Study on Damage Detection Resolution Based on Lamb Wave and TR-MUSIC Algorithm

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ABSTRACT

Lamb wave-based structural health monitoring technology by using piezoelectric transducer array has been greatly developed for damage detection of aircraft plate-type structures. In the existing damage quantification algorithms, low exciting frequency (small than 50kHz) is preferred for that there is only a single wave mode, i.e., A0 mode, and it is easier to illustrate Lamb wave signals. However, wavelength for low frequency Lamb wave is so large that these algorithms have a poor resolution. In this paper, a damage imaging method, based on the decomposition of the time reversal operation with multiple signal classification (TR-MUSIC), has been developed for super-resolution damage location. In this method, the eigenvectors of time reversal operation could be divided into two parts, in which one part with non-zero eigenvalues is related to the signal subspace, and another with zero eigenvalues is named as noise subspace. Signal and noise subspaces are orthogonal to each other, which denotes that the noise subspace is orthogonal to the background Green function vector. In this paper, the spatial Green function are not obtained by theoretical solution but experimental measurement, by which spatial pseudo-spectrum, i.e., reciprocal of product between signal and noise subspaces, is constructed to obtain damage contour of the plate-type structure. In the paper, damage identification resolutions of TR-MUSIC and conventional Delay-And-Sum(DAS) algorithm have been compared with each other on aluminium plate. The simulation results show that TR-MUSIC algorithm can accurately differentiate neighbour damages with a smaller positioning error and a higher resolution than DAS algorithm.

1. Introduction

Structural Health Monitoring (SHM) technology has been paid close attention in the fields of aerospace, marine, civil infrastructure, et al, during the last two decades[1]. Lamb wave-based damage detection algorithm is one of research hotspots for aircraft SHM because of its wide monitoring area and high sensitivity to small damage in the plate-type structures. This method makes use of piezoelectric transducer not only as a transmitter to excite Lamb waves in the
structure, but also as a receiver to acquire Lamb wave signals which carry damage scattering information. By extracting the damage characteristics in the Lamb wave signals and combining with damage location or imaging algorithm, the present, location and severity of damage can be identified. In the existing Lamb wave-based damage detection algorithms, most of damage imaging algorithms only focus on the accuracy of the damage location and damage size, although the relation between the identified damage size and the real one is not stable. Based on probabilistic damage imaging, Su et al. realized delamination imaging in composite plates and quantitative analysis of specific damage in aluminum plates\([2,3]\). Wu et al. achieved damage localization of stiffened composite plates based on an improved probabilistic damage imaging method\([4]\). Ng et al. improved the Delay-And-Sum (DAS) algorithm to realize damage imaging of composite plates by using different group velocities along different directions \([5]\). Shan et al. proposed an adaptive data extraction strategy, together with DAS algorithm, to realize multi-damage localization in complex structures \([6]\). Based on the decomposition of the time reversal operation with multiple signal classification method (TR-MUSIC), Yuan et al. discussed the ability of subwavelength damage detection in plate-type structures \([7]\). However, Yuan et al. only got a clearer imaging for multiple damages with a long distance, in which Lamb wave scattering from damages have no influence with each other. When damages are closer, it is different that Lamb wave scattering from damages interacts with each other, and there may need more signal sub-space dimension. But, for Lamb wave-based MUSIC-type algorithms, there are few studies on the resolution of damage identification, that is, ability of identifying multiple damages with a very close distance. This paper mainly studies the double-damage imaging with a close distance in aluminum plate. Firstly, in section 2 and 3, the theoretical basis is introduced for TR-MUSIC and DAS algorithm, respectively. In section 4, damage contours of the two methods are compared with each other by using the finite element simulation signals. Finally, some conclusions are drawn.

### 2. The Theoretical Basis of TR-MUSIC

This section mainly describes the implementation process of Lamb wave-based TR-MUSIC algorithm. For piezoelectric arrays-based structural health monitoring, one piezoelectric transducer is difficult to be used as both a transmitter and a receiver. Thus, one piezoelectric transmitter array and another piezoelectric receiver array are mounted on the surface of the structure, separately. As shown in Figure 1, a piezoelectric transmitter array with \( M \) sensors is arranged on the left side of the structure, and its coordinate position is \( r_i^A, i = 1, 2, \ldots, M \), meanwhile, a piezoelectric sensor array with \( N \) sensors is arranged on the lower side of the structure, and its coordinates are \( r_j^D, j = 1, 2, \ldots, N \). The structure contains \( L \) damages, and its coordinates are \( r_l^S, l = 1, 2, \ldots, L \).
2.1 Transfer Function Matrix

In the Lamb wave-based TR-MUSIC imaging algorithm, while each element in the piezoelectric transmitter array excites the Lamb wave signal in turn, all elements in the piezoelectric receiver array receives the scattering signal. The transfer function matrix can be obtained by the receiving signals divided by transmitting signal in frequency domain, which describes the impulse response between different elements in two arrays. As shown in Figure 1, the $i$th array element in the piezoelectric transmitter array emits a signal, which forms Lamb wave and propagates in the medium, the signal received by the $j$th array element of the piezoelectric receiver array is expressed as

$$r_j(t) = k_{ji}(t) \otimes e_i(t)$$  \hspace{1cm} (1)

where $\otimes$ represents the signal convolution in time domain, corresponding to the signal product in frequency domain. By using Fourier transform, Equation (1) is transformed from the time domain to the frequency domain as shown in Equation (2)

$$R_j(\omega) = K_{ji}(\omega)E_i(\omega)$$  \hspace{1cm} (2)

where $R_j(\omega), E_i(\omega)$ and $K_{ji}(\omega)$ corresponds to the Fourier transform of the time domain signals $r_j(t), e_i(t)$ and $k_{ji}(t)$, respectively.

Equation (2) is expressed in a matrix form as shown in the Equations (3) and (4).

$$
\begin{bmatrix}
R_{11}(\omega) & R_{12}(\omega) & \cdots & R_{1M}(\omega) \\
R_{21}(\omega) & R_{22}(\omega) & \cdots & R_{2M}(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
R_{N1}(\omega) & R_{N2}(\omega) & \cdots & R_{NM}(\omega)
\end{bmatrix}
\begin{bmatrix}
K_{11}(\omega) & K_{12}(\omega) & \cdots & K_{1M}(\omega) \\
K_{21}(\omega) & K_{22}(\omega) & \cdots & K_{2M}(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
K_{N1}(\omega) & K_{N2}(\omega) & \cdots & K_{NM}(\omega)
\end{bmatrix}
\begin{bmatrix} E_1(\omega) \\
E_2(\omega) \\
\vdots \\
E_M(\omega)
\end{bmatrix}
= R(\omega)
$$  \hspace{1cm} (3)

$$R(\omega) = K(\omega)E(\omega)$$  \hspace{1cm} (4)
where \( E(\omega) \) represents the matrix of the exciting signal, \( R(\omega) \) represents the matrix of receiving signals, and \( K(\omega) \) represents the matrix of transfer functions.

### 2.2 Time Reversal Matrix and Its Eigenvalues

The time reversal operator is the basis of the TR-MUSIC method and can be calculated from the transfer function matrix. Yuan et al has given the definition of the time reversal operator and its eigenvectors\(^7\) as shown in Equations (5)-(6).

\[
T_1 = K^H K
\]  
\[
T_2 = KK^H
\]

where the symbol ‘\( H \)’ represents the adjoint operator of the matrix, that is the conjugate transpose of the matrix.

\[
\bar{g}_i^A = \lambda_i g_i^A, \quad i = 1, 2, \cdots, L
\]

\[
\bar{g}_j^P = \lambda_j g_j^P, \quad j = 1, 2, \cdots, L
\]

\[
g_i^A = [g(r_i^x;r_i^A), g(r_i^x;r_2^A), \cdots, g(r_i^x;r_d^A)]^T
\]

\[
g_j^P = [g(r_i^x;r_i^A), g(r_i^x;r_2^P), \cdots, g(r_i^x;r_n^P)]^T
\]

where \( g_i^A \) represents the Green function between damage \( r_i^x \) and piezoelectric transmitter \( r_i^A \), \( g_j^P \) represents the Green function between damage \( r_i^x \) and piezoelectric receiver \( r_j^P \) and \( L \) represents the number of signal subspaces. The symbol “\( \bar{\cdot} \)” represents the complex conjugation of a vector, and “\( T \)” represents the transpose of a vector. In this paper, the value of the \( g^A \) and \( g^s \) is obtained by means of actual measurement.

### 2.3 Imaging Function

When multiple sites of damage are not too close to each other, the interaction between different sites of the damage is weak and can be classified as well-resolved targets. After eigenvalue decomposition of the time reversal operator, the L non-zero eigenvalues, which are equal to the number of damages in the structure, and (\( N-L \)) zero eigenvalues correspond to the signal subspace and the noise subspace respectively. The noise subspace is orthogonal to the signal subspace, so the noise subspace obtained by the time reversal operator after eigenvalue decomposition is orthogonal to the background Green's function. If the damage location in the structure is too close, the number of non-zero eigenvalues will not be equal to the number of
damages. Therefore, it is necessary to select the appropriate criteria to divide the signal subspace and the noise subspace.

\[ \mu_i \star g_i^A = 0, \lambda_i = 0 \]  \hspace{1cm} (11)  

\[ \mu_j \star g_j^B = 0, \lambda_j = 0 \]  \hspace{1cm} (12)  

where \( \lambda_i \) and \( \lambda_j \) are the \( i \)th and \( j \)th zero eigenvalues after the eigenvalue decomposition of the time reversal operators \( T_i \) and \( T_j \) respectively, \( \mu_i \) is the eigenvector corresponding to \( \lambda_i \) and \( \mu_j \) is the eigenvector corresponding to \( \lambda_j \).

The spatial spectrum of TR-MUSIC can be defined as

\[ P(r_q^A) = \frac{1}{\sum_{i=1}^{M} \left( \mu_i \cdot \bar{g}_q^A \right)^2} \]  \hspace{1cm} (13)  

\[ P(r_q^B) = \frac{1}{\sum_{j=1}^{N} \left( \mu_j \cdot \bar{g}_q^B \right)^2} \]  \hspace{1cm} (14)  

where \( r_q \) represents the discrete point of the structure and \( r_i \) represents the position of the scattering target. When \( r_q = r_i \), the inner product is zero at the scattering position of the target, then the spatial spectrum of the structure appears peak value at the target point, thus realizing the damage super-resolution imaging.

3. Delay-And-Sum Algorithm

Delay-And-Sum (DAS) algorithm is a damage detection algorithm based on the time of flight (TOF). In this method, the scattering signals of all excitation-sensing paths in the monitoring area are delayed by appropriate time-shift rules and superimposed to characterize the probability of damage occurrence in the monitored area. The implementation of the method can be divided into four steps.

The first step is to calculate the flight time from the piezoelectric transmitter to the discrete point in the monitored area and then to the piezoelectric receiver, as shown in Equation (15).

\[ t_q \left( r_q^A \right) = t_{off} + \sqrt{\frac{\left| r_q^A - r_q^S \right|^2}{\nu} + \frac{\left| r_q^S - r_q^B \right|^2}{\nu}} \]  \hspace{1cm} (15)  

where \( \nu \) is the group velocity of Lamb wave.

The second step is normalization. Because different transmitting-receiving paths are different, the scattering signal is normalized according to Equation (16).

\[ \hat{R}_q(t) = \frac{R_q(t)}{R_q(t)_{\text{max}}} \]  \hspace{1cm} (16)
where $R_{ij}(t)$ represents the scattering signal when the $i$th piezoelectric element acts as transmitter and the $j$th piezoelectric element acts as receiver.

Thirdly, Gabor wavelet transform is applied to the normalized scattering signal to extract damage feature information for further imaging processing.

$$E(b) = \text{abs}(CWT_{R_{ij}}(a,b))$$ (17)

Finally, the probability of damage occurring at discrete points in the monitored area is calculated and the corresponding imaging processing is performed. For the discrete points $r_q^S$ of the monitored area, the scattered signals of each transmitting-receiving paths are subjected to corresponding time delay according to the calculation results of $t_q(r_q^S)$, and superimposed to characterize the probability of damage occurrence at the pixel point.

### 4. Damage Imaging

In order to compare the accuracy and resolution of damage detection between TR-MUSIC algorithm and DAS algorithm, four cases, containing single damage, double damages with a long distance, double damages with a short distance and double damages with a half-wavelength distance, are considered and simulated by finite element method to get sensing signals, which are used to image the damage by using two algorithms.

An aluminium plate with dimensions $600\,\text{mm} \times 600\,\text{mm} \times 1\,\text{mm}$, elastic modulus $70\,\text{GPa}$, Poisson's ratio 0.3 and density of $2700\,\text{kg/m3}$ are considered in the simulation. As shown in Figure 1, eleven piezoelectric elements are set on the left side of the plate as transmitters with horizontal coordinate $-150\,\text{mm}$ and vertical coordinate ranged from $-25\,\text{mm}$ to $25\,\text{mm}$ in steps of 5mm. Similarly, the piezoelectric receiver array is set on the low side of the plate with vertical coordinate $-150\,\text{mm}$ and horizontal coordinate ranged from $-25\,\text{mm}$ to $25\,\text{mm}$ in steps of 5mm. A five-peak Hanning-windowed sine toneburst with centre frequency of $50\,\text{kHz}$ and a wavelength of $13.6\,\text{mm}$ is selected as the excitation signal. The simulated damage related parameters in the four cases are shown in Table 1, and the dimensions of the simulated damage are all $4\,\text{mm} \times 4\,\text{mm}$.

<table>
<thead>
<tr>
<th>Case</th>
<th>Damage center coordinates</th>
<th>Number of damages</th>
<th>Damage distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(15,15)$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$(-15,-15)$</td>
<td>$(15,15)$</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>$(5,5)$</td>
<td>$(15,15)$</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>$(8,8)$</td>
<td>$(15,15)$</td>
<td>2</td>
</tr>
</tbody>
</table>

As shown in Figure 2, TR-MUSIC and DAS algorithm can accurately locate the damage target for a single structural damage. In TR-MUSIC imaging processing, the dimension of signal
subspace and noise subspace is selected by appropriate criterion, and the eigenvalues decomposed by transfer function matrix are logarithmized and normalized. The eigenvectors corresponding to the eigenvalues exceeding 0.2 are selected as signal subspace and the rest as noise subspace. The Green function required by the spectrum is obtained by the actual measurement. According to Equations (13)-(14), the spatial spectrum is obtained and multiplied for damage imaging. Because the measured Greens function data of the whole structure is too large, a target area of 100 mm × 100mm is selected for measuring the Green function and damage imaging.

(a) TR-MUSIC  (b) DAS algorithm

Figure 2. The single damage location image

(a) TR-MUSIC  (b) DAS algorithm

Figure 3. Double-damage imaging with a long distance
As shown in Figure 3, both TR-MUSIC and DAS algorithm can identify the damage and locate the image for double damages with a long distance, but the accuracy of TR-MUSIC is much higher than that of DAS algorithm. Figure 3(a) is TR-MUSIC double-damage contour. It can be seen from the figure that the relative positioning error of the plate structure with the size of 600 mm * 600 mm can be neglected. Figure 3(b) is the damage image of the DAS algorithm. It can be seen that although this method can also locate and image multiple damages over a long distance, there are still some errors between the identified damage location and the actual damage location.

For the case of double damages with a short distance, as shown in Figure 4, the DAS algorithm can still perform damage imaging, but its localization accuracy is obviously lower than the TR-MUSIC algorithm. The closest distance between the two damages is only 8.49 mm. As can be seen from Figure 4(a), TR-MUSIC can locate and identify double damages, and achieve high-resolution location imaging of damage location. But the positioning error of DAS algorithm is relatively large. It can be seen from the Figure 4(b) that the distance between the identified double damages is larger than the actual one.

In order to further compare the accuracy of damage location between TR-MUSIC algorithm and DAS algorithm, the case of double damages with a half-wavelength distance is also considered in this paper. The closest distance between the two damages is only 4.24mm, which is smaller than the half-wavelength of A0 wave with frequency 50kHz. As shown in Figure 5, for multi-damage cases within half-wavelength distance, TR-MUSIC algorithm can still accurately locate and image damages with a small error. As for the DAS algorithm, only one single damage can be identified, and the super-resolution imaging of multi-damage within half-wavelength distance cannot be realized.

5. Conclusions
For Lamb wave-based structural health monitoring, in order to reduce the difficulty of signal processing caused by multiple modes of Lamb wave, single mode A0 wave with low frequency is often used to locate the damage, but the resolution of damage identification is very low. It has important value of practical applications to improve the resolution of damage imaging based on Lamb wave. In this paper, by simulating the multi-damage situation of plate-type structure with different distance and comparing the damage contour of TR-MUSIC algorithm and DAS algorithm, it can be concluded that TR-MUSIC algorithm can achieve ultra-high-resolution damage localization under low frequency excitation, and the localization accuracy is good enough.

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References