$ at $(x, y)$ can be geometrically determined (seen in Fig. 1) and the relevant time delay $t_{ij}(x, y)$ is$^{[14]}$

$$t_{ij}(x, y) = \frac{\sqrt{(x-x_i)^2+(y-y_i)^2/c_{p0}} + \sqrt{(x-x_j)^2+(y-y_j)^2/c_{p0}}}{c_{p0}}$$

Figure 1. Illustration of damage imaging

If $s_{ij}(t)$ is the damage scattered signal of $P_{ij}$, then $s_{ij}(t_{ij}(x, y))$ corresponds to the amplitude of the signal scattered from the point $O$. The scattered signals measured by all PZT pairs $P_{ij}(i \neq j, 1 \leq i, j \leq N)$ are time-shifted and summarized to get an averaged energy of point $O$. That is

$$E(x, y) = \left[ \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} s_{ij}(t_{ij}(x, y)) \right]^{1/2}$$

Every point of the tested structure is considered as a potential flaw and the result of equation (14) is taken as the brightness in the final image. The local area in the image with higher intensities will probably correspond to actual defect. To eliminate the dispersion effect on the imaging result, LDSC
is firstly performed for $s_i(t)$ before delay-and-sum imaging. The pixel value at point $O$ is then calculated as

$$E'(x, y) = \left[ \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} s'_i(t_i(x,y)) \right]^2$$  \hspace{1cm} (15)$$

where $s'_i(t)$ is the LDSC-processed scattered signal.

In equation (15), since the formerly elongated and distorted wavepackets have been properly recovered by LDSC in $s'_i(t)$, the spatial resolution of damage imaging can be greatly improved.

5. Experimental Validation

An experimental study is carried out on a 600mm×600mm×2mm quasi-isotropic glass fiber reinforced composite plate with unknown material properties. To monitor the entire plate, eight PZT wafers $P_1 \sim P_8$ (PZT-5, 8mm in diameter and 0.5mm in thickness) are bonded to form a familiar square transducer array, as shown in Fig. 2. Two identical hexagonal hollow screws denoted as $D_1$ and $D_2$, are mounted near to each other on the plate to simulate adjacent damages. The exact positions of PZT wafers and damages in the orthogonal coordinate (seen in Fig. 2) are listed in Table 1. To generate the $A_0$ mode dominant sensor signal, the central frequency of 3-cycle sine burst excitation signal $v_a(t)$ is selected as 50kHz, as shown in Fig. 3(a).

The relative wavenumber curve $K'_0(\omega)$ is firstly measured on the composite plate using the sensor signal $v_{68}(t)$ (Seen in Fig. 3(b)) of $P_{6, 8}$. With $T, t_a, t_1$ and $t_2$ in equation (10) individually decided as 600μs, 5μs, 265.5μs and 412.7μs, a time window is built in Fig. 3(c) to extract the sensing wavepacket of $A_0$ mode direct arrival from $v_{68}(t)$. Fig. 3(d) gives the picked-up sensing wavepacket. The phase difference $\Phi(t)$ between $v_a(t)$ and the sensing wavepacket is computed using equation (11). After phase-unwrapped, the relative phase difference spectrum $\Phi'(\omega)$ can be obtained in Fig. 3(e). Then, the relative wavenumber curve $K'_0(\omega)$ is calculated with equation (12), as Fig. 3(f) shows.

![Figure 2. Monitored glass fiber reinforced composite plate](image)

<table>
<thead>
<tr>
<th>PZT</th>
<th>(x, y)/(mm)</th>
<th>PZT or Defect</th>
<th>(x, y)/(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>(200, 200)</td>
<td>$P_6$</td>
<td>(-200, 0)</td>
</tr>
<tr>
<td>$P_2$</td>
<td>(-200, 200)</td>
<td>$P_7$</td>
<td>(0, -200)</td>
</tr>
<tr>
<td>$P_3$</td>
<td>(-200, -200)</td>
<td>$P_8$</td>
<td>(200, 0)</td>
</tr>
<tr>
<td>$P_4$</td>
<td>(200, -200)</td>
<td>$D_1$</td>
<td>(60, 70)</td>
</tr>
<tr>
<td>$P_5$</td>
<td>(0, 200)</td>
<td>$D_2$</td>
<td>(10, -30)</td>
</tr>
</tbody>
</table>
Figure 3. Procedure of relative wavenumber curve measurement

With the measured relative $K'_r(\omega)$, the linearized wavenumber curve $K'_r(\omega)$ is obtained with equation (5). The interpolation mapping sequence $\Omega'_e(\omega)$ can be then decided using equation (7), as shown in Fig. 4. By subtracting the $A_0$ mode sensor signals of the health plate from those of the damaged plate, the damage scattered signals can be obtained. Fig. 5(a) shows the typical original damage scattered signal $s_{58}(t)$ from $D_1$ and $D_2$ measured by $P_{58}$. Due to the dispersion effect, the two damage scattered wavepackets are elongated and overlapped with each other. After LDSC with equation (6), every wavepacket in $s_{58}(t)$ is recompressed to the incipient excitation waveform, as shown in Fig. 5(b). The dispersion effect on $s_{58}(t)$ is successfully removed by LDSC with measured relative $K'_r(\omega)$. With equation (14) or (15), the damage image can be constructed with all the original or LDSC-processed scattered signals, respectively. Due to the dispersion effect (seen in Fig. 5(a)), the two flaw spots in the image are enlarged and mixed together, as shown in Fig. 6(a) where the symbol “X” denotes the actual damage position. Because the dispersion effect is well compensated (seen in Fig. 6(a)), the two adjacent flaws can be easily identified as the bright focalized spots in the LDSC-based imaging result, as Fig. 6(b) shows. The high spatial resolution is achieved by the proposed LDSC-based imaging method.

Figure 4. Interpolation mapping sequence $\Omega'_e(\omega)$
6. Conclusions

To efficiently compensate the dispersion effect of Lamb waves with theoretical wavenumber relations unobtainable, the methods of relative wavenumber curve measurement and LDSC with measured relative wavenumber curves are introduced. Furthermore, a high spatial-resolution Lamb wave imaging based on LDSC is proposed. The effectiveness of LDSC and LDSC-based imaging methods are verified by the experiment conducted on a glass fiber reinforced composite plate with unknown material parameters.

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