The Interaction of the Fundamental Symmetric and Antisymmetric Lamb Wave Modes with Material Discontinuity - A 3D Finite Element Analysis

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ABSTRACT

This study investigates the propagation behavior of the fundamental Lamb-wave modes upon interaction with welded joints of dissimilar materials. A plate with an intact AA6061-T6/AZ31B dissimilar joint was employed, and the interaction of the propagating wave with the material interface was scrutinized using “COMSOL Multiphysics” finite element software. The effect of the angle of incidence on the reflections and transmissions of the waves from the joint was analyzed. The study was conducted as the wave propagated from AA6061-T6 to AZ31B (forward direction) and when the propagation direction was reversed (backward direction). Lamb waves were excited at a central frequency of 200 kHz at different incidence angles varying from 0 to 80 degrees. Reflections from the material interface were extracted by comparing waves propagating in dissimilar-material plates to waves propagating in single-material plates (without the material discontinuity). The reflection and transmission coefficients of the studied modes (symmetric (S₀₀), antisymmetric (A₀₀), and shear-horizontal (SH₀₀)) were assessed through a comparison of the wave amplitude in single-material propagation (along the incidence direction) and in dissimilar-material propagation (along the expected reflection and transmission directions). The SH₀₀ evolved upon the interaction of the obliquely-incident S₀ mode with the interface. The reflection of the S₀ mode varied, with the incidence angle, between 0 and 20% in both the forward and backward propagation directions. The reflection of the A₀ mode increased from 6 to 16% when the wave propagated from AA6061-T6 to AZ31B, while it increased from about 3.5 to 12% in the reversed direction. For the used excitation frequency, changing the incidence angle did not have any effect on the transmission coefficients of the S₀ and A₀ modes. The total reflection was not observed at any of the examined propagation directions.

1. Introduction

Lamb waves (LWs) have proven their potential in the non-destructive evaluation (NDE) of composite and metallic structures including traditional and friction stir welded (FSW) joints[1-3]. The interaction behavior of LWs with material discontinuity along a dissimilar weld line is not yet well understood. Clear understanding of the wave propagation behavior would allow the integration of such technique in structural health monitoring (SHM) systems. As an example, the transmission of a guided wave through features including welds, stiffeners, or bends may indicate its sensitivity to a defect beyond those features[4].

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Analytical solutions providing the reflection and transmission coefficients of ultrasonic bulk waves, at normal and oblique incidence and between different types of materials, can be readily found in textbooks[5]. The problem is more complex when dealing with guided waves due to the need of numerical methods for solving the analytically determined equations[4]. In the early 80’s, Gregory and Gladwell[6] have developed an analytical solution to determine the reflection of a normally incident symmetric Rayleigh-Lamb wave from the fixed or free edge of a semi-infinite plate. The energy distribution between the various reflected modes was then numerically determined, for a fixed Poisson ratio, and was analyzed for a range of wavenumbers. The authors had expressed the displacement and stress fields of a semi-infinite plate as a superposition of the eigen modes of an infinite plate including the suitable boundary conditions. After that, a biorthogonality relationship was applied between the eigen modes to determine the modal coefficients. In 1990, Scandrett and Vasudevan[7] have addressed an in-depth theoretical study on the propagation behavior of normally incident Rayleigh-Lamb waves in perfectly bonded dissimilar materials. Using a similar approach as in[6], they have presented and analyzed the energy distributions of the reflected and transmitted modes over a range of excitation frequencies. Symmetric and antisymmetric incident fields were studied for different material combinations.

Unlike normal incidence problems, Lamb waves and shear-horizontal (SH) waves cannot be analyzed separately in oblique incidence problems due to the existence of mode conversions between them[8]. Gunawan and Hirose[8] analyzed the edge-reflection problem of obliquely-incident guided waves in a plate using the mode decomposition semi-analytical method. Reflection coefficients of different modes were plotted, against non-dimensionalized frequency, for various cases of symmetric and antisymmetric incident Lamb modes. Experimental validation was performed on a steel plate with different incident angles showing very good agreement with the numerical results. Wilcox et al.[4] have used a semi-analytical finite element (SAFE) method to model the scattering of obliquely-incident guided waves from an infinitely-long feature located in the waveguide. Transmission and reflection coefficients were calculated and plotted for different incidence angles and different frequencies. The authors compared their simulation solutions to experimental data, for an adhesively-bonded stiffener, showing a good agreement. Other theoretical and experimental work can be found, in literature, studying the behavior of Lamb waves when crossing through material interfaces[9, 10]. However, no work was found to study the effect of the Lamb wave’s angle of incidence on its interaction with dissimilar-material interfaces using full 3D finite element (FE) models.

Experimental and FE studies were initiated by the authors of this article towards a better understanding of LWs propagation through similar and dissimilar welds[1, 2, 11-13]. It was shown that the wave behavior is highly dependent on the elastic properties of the mediums it is propagating in, as well as the quality of the material interface at the location of the joint.

This work presents a numerical investigation of the propagation behavior of the fundamental guided-wave modes (A\(_0\), S\(_0\), and SH\(_0\)) when crossing from one medium to another, through a perfect dissimilar-material interface, and at different angles of incidence.

2. Finite Element Modeling

“COMSOL Multiphysics” FE software was used to create two adjoining metallic plates of 3 mm thickness and of the same size along the common interface (Figure 1(a)). The two plates joined together represent a dissimilar-material welded plate assuming a perfect joint. The material properties assigned to each of the half-plates were those of AA6061-T6 and AZ31B alloys[14] (AA6061-T6: \(\rho = 2700 \text{ kg/m}^3, E = 69 \text{ GPa}, \text{ and } v = 0.33\); AZ31B: \(\rho = 1770 \text{ kg/m}^3, E = 45 \text{ GPa}, \text{ and } v = 0.35\)). Homogeneous isotropic linearly-elastic materials were assumed.

Circular Lead Zirconate Titanate (PZT-5H) piezoelectric transducers, 10 mm in diameter and 1 mm in thickness, were modeled based on a solid-mechanics/electrostatics multi-physics solver and used to excite the Lamb waves. Five-cycle-Hanning-windowed signals, of 200 kHz central frequency and
voltage of 240 volts peak-to-peak, were fed into the poles of the PZT wafer. The actuator was placed at 115 mm from the joint, and 9 incidence directions were used for excitation, namely 0, 10, 20, 30, 40, 50, 60, 70, and 80 degrees with the normal to the material interface (as shown in Figure 1(a)). Forward and backward directions were simulated (AA6061-T6 to AZ31B and reversed) for each incidence angle. Similar analyses were completed for single-material plates (AA6061-T6 plate and AZ31B plate) for comparison. The width and length of the two half-plates were changed, as convenient, based on the position of the transducer and the sensing points upon changing the angle of incidence and propagation direction. The mesh used was a free tetrahedral mesh with a maximum element size of 2 mm. The outer side-edges of the plate were assigned low reflecting boundaries to minimize the undesired boundary reflections.

Both symmetric and antisymmetric modes will be excited at the selected central frequency of 200 kHz. Based on theoretical dispersion curves of the used materials (provided by “Wavescope”\(^{[15]}\)), the 200 kHz frequency will result in a dominant \( S_0 \) mode in the incident waves. Since the finite element model is assuming a perfect joint, no scattering of the wave is expected but ideal transmissions and reflections. Upon interaction with the material interface, the energy of the incident modes is distributed among the reflected and transmitted (or refracted) wave modes. Since the only antisymmetric mode existing at the used excitation frequency is the \( A_0 \) mode, this mode will not undergo any mode conversion. On the other hand, a portion of the incident symmetric mode (\( S_0 \)) may undergo mode conversion into the symmetric shear-horizontal mode (\( SH_0 \)) when obliquely interacting with the joint \(^{[16]}\). The relation between the angles of the incident modes and their corresponding reflected/transmitted modes is defined by Snell’s law\(^{[5]}\) (traction free boundary condition at the material interface):

\[
k_1 \sin(\theta_1) = k_2 \sin(\theta_2).
\]  

(1)

where:

- \( k_1 \): is the wavenumber of the incident mode
- \( k_2 \): is the wavenumber of the reflected/transmitted mode
- \( \theta_1 \): is the angle between the incident mode’s propagation direction and the normal to the material interface
- \( \theta_2 \): is the angle between the reflected/transmitted mode’s propagation direction and the normal to the material interface

It was necessary to take measurements on the theoretical propagation direction of each of the expected existing modes. The angles defining those directions are denoted on Figure 1(a) as \( \hat{t} \), \( \hat{r}_{\text{mode}} \), and \( \hat{t}_{\text{mode}} \) for the incidence, reflection, and transmission angles, respectively. Those angles were derived based on Snell’s law as follows:

\[
k_1 \sin(\theta_1) = k_2 \sin(\theta_2).
\]

\[
\Rightarrow \omega \frac{C_1}{c_t} \sin(\theta_1) = \omega \frac{C_2}{c_t} \sin(\theta_2).
\]

\[
\Rightarrow \sin(\theta_2) = \frac{C_2}{C_1} \sin(\theta_1).
\]

\[
\Rightarrow \theta_2 = \arcsin\left[\frac{C_2}{C_1} \sin(\theta_1)\right].
\]  

(2)

where:

- \( \omega \): is the angular frequency
- \( C_1 \): is the phase velocity of the incident mode
- \( C_2 \): is the phase velocity of the reflected/transmitted mode

Based on equation (2), because the phase velocity of a certain mode is constant when propagating in the same material, then its reflecting angle is the same as its incidence angle. This implies that \( \hat{t} = \hat{r}_{S_0} = \hat{r}_{A_0} \) as designated on Figure 1(a). The phase velocities of the Lamb-wave modes were determined using
“Wavescope”\cite{15}, while that of the SH\textsubscript{0} mode was found to be equal to the bulk-wave shear velocity based on its theoretical formulation from literature \cite{16}.

Fifty sensors were placed along each of the propagation directions and an additional sensor was placed right on the material interface (central point). This gave a total of three hundred and one measurement points that are allocated on the surface of the plate based on the incidence angle and propagation direction across materials (using equation (2)). All the sensors were equally spaced, with 2 mm spacing (as seen in Figure 1(a)). They were distributed along a distance of 100 mm before and 100 mm after the weld and along the expected propagation directions. This will be indicated, later in this paper, as -100 mm to 100 mm in the time-spatial plots of the waves.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{(a) A schematic of the FE model (all dimensions are in mm) (b) An illustrating example of S\textsubscript{0} and SH\textsubscript{0} displacement fields’ calculations}
\end{figure}

3. Results and Discussions

3.1 Mode Identification

Lamb waves are elastic waves guided between the free top and bottom surfaces of thin plates\cite{17}. They are distinguished by the longitudinal and shear-vertical vibrations of particles. A longitudinal wave oscillates the plate particles back and forth in the direction of wave propagation, while the shear-vertical wave oscillates the particles vertically (normal to the plate surface) and perpendicular to the direction of wave propagation. The fundamental symmetric (S\textsubscript{0}) and antisymmetric (A\textsubscript{0}) modes are, respectively, longitudinal and shear-vertical Lamb-wave modes. The fundamental shear-horizontal mode (SH\textsubscript{0}) is another guided-wave mode that vibrates the particles in the horizontal plane (parallel to the plate’s surface) and perpendicular to the wave propagation direction (check Figure 1(b)). To have a better understanding of the concepts of guided wave modes and their vibration directions, reader may refer to comprehensive textbooks as\cite{5} and\cite{17}.

In-plane displacements in the $x$ and $y$ directions ($u$ and $v$ respectively) as well as the out-of-plane displacements in the $z$ direction ($w$) were recorded from all sensing points. All measurements were taken at a sampling rate of 20 MHz giving a constant time step of $5 \times 10^{-8}$s. Based on the previous discussion, $w$ displacements correspond to the A\textsubscript{0} mode displacements. Referring to the schematic shown in Figure 1(b), the S\textsubscript{0} and SH\textsubscript{0} displacements can be calculated according to equations (3) and (4) respectively:

$$U_{S0} = u \cos(\phi_{S0}) + v \sin(\phi_{S0}).$$

$$U_{SH0} = v \cos(\phi_{SH0}) - u \sin(\phi_{SH0}).$$
where:

\( U_m \): is the displacement field of the mode “m”

\( \phi_m \): based on the wave propagation direction and the quadrant in which the sensing points are located, \( \phi \) of a certain mode varies according to the following geometrical cases:

1st or 3rd quadrant, wave propagating from left to right: \( \phi_m = \theta_m \)

1st or 3rd quadrant, wave propagating from right to left: \( \phi_m = 180^\circ + \theta_m \)

2nd or 4th quadrant, wave propagating from left to right: \( \phi_m = -\theta_m \)

2nd or 4th quadrant, wave propagating from right to left: \( \phi_m = 180^\circ - \theta_m \)

\( \theta_m \): is the acute angle between the propagation direction of the mode “m” and the normal to the material interface.

In addition to their particle-oscillation directions, the existing modes (S\(_0\), A\(_0\), and SH\(_0\)) were also identified in the measured wave fields based on their group velocities \((C_g)\). The theoretical group velocities of the S\(_0\) and A\(_0\) modes, were determined using “Wavescope”\([15]\), while the group velocity of the SH\(_0\) mode was calculated based on its theoretical formulations\([16]\). Table 1 shows the theoretical group velocities of the modes of interest at the excitation frequency of 200 kHz.

Signals obtained at all the sensors along a certain measurement direction can be plotted in the time-spatial domain. This can be done by accumulating all the vertical time plots of the signals side-by-side along the spatial axis (horizontal axis). Figures 2(a), 2(c), and 2(e) show the wave fields obtained for the wave propagating from AA6061-T6 to AZ31B, at \( \hat{i} = 20^\circ \), for the S\(_0\), A\(_0\), and SH\(_0\) modes respectively. Different modes were identified based on their measurement directions and velocities as explained earlier. Figures 2(b), 2(d), and 2(f) show the same wave fields, measured in pure aluminum (AA6061-T6 to AA6061-T6). The SH\(_0\) transmission (in Figure 2(f)) is shown along the same transmission direction as that of Figure 2(e) for comparison. It can be seen that the S\(_0\) and A\(_0\) modes were dominant in the wave before it interacted with the material interface. The average travelling speeds of the two modes were, respectively, 5321 m/s and 3074 m/s (Figure 2(a) and Figure 2(c)). The SH\(_0\) transmission did not appear clearly in Figure 2(e) due to its low amplitude when compared to the other wave fields propagating in the plate. This will be discussed further in section 3.3.

Table 1. Theoretical group velocities of the existing guided-wave modes

<table>
<thead>
<tr>
<th>Wave mode</th>
<th>Frequency (kHz)</th>
<th>Theoretical ( C_g ) in AA6061-T6 (m/s)</th>
<th>Theoretical ( C_g ) in AZ31B (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(_0)</td>
<td>200</td>
<td>2983.01</td>
<td>2970.29</td>
</tr>
<tr>
<td>S(_0)</td>
<td>200</td>
<td>5267.45</td>
<td>5278.67</td>
</tr>
<tr>
<td>SH(_0)</td>
<td>200</td>
<td>3099.6</td>
<td>3068.6</td>
</tr>
</tbody>
</table>

(a) (b)
Figure 2. Time spatial plots of the wave fields measured along the incident and transmission directions of different modes for a wave propagating at $i = 20^\circ$.

3.2 Reflection Separation

Figure 3(a) shows the wave field measured along the reflection direction of the S$_0$ mode, for the same simulation of Figure 2. Waves received at those sensors are the superposition of the direct transmissions from the actuator, boundary reflections, and reflections from the material interface. This gives the wave field its complex shape, and thus an additional technique is needed to separate the reflections caused due to the presence of the interface.

To separate interface reflections, measurements from the same actuator-sensor configurations were recorded in pure material for the same plate size. The obtained wave fields measured at the reflection directions (e.g. Figure 3(b)) were then subtracted from the dissimilar-material reflection wave fields (e.g. Figure 3(a)). Then, the obtained fields are those resulting from the wave interaction with the dissimilar-material interface (e.g. Figure 3(c)). To minimize possible numerical variations, the measurements were taken at the same exact time steps. To isolate the reflections in AA6061-T6 to AZ31B measurements, the AA6061-T6 single-material measurements should be subtracted, while to isolate reflections in the inverse propagation direction, AZ31B single-material measurements are to be used. Equation (5) summarizes the followed reflection separation method for an incident wave propagating from material X to material Y:

$$U_{R(X/Y)} = U_{RD(X/Y)} - U_{RD(X/X)}.$$  \hspace{1cm} (5)

where:

$U_{R(X/Y)}$ : is the wave field, reflected from the material interface, from an incident wave propagating from material X to material Y at an angle of incidence $i$. 


$U_{RD(X/Y)}$: is the wave field, measured along the reflection direction, from an incident wave propagating from material $X$ to material $Y$ at an angle of incidence $i$

$U_{RD(X/X)}$: is the wave field, measured along the reflection direction, from an incident wave propagating from material $X$ to material $X$ at an angle of incidence $i$

In order to illustrate this, Figure 3 presents the measurements of the $S_0$ mode along its reflection direction for an incident wave propagating from AA6061-T6 to AZ31B at $i = 20^\circ$ (Figure 3(a)), and also in the AA6061-T6/AA6061-T6 single-material propagation (Figure 3(b)). After subtracting these two wave fields, the wave field in Figure 3(c) was obtained, where a clear reflection field of the $S_0$ mode is present. The reflection has an average group velocity of 5333 m/s which is in a good agreement with the AA6061-T6 theoretical value presented in Table 1 (5267.45 m/s).

The separated reflection fields of the other modes ($A_0$ and $SH_0$) of the same wave are presented in Figure 4. As mentioned earlier, $SH_0$ was not significant before the wave interaction with the material interface, thus, its noticeable appearance in the reflection field is due to $S_0$-$SH_0$ conversion upon $S_0$ interaction with the joint.16 This is also clear based on the amplitude of the $SH_0$ mode in the reflected field (Figure 4(b)), which is higher than its amplitude in the incident field (Figures 2(e) and 2(f)). The average group velocities of the obtained $A_0$ and $SH_0$ reflections were determined to be 3062 m/s and 3094 m/s respectively, and they are both close to the theoretical values shown in Table 1 (2983.01 m/s and 3099.6 m/s respectively).

**Figure 3.** $S_0$ reflection field separation method, computed from measurements along $S_0$ reflection direction for a wave propagating from AA6061-T6 to AZ31B at $i = 20^\circ$
3.3 $S_0$ - $SH_0$ Transmission

The transmission of the $S_0$ mode into an $SH_0$ mode has been shown in literature to have an extremely low transmission coefficient\[^4\] with low amplitude when compared to other measured wave fields in the plate. The measured $S_0$-$SH_0$ transmission along its expected propagation direction was found to be masked by other measured transmissions and reflections (e.g. Figure 2(e)). A good approximation of the transmitted $S_0$-$SH_0$ wave requires the use of a separation procedure similar to the one described earlier.

In the previous separation method, wave fields measured along the reflection directions would have propagated only in material X, which makes their direct subtraction a reasonable solution. On the other hand, the wave field measured along the transmission direction, would have propagated only in material X in the single-material propagation, but passed from material X to material Y in the dissimilar-material propagation. A wave field of a certain energy would have different vibration amplitudes when propagating in different materials. For this reason, the measured signals in pure material were normalized to those in dissimilar materials before subtraction. Since the first wave-pack was noticed to be common between the two subtracted wave fields, the amplitude of the 3rd peak in this wave-pack was used for amplitude normalization; using another peak may have given negligibly different results. This procedure is summarized in Equation (6):

$$U_{S_0-SH_0}(X/Y) = U_{SH_0D}(X/Y) - a \times U_{SH_0D}(X/X).$$

where:
- $U_{S_0-SH_0}(X/Y)$: is the refracted $SH_0$ wave field, converted from the $S_0$ mode of an incident wave propagating from material X to material Y at an angle of incidence $i$.
- $U_{SH_0D}(X/Y)$: is the $SH_0$ wave field, measured along the $SH_0$ transmission direction, from an incident wave propagating from material X to material Y at an angle of incidence $i$.
- $U_{SH_0D}(X/X)$: is the $SH_0$ wave field, measured along the same direction as $U_{SH_0D}(X/Y)$, from an incident wave propagating from material X to material X at an angle of incidence $i$.
- $a$: is the coefficient that normalizes the amplitudes of $U_{SH_0D}(X/X)$ to those of $U_{SH_0D}(X/Y)$, determined from the 3rd detected peak at the last sensor position (+100 mm).

Figure 5 illustrates the separation of the $SH_0$ transmission for the same wave simulation of the previous figures. The average group velocity of the obtained $S_0$-$SH_0$ transmission is 3021 m/s which is very close to the theoretical value presented in Table 1 (3068.6 m/s).
3.4 Transmission and Reflection Coefficients

The transmission and reflection coefficients of the wave modes can be determined by direct comparison of the amplitudes of the transmitted and reflected fields with the transmissions while the wave is propagating in single-material. Equations (7) and (8) were used to compute the transmission and reflection coefficients for the $A_0$ and $S_0$ modes, respectively. Because the studied SH$_0$ mode is a conversion from the $S_0$ mode, the transmission and reflection coefficients of the SH$_0$ mode were determined using Equations (9) and (10).

\[
T_{(S_0 \text{ or } A_0)}^i = \frac{A_{tr}^i (S_0 \text{ or } A_0)}{A_{tr-s}^i (S_0 \text{ or } A_0)} \\
R_{(S_0 \text{ or } A_0)}^i = \frac{A_{refl}^i (S_0 \text{ or } A_0)}{A_{tr-s}^i (S_0 \text{ or } A_0)} \\
T_{(S_0 - SH_0)}^i = \frac{A_{tr}^i (S_0 - SH_0)}{A_{tr-s}^i (S_0)} \\
R_{(S_0 - SH_0)}^i = \frac{A_{refl}^i (S_0 - SH_0)}{A_{tr-s}^i (S_0)}
\]

where:
$T_{(m)}^i$ : transmission coefficient of mode “m” at incidence angle $i$
$R_{(m)}^i$ : reflection coefficient of mode “m” at incidence angle $i$
$A^i_{tr} (m)$: amplitude of the transmitted mode “m” after travelling a distance $d$ from material X to material Y; incident wave propagating from material X to material Y at an angle of incidence $\hat{i}$

$A^i_{refl} (m)$: amplitude of the reflected mode “m” after travelling a distance $d$ in material X; incident wave propagating from material X to material Y at an angle of incidence $\hat{i}$

$A^i_{tr-s} (m)$: amplitude of the transmitted mode “m” after travelling a distance $d$ in a single material X at an angle of incidence angle $\hat{i}$

It should be noted that the transmission and reflection coefficients calculated are based on the amplitudes of node displacements. Amplitudes of the single-material propagations (appearing in the denominators of equations (7) to (10)) could be taken at any incidence angle since there is no material interface. However, it was preferred to take them at the same incidence angles as those of the dissimilar-material propagations (in the numerators) to eliminate any possible numerical errors due to wave propagations in different mesh shapes.

Figure 6 shows the transmission and reflection coefficients of the three studied wave modes for both the forward (AA6061-T6 to AZ31B) and backward (AZ31B to AA6061-T6) propagation directions.

The transmissions of the $S_0$ and $A_0$ modes remained approximately constant and unaffected by the change of the incidence angle. However, this trait may not be generalized and must be tested for different types of materials and different excitation frequencies.

The values of the $A_0$ and $S_0$ transmission coefficients were above unity ($\equiv 1.2$ for both modes) when the wave propagated from AA6061-T6 to AZ31B. This means that the amplitude of the propagating wave was amplified by 20% when it passed from one medium to the other. On the other hand, the transmission coefficients of the Lamb modes were about 0.8 when the wave propagated in the inverse direction. Thus, the wave amplitude dropped by 20% when passing from AZ31B to AA6061-T6. This indicates that the wave field is amplified when passing from a stiff medium to a softer medium, while it is attenuated when the propagation direction of the wave is reversed.

The $S_0$-$SH_0$ transmission coefficient was very low, in both propagation directions, maintaining a value less than 3% for the various tested incidence angles, excluding $\hat{i} = 80^\circ$ where it increased to a maximum of 7% in the forward direction. $S_0$-$SH_0$ transmission shows a local minimum at $\hat{i} = 60^\circ$.

On the other hand, the reflection coefficients of the three studied modes were approximately equal for both propagation directions.

The $S_0$ reflection decreased from 21% at normal incidence to reach a local minimum of 1.5 to 2% at $\hat{i} = 60^\circ$. It then increased again to reach about 8 to 10% at $\hat{i} = 80^\circ$.

In an opposite manner, the $S_0$-$SH_0$ reflection increased from zero at normal incidence to reach a peak of about 28% at $\hat{i} = 50^\circ$, after which it started to decrease again until it reached about 18% at $\hat{i} = 80^\circ$.

Finally, the $A_0$ reflection increased monotonically with the increase of the incidence angle. It started from about 5 to 6% at normal incidence, then it increased gradually up to a maximum of about 16% at $\hat{i} = 80^\circ$. 
4. Conclusions

The behavior of the fundamental symmetric and antisymmetric Lamb-wave modes, upon the oblique interaction with dissimilar solid mediums, was numerically studied. An intact AA6061-T6/AZ31B joint was used for this purpose. Reflections from the material interface, as well as the converted $S_0$-$SH_0$ transmissions, were separated from other superpositions appearing in the measured wave fields. The transmission and reflection coefficients of the studied modes ($S_0$, $A_0$, and $SH_0$) were then assessed and analyzed. The existence of an $S_0$-$SH_0$ mode conversion was verified when an obliquely-incident $S_0$ mode interacts with the material interface. The converted $SH_0$ mode was more significant as a reflection from the interface rather than a transmission through the joint. While reflection coefficients of the $S_0$ and $A_0$ Lamb-wave modes varied with the incidence angle, their transmissions were not affected. The results presented in this paper are to be experimentally validated in a future study. Future work may also include investigating the possible existence of negative reflections and refractions.

**Figure 6.** Transmission and reflection coefficients of the studied modes

(a) AA6061-T6 to AZ31B

(b) AZ31B to AA6061-T6
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References and Footnotes


[15] Laboratory for Active Materials and Smart Structures (LAMSS), "Wavescope 2.5: dispersion curves, group velocities and tuning for metallic structures", University of South Carolina, USA.
