A novel MOALO-based algorithm for structural damage detection

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ABSTRACT

Structural damage detection (SDD) is the first-type inverse problem in structural dynamics, and it is usually converted to a kind of constrained optimization problem in mathematics. Exploring a highly accurate and robust solution to the SDD problem is a challenging task in the field of structural health monitoring (SHM). In this study, a novel SDD method is proposed based on multi-objective ant lion optimizer (MOALO) algorithm, in which multi-objective functions are defined for the typical SDD inverse problem based on the model updating theory. The modal parameters, such as natural frequencies and mode shapes, are adopted into the objective functions whilst considering weighted coefficients corresponding to different modal parameters at the same time. The weighted strategy and sparse regularization are also taken into consideration for accurate SDD results with good robustness to noise. All illustrated results show that the proposed MOALO method is able to accurately locate and quantify damage of structures, which provides a potential SDD tool for real structures onsite.

1. Introduction

Civil infrastructures, such as bridges and buildings, begin to deteriorate once they are built and used[1]. In order to evaluate safety and reliability of structures during their service life, structural health monitoring (SHM) technology has been proposed to provide a way for solving the interested issues. SHM technology has already succeeded in applying to understand the loads, environment actions, and behaviors of a structure subjected to various actions through solving a reverse problem[2]. As a vital part of SHM technology, structural damage detection (SDD) methodologies have been developed to monitor the potential damages occurring in structures[3]. In recent decades, extensive studies have been conducted for SDD methodologies to quantify the damage occurring in civil, mechanical, and aerospace structures, etc.
Over the past decades, SDD technology has enjoyed highly attention by researchers around the world and a great number of SDD methods have been developed. The responses based SDD methods has directly employed dynamic signals to identify structural damage, including natural frequency, mode shape, mode shape curvature, modal strain energy, etc. The existing damages and their locations can be accurately identified and located utilizing this kind of SDD method, which is implemented by defined some relative indexes; however, this kind of SDD method is difficult to quantify damage effectively. Thus model-based SDD methods have been proposed to meet the requirements of accurate and effective quantitative damage. Establishing a rational relationship between changes in measured responses and structural physical properties is vital to the model-based SDD methods. Once the change in structural physical properties, then dynamic characteristics of structures will change as well. In order to meet the requirements of identifying structural damages accurately, a precise finite element model (FEM) of structure should be established. The model-based SDD methods usually transform the SDD problem into an optimization problem in mathematics and it is expected to update the initial FEM until discrepancy of responses between model and
measurement is minimized\textsuperscript{[4]}. This kind of methods can be usually divided into sensitivity-based FEM updating approach\textsuperscript{[5]} and swarm intelligence (SI) algorithm. Both methods always involve ill-conditioned problems and complex optimization problems. However, SI algorithm, a trial-and-error method, can overcome these deficiencies due to its advantages. Over the past 20 years, numerous SI algorithms have been proposed and most of them have been widely utilized to solve the SDD optimization problems in large-scale infrastructures.

SI algorithm, an optimization technique, has been considered as one of the best techniques for finding optimal designs using machines. However, local optima stagnation, a major drawback of optimization algorithms, makes numerous traditional algorithms shrinkage\textsuperscript{[6]}. Therefore, a series of stochastic optimization techniques become the major alternatives for designers. Although these methods have several superiorities as compared to conventional optimization algorithms, applying to practical engineering problems needs to solve various challenges, such as constraints, uncertainties, deceptive global solutions, local solutions, multiple-objectives, etc. To dispose these conundrums, corresponding operators\textsuperscript{[7]} should be adopted by optimization algorithms. Subsequently, multi-objective optimization, a technique for seeking solutions of the issues with more than one objective, has been proposed to fulfil this work\textsuperscript{[8]} . The core of the issues in multi-objective optimization is that the multiple objective problems are difficult to handle for its conflicting. And stochastic multi-objective optimization algorithms aim to determine the set of best trade-offs between the objectives, the so called Pareto optimal set\textsuperscript{[7]}.

Existing studies of multi objective optimization using stochastic optimization techniques can be mainly divided into posteriori versus priori\textsuperscript{[9]-[10]}. For the priori methods, a multi-objective optimization problem is converted to a single-objective one by defining a set of weights to aggregate the objectives. But such methods have several drawbacks: (i) algorithms must run multiple times to ascertain the Pareto optimal set, (ii) the weights need to consult with an expert in the problem domain and (iii) this method is unable to determine some special Pareto optimal fronts\textsuperscript{[11]-[12]}, etc. In contrast, the posterior approaches, benefiting from maintaining multi-objective formulation of a multi-objective problems, have two advantages: (i) the Pareto optimal set can be found in just one run and (ii) this approach can determine any kind of Pareto front. Meanwhile, the posterior approaches require higher computational cost and addressing multiple objectives at the same time. Such methods have been widely applied to practical problems in various fields and the most recognized algorithms for the researchers include: non-dominated sorting genetic algorithm (NSGA) and multi-objective particle swarm optimization (MOPSO). Several algorithms proposing in recent years include multi-objective bee algorithm, multi-objective bat algorithm, multi-objective grey wolf optimizer (MOGWO), etc. According to the no-free lunch (NFL) theorem\textsuperscript{[13]}, any algorithm cannot address all optimization problems. Therefore, Mirjalili et al. proposed a new algorithm, multi-objective ant lion optimizer (MOALO)\textsuperscript{[7]}, to better solve some existing or emerging engineering problems. Meanwhile, the performance of MOALO algorithm in benchmark show that its convergence is reasonable and coverage is extremely high and almost uniform. In short, comparing with the ALO algorithm, the MOALO algorithm can effectively solve the multi-objective problems. Meanwhile, the MOALO algorithm has an excellent coverage that the ALO algorithm lacks.

In order to effectively solve the SDD optimization problems, which are often required to define multiple objective functions, the MOALO is introduced in this study as an attempt. The quantitative function of natural frequencies and mode shapes are employed for defining the objective functions because the data is usually incomplete in practice. Therefore, more dynamic signals used as possible as damage indexes are helpful in addressing this issue. In addition, structural damages are generally sparse and the measured data are easily contaminated by noise in real infrastructures. The sparse regularization, which describes the sparsity of true damages in the physical space and enhances the noise robustness of SDD methods, is essential for SDD methods\textsuperscript{[14]}. A simple superposition of two dynamic parameters in objective function is often irrational. A weighted strategy is also introduced in this study, which provides potential for appropriate weighted coefficients.

2. Theoretical background

2.1 Structural damage model
Comparing with the change in stiffness of a structure, the change in mass can be negligible\(^{[15]}\), therefore, stiffness reduction theory is usually used to quantitatively structural damage. In FEM, the structural stiffness matrix can be expressed in the following forms:

\[
\mathbf{K}(\mathbf{\theta}) = \sum_{i=1}^{N_{ele}} (1 - \theta_i) \mathbf{K}_i
\]

in which \(N_{ele}\) is total number of elements and \(\mathbf{\theta}\) is damage factor vector with the length of \(N_{ele}\). \(\mathbf{\theta}\) ranges from 0 to 0.99 and \(\theta_i\) indicates the damage factor at the \(i\)-th element. Particularly, \(\theta_i = 0\) means the \(i\)-th element is healthy. \(\mathbf{K}\) and \(\mathbf{K}_i\) represent structural global stiffness matrix and the \(i\)-th element stiffness matrix respectively.

### 2.2 Objective function

In this study, natural frequencies and mode shapes, two modal parameters most universally employed as damage indicators, are used for describing the objective functions. The objective functions are defined by the two modal parameters as follows:

\[
O_1 = \arg \min_{\mathbf{\theta}} \{ \omega(\mathbf{\theta}) \} \quad \text{s.t.} \quad 0 \leq \theta_i < 1
\]

\[
O_2 = \arg \min_{\mathbf{\theta}} \{ \varphi(\mathbf{\theta}) \} \quad \text{s.t.} \quad 0 \leq \theta_i < 1
\]

in which \(O_1\) and \(O_2\) are different single objective functions based on natural frequencies and mode shapes respectively; \(\mathbf{\theta}\) represents the damage factor vector according to measured responses of structures. The discrepancy of \(O_1\) and \(O_2\) between measured and calculated dynamic characteristics are different based on different damage factor vectors \(\mathbf{\theta}\), defining in two forms as follows:

\[
\omega(\mathbf{\theta}) = \sum_{i=1}^{N_m} \frac{\omega_i^t - \omega_i^a}{\omega_i^t}
\]

\[
\varphi(\mathbf{\theta}) = \sum_{i=1}^{N_m} (1 - MAC(\varphi_i^t, \varphi_i^a))
\]

in which \(\omega\) and \(\varphi\) represent frequency and mode shape respectively. The subscript \(i\) means the \(i\)-th vibration mode. \(N_m\) indicates the total modal number considered. Meanwhile, \(\omega\) and \(\varphi\) are both dynamic properties but \(\omega\) is a scalar while \(\varphi\) is a column vector. The superscripts \(t\) and \(a\) mean the test dynamic characteristic and the analytical ones by FEM, respectively. The specific formula for the function \(MAC(\varphi_i^t, \varphi_i^a)\) is as follows:

\[
MAC(\varphi_i^t, \varphi_i^a) = \frac{(\varphi_i^{Tt} \cdot \varphi_i^{Ta})^2}{(\varphi_i^{Ta} \cdot \varphi_i^{Ta})(\varphi_i^{Tt} \cdot \varphi_i^{Tt})}
\]

where superscript \(T\) denotes the matrix transpose operator.

In fact, measured data are usually contaminated by noise in practical structures. In order to assess the noise robustness of the proposed SDD method, noise is added into frequencies and mode shapes. According to the suggestion in reference\(^{[16]}\), the noise level is set to be 1%.

Meanwhile, the equation of noise contaminated in measured modal parameters are formulated as follows:

\[
\mathbf{D}_n = \mathbf{D} + \lambda\mathbf{R}
\]

where \(\mathbf{D}_n\) and \(\mathbf{D}\) are measured data with and without noise, respectively. \(\lambda\) is the noise level ranging from 0% to 1% and \(\mathbf{R}\), a random vector, obeys the distribution \(N(0,1)\). \(\Omega\) represents the value of frequency for frequency data or it indicates mode shapes considering \(N_n\) nodes, which is calculated by the following equation:

\[
\Omega = \sqrt{\frac{1}{N_m N_n} \sum_{i=1}^{N_m} \sum_{n=1}^{N_n} \varphi_{in}^2}
\]
2.3 Multi-objective ant lion optimizer (MOALO)

MOALO algorithm, a posteriori approach, originates in the ant lion optimizer (ALO) algorithm and inherits its convergence. However, finding the Pareto optimal solution set should be equipped with preferable convergence (accuracy) and coverage (distribution) at the same time. Therefore, MOALO algorithm should improve the distribution of the non-dominated solutions to cover the entire true Pareto optimal front. Mirjalili et al. has introduced the leader selection and archive maintenance into the original ALO algorithm; meanwhile, two mechanisms similarly to those in MOPSO have been considered to improve the distribution of the solutions in the archive[7]. The more details about the ALO algorithm can be found in reference[17].

2.3.1 Archive mechanism

To store Pareto optimal solution, an archive mechanism is employed on the basis of ALO algorithm. In the first place, the antlions are chosen from the solutions with the least populated neighbourhood. The probability of selecting a solution in the archive is defined as follows:

\[ P_i = \frac{c}{N_i} \]

where \( c \) is a constant greater than 1 and \( N_i \) is the number of solutions in the vicinity of the \( i \)-th solution. In order to store new solutions, however, the solutions with most populated neighbourhood will be removed from the archive when the solutions fill up the archive:

\[ P_j = \frac{N_j}{c} \]

where \( N_j \) is the number of solutions in the vicinity of the \( j \)-th solution.

For the ALO algorithm, the equation (11) is used to simulate catching the ant and reconstructing the pit:

\[ AL_j = \text{Ant}_i' \quad \text{if} \quad f(\text{Ant}_i') < f(AL_j) \]

where \( AL_j \) represents the position of selected \( j \)-th antlion at \( t \)-th iteration, and \( \text{Ant}_i' \) shows the position of \( i \)-th ant at \( t \)-th iteration. To adapt the nature of multi-objective issues, equation (11) should be modified.

The solution of this mechanism is a cycle elimination process. Firstly, the better solutions are put into the archive by equation (9). When the archive is full, some worse solutions are deleted by a Roulette wheel and equation (10) from the archive in order to accommodate new solutions.

2.3.2 Antlion selection mechanism

Antlion selection mechanism is used for selecting random antlions and elite in the following equation:

\[ \text{Ant}_i' = \frac{R_A + R_E}{2} \]

Then, a non-dominated solution is chosen from the archive through the Roulette wheel and equation (11).

3. Methodologies

3.1 Weighted strategy

In order to study the multi-objective optimization of the proposed method, two simple objective functions are aggregated reasonably by weighted strategy in the multi-objective case:

\[ O_1 = \Delta_1 O_1 = \arg \min_{\theta} \{ \Delta_1 \omega(\theta) \} \quad \text{s.t.} \ 0 \leq \theta < 1 \]

\[ O_2 = \Delta_2 O_2 = \arg \min_{\theta} \{ \Delta_2 \varphi(\theta) \} \quad \text{s.t.} \ 0 \leq \theta < 1 \]

where both \( \Delta_1 \) and \( \Delta_2 \) represent the weighted coefficients.
In this study, two weighted coefficients are employed to balance the information from \( O_1 \) and \( O_2 \) in the multi-objective case [18]. In fact, natural frequency and mode shape are two different types of modal parameters; meanwhile, the numerical values of \( O_1 \) and \( O_2 \) are completely different because they are composed of frequencies and mode shapes, respectively. Therefore, weighted coefficients \( \Delta_1 \) and \( \Delta_2 \) are used to narrow the gap of \( O_1 \) and \( O_2 \) in numerical values. The principle is based on the difference of sensitivity between natural frequency and mode shape in SDD. The weighted average of two function is used to achieve the goal of obtaining the best solution more accurately. Firstly, \( O_1 \) and \( O_2 \) for different damage extents are calculated in the case of single damage, respectively. Each running is calculated 10 times in each case and the average result is taken as the final result. Secondly, equation (14) is used to calculate the multiplier \( n_i \) between \( O_1 \) and \( O_2 \), then the average value of \( n_i \), i.e. average multiplier \( \bar{n} \) is calculated. Finally, the average value of \( \bar{n} \) is calculated below:

\[
\bar{n} = \frac{\sum_{i,j=1}^{N_{\text{ele}}} O_i(w(\theta_j))}{\sum_{i,j=1}^{N_{\text{ele}}} O_i(\phi(\theta_j))}, \quad N_{\text{ele}} = \frac{\sum_{i,j=1}^{N_{\text{ele}}} N_i}{N_{\text{ele}}^2}
\]

where \( \theta_i \) is set to be 0.1, 0.2, ..., 0.8, 0.9 and 0.99 respectively. The functions of \( w(\theta_j) \) and \( \phi(\theta_j) \) represent the value of \( w(a) \) and \( \phi(a) \) at the \( j \)-th element when the factor is \( \theta_j \). And \( \bar{n} \) is a sensitivity multiplier of frequency and mode shape for SDD. The values of \( \Delta_1 \) and \( \Delta_2 \) are calculated as follows:

\[
\Delta_1 = \frac{\bar{n}}{\bar{n} + 1}, \quad \Delta_2 = 1 - \Delta_1
\]

3.2 Trace Lasso

To remove the negative effect from noise, the trace Lasso [19] has been employed to eliminate misidentified damaged elements in the solution. In addition, the sparsity of the trace Lasso, which is equivalent to the Tikhonov and the \( l_1 \)-norm under two particular cases, contributes to the sparse representation of true damages in the physical space. The detail of the trace Lasso can be found in the reference [19]. Therefore, to enhance the accuracy of SDD, the trace Lasso is introduced into equations (2), (3) and (13) as follows:

\[
\begin{align*}
O_1^* &= \arg \min_{\theta} \left\{ \omega(\theta) + \mu_1 \| M \| \right\} \quad s.t. 0 \leq \theta_i < 1 \\
O_2^* &= \arg \min_{\theta} \left\{ \phi(\theta) + \mu_2 \| M \| \right\} \quad s.t. 0 \leq \theta_i < 1 \\
O_3^* &= \left\{ \begin{array}{ll}
\arg \min_{\theta} \left\{ \Delta_1 \omega(\theta) + \mu_1 \| M \| \right\} & s.t. 0 \leq \theta_i < 1 \\
\arg \min_{\theta} \left\{ \Delta_2 \omega(\theta) + \mu_2 \| M \| \right\} & s.t. 0 \leq \theta_i < 1
\end{array} \right.
\end{align*}
\]

where \( \mu_i \) is a regularization parameter. Matrix \( M = XD_{\text{diag}}(\theta) \) and \( X \) is a design matrix defined as follows:

\[
X = R_{N_{\text{ele}}}, E_{N_{\text{ele}}}
\]

where \( R_{N_{\text{ele}}} \), a random matrix, obeys the distribution \( N(0,1) \) and \( E_{N_{\text{ele}}} \) is a unit matrix. The flow chart of the proposed method is shown in Figure 1.

4. Numerical simulations

4.1 Two-storey rigid frame structure

As shown in Figure 2, a two-storey rigid frame structure with height and width of 1.41 m at each storey is adopted to assess the performance of the proposed method. The structure is divided into 18 elements with
equal length and each element has two nodes and six degrees of freedom (DOFs). The structure is simulated with the parameters as listed in Table 1 and the parameters of MOALO algorithm are listed in Table 2. It needs to explain that the mode shape only in horizontal DOFs at each element node of the column is considered, whilst those only in vertical DOFs at node of the beam is considered. In this study, the first five natural frequencies and mode shapes of the structure are employed for SDD. Each running is calculated 10 times in each case and the average value of the total 10 results is taken as the final result. The damage factor ranges \( \theta_i \in [0,0.99] \) and the threshold value is set to be \( \varepsilon = \max(\theta_i) \times 15\% \) [16]. The elements are defined as false positive (FP) when \( \theta_i \geq \varepsilon \), which represents that the healthy state of structure is deemed as the damage state[20]. On the contrary, the elements are considered as false negative (FN) when \( \theta_i < \varepsilon \), which indicates that the damaged elements are deemed as the health elements.

![Flow chart of proposed method](image1)

![FEM of two-storey rigid frame structure](image2)

**Figure 1.** Flow chart of proposed method

**Figure 2.** FEM of two-storey rigid frame structure

**Table 1.** Simulation parameters of two-storey rigid frame

<table>
<thead>
<tr>
<th>Type</th>
<th>Elastic modulus</th>
<th>Moment of inertia</th>
<th>Cross-sectional area</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>(2 \times 10^{11} \text{ N/m}^2)</td>
<td>(2.36 \times 10^{-5} \text{ m}^4)</td>
<td>(3.2 \times 10^{3} \text{ m}^2)</td>
<td>7593 \text{ kg/m}^3)</td>
</tr>
<tr>
<td>Column</td>
<td>(2 \times 10^{11} \text{ N/m}^2)</td>
<td>(1.26 \times 10^{-5} \text{ m}^4)</td>
<td>(2.98 \times 10^{3} \text{ m}^2)</td>
<td>8590 \text{ kg/m}^3)</td>
</tr>
</tbody>
</table>
Table 2. Parameters of MOALO algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable dimension</th>
<th>Population size (ant and antlion)</th>
<th>Max iteration</th>
<th>Archive size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>18</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

4.2 Case study

Four damage cases are set in Table 3, including asymmetric damage and symmetric damage cases in 18-element two-storey rigid frame. It should be noted that the weighted coefficients, i.e., $\Delta_1 = 0.015$ and $\Delta_2 = 0.985$ are adopted for both the MOALO algorithm and the ALO algorithm, and the effect of the trac Lasso is best when $\mu_1 = 0.001$ and $\mu_2 = 0.0001$ just for the MOALO algorithm. In addition, the noise level of 1% is added to study the robustness performance of the proposed method, meanwhile, in order to evaluate the proposed method, three different scenarios are considered in two different parts of single-objective optimization and one multi-objective optimization as follows:

Single-objective optimization with the MOALO algorithm:

Scenario 1: Optimizing $O^*_1$ using natural frequencies only
Scenario 2: Optimizing $O^*_2$ using mode shapes only

Multi-objective optimization:

Scenario 3: Optimizing $O^*_3$ with the MOALO algorithm
Scenario 4: Optimizing $O^{**}_3$ with the ALO algorithm

It needs to be explained that $O^{**}_3$ represents:

$$O^{**}_3 = \arg \min_\theta \{ \Delta_1 \omega(\theta) + \Delta_2 \omega(\theta) + \mu_3 \| \mathbf{M} \| \} \quad s.t. \theta_i < 1$$

(21)

where $\mu_3 = 0.0001$ is used in this study.

Table 3. Damage cases

<table>
<thead>
<tr>
<th>Cases</th>
<th>Type</th>
<th>Damage extent @ element</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>10%@17</td>
</tr>
<tr>
<td>2</td>
<td>Asymmetric</td>
<td>10%@8, 10%@17</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>10%@8, 20%@11, 15%@17</td>
</tr>
<tr>
<td>4</td>
<td>Symmetric</td>
<td>10%@5, 10%@11</td>
</tr>
</tbody>
</table>

4.2.1 Single-objective optimization

The SDD results in scenarios 1 and 2 and noise-free are listed in Tables 4 and 5 respectively. From them, it can be seen that when natural frequencies are used as damage index in scenario 1, the FP numbers are more than those when the mode shapes used as damage index in scenario 2. Meanwhile, it can be also seen that the SDD results are closer to true damage when mode shapes are used as damage index except in case 1. In addition, in the case of single damage, the SDD results in scenario 2 has one FP, but there is no FP in scenario 1. Therefore, the next section will combine the advantages of natural frequency and mode shape to carry on multi-objective optimization.

Table 4. SDD results for scenario 1

<table>
<thead>
<tr>
<th>Cases</th>
<th>Damage extent @ element</th>
<th>FP (noise-free)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.12%@17</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>8.28%@8, 7.94%@17</td>
<td>2.21%@11</td>
</tr>
<tr>
<td>3</td>
<td>8.74%@8, 15.43%@11, 12.20%@17</td>
<td>3.49%@5, 3.33%@9</td>
</tr>
<tr>
<td>4</td>
<td>4.73%@5, 8.19%@11</td>
<td>2.32%@8, 2.91%@16</td>
</tr>
</tbody>
</table>

Table 5. SDD results for scenario 2

<table>
<thead>
<tr>
<th>Cases</th>
<th>Damage extent @ element</th>
<th>FP (noise-free)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.12%@17</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>8.28%@8, 7.94%@17</td>
<td>2.21%@11</td>
</tr>
<tr>
<td>3</td>
<td>8.74%@8, 15.43%@11, 12.20%@17</td>
<td>3.49%@5, 3.33%@9</td>
</tr>
<tr>
<td>4</td>
<td>4.73%@5, 8.19%@11</td>
<td>2.32%@8, 2.91%@16</td>
</tr>
</tbody>
</table>
Table 5. SDD results for scenario 2

<table>
<thead>
<tr>
<th>Cases</th>
<th>Damage extent @ element</th>
<th>FP (noise-free)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.26%@17</td>
<td>2.28%@8</td>
</tr>
<tr>
<td>2</td>
<td>9.94%@8, 9.53%@17</td>
<td>None</td>
</tr>
<tr>
<td>3</td>
<td>11.5%@8, 20.63%@11, 13.81%@17</td>
<td>None</td>
</tr>
<tr>
<td>4</td>
<td>9.79%@5, 9.89%@11</td>
<td>1.89%@6, 1.88%@10</td>
</tr>
</tbody>
</table>

4.2.2 Multi-objective optimization

The SDD results for multi-objective optimization in scenario 3 and scenario 4 are listed in Table 6 and 7 respectively, and the SDD results of two scenarios under 1% noise level are shown in Figure 3. By weighted strategy, the weight of the two objective functions is more reasonable in the multi-objective optimization process, and then the optimal solution is obtained. From Table 6, it can be seen that the SDD results are all really close to the truth damage and the FP is completely disappeared. And from Table 7, it can be found that although the SDD results are also quite well, the SDD results in scenario 3 are better overall than those in scenario 4. In other words, the proposed MOALO algorithm is better than the ALO in these cases.

Table 6. SDD results for scenario 3

<table>
<thead>
<tr>
<th>Cases</th>
<th>Damage extent @ location</th>
<th>FP (noise-free)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.23%@17</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>9.51%@8, 9.61%@17</td>
<td>None</td>
</tr>
<tr>
<td>3</td>
<td>9.73%@8, 19.44%@11, 14.88%@17</td>
<td>None</td>
</tr>
<tr>
<td>4</td>
<td>9.28%@5, 9.25%@11</td>
<td>None</td>
</tr>
</tbody>
</table>

Table 7. SDD results for scenario 4

<table>
<thead>
<tr>
<th>Cases</th>
<th>Damage extent @ location</th>
<th>FP (noise-free)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.82%@17</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>9.50%@8, 7.93%@17</td>
<td>None</td>
</tr>
<tr>
<td>3</td>
<td>9.28%@8, 19.90%@11, 13.89%@17</td>
<td>None</td>
</tr>
<tr>
<td>4</td>
<td>8.91%@5, 8.95%@11</td>
<td>None</td>
</tr>
</tbody>
</table>

From the SDD results in Figure 3, it can be seen that although adding 1% noise in four cases will cause several FP in SDD results, those FP values are much lower than the true damages. The main reason is that the mode shapes are not sensitive to the minimal changes in structures, but the mode shape component in objective function is sensitive to the noise, the extreme point of the objective function easily shifts if the noise pollution is considered[15]. It can be also found that the SDD results are all pretty close to true damages when both the MOALO algorithm and the ALO algorithm are employed; however, the SDD results obtained from the MOALO are significantly better than those from the ALO. In conclusion, the effect by adopting multi-objective for SDD is better than those by single objective, the performance of the proposed MOALO algorithm is superior to that of the ALO algorithm in this study. It can be clearly seen that the proposed method can accurately locate the damage and quantify the damage extent. Meanwhile, the illustrated simulations show that the proposed method is efficient and robust.

5. Conclusions

In this study, a novel multi-objective ant lion optimizer (MOALO) algorithm is proposed for structural damage detection (SDD). The MOALO algorithm is equipped with weighted strategy and the trace Lasso
for enhancing the SDD potential of MOALO algorithm and the robustness to noise. In order to verify the effectiveness and feasibility of the proposed method, a two-storey rigid frame structure with 18 finite elements is taken as an example for numerical simulations. Some conclusions can be made as follows:

1. The introduction of the MOALO algorithm, the weighted strategy and the trace Lasso into the SDD problem is feasible and effective, which provides a potential SDD tool for real structures onsite.

2. The performance of the proposed MOALO algorithm is better than that of the ALO algorithm in this study.

3. The SDD results show that the proposed method can accurately locate the structural damages and quantify damage extents, even in the face of multiple lower damages and measurement noise environments. It shows that the trace Lasso can provide noise robustness and excellent SDD accuracy for the proposed method.

4. The identification accuracy of the proposed method will be reduced if the noise is considered. There are more FP values in the SDD results if the damage occurs in symmetric elements.

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