Study on Instantaneous Frequency Analysis of GPR Signal Using Variational Mode Decomposition

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ABSTRACT

The basic principle of the variational mode decomposition in Hilbert-Huang transform is introduced, and instantaneous frequency is obtained by the intrinsic mode function. Based on the Hilbert-Huang transform theory, the variational mode decomposition method is used to study a specific synthetic signal and the ground penetrating radar forward model. The multi-resolution characteristics of the intrinsic mode function are discussed. From a comparison study, its effects are clearly more superior to those of the traditional empirical mode decomposition and the ensemble empirical mode decomposition. Finally, the variational mode decomposition is used to analyze one ground penetrating radar profile of a highroad exploration. The proposed method can handle ground penetrating radar data and highlight abnormal characteristics in the profile, so as to improve the effect of the GPR signal analysis.

1. Introduction

The results of ground penetrating radar (GPR) signal processing determine its detection effects, while the spatial distribution of underground media is as a result of analyzing the characteristics of effective reflection electromagnetic waves. The extraction of echo signals is one of the most critical technologies. It is directly related to the accurate detection and reliability of the underground target [1]. Instantaneous frequency is a powerful tool in GPR signal analysis. It is very important in the recognition of underground media [2,3]. Since the conventional GPR is a broadband signal, the traditional instantaneous frequency analysis method is based on Hilbert transform and is unable to separate signals at different scales [4,5]. Since Norden E. Huang proposed the Hilber-Huang Transform (HHT), the instantaneous spectrum of intrinsic mode function (IMF) of GPR signal can been obtained by using the empirical mode decomposition (EMD) [6-10]. Due to some problems of the EMD methods such as mode aliasing and endpoint effects, the HHT method and its use is limited to a certain extent [11-14]. In order to overcome this problem, some scholars have proposed that the ensemble empirical mode decomposition (EEMD) and the complete ensemble empirical mode decomposition (CEEMD) be used in GPR signal analysis [15-17]. However, these methods also still have limitations of their own. Recently, the variational mode decomposition (VMD), which encompasses multiple adaptive Wiener filter groups, was discovered to have good robustness with the ability to overcome the disadvantages of mode aliasing, small end effects and pseudo components as compared to other conventional algorithms [18,19]. The VMD algorithm has been applied to some fields such as in mechanical fault diagnosis, structural damage identification, and it has achieved significant results [20-22]. The VMD has also been applied to analyze seismic data [23,24]. However, relevant reports on utilizing VMD to analyze GPR data are lacking.

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Therefore, investigating the application of VMD theory in GPR data processing is academically significant. In this study, VMD algorithm was used to calculate IMF, and the Hilbert transform was used to compute the instantaneous frequency of IMF. The instantaneous frequency of the GPR was established based on VMD. Taking one synthetic signal and one simulation GPR signal for analysis, the VMD, EMD and EEMD algorithms were used to solve the time-frequency spectrum. The effect of the analysis was that it compared the three methods against each other based on the time-frequency spectrum. Finally, the VMD method was applied to detect the geological hazards of highway surveys and to test the effects of its application.

2. VMD Theory and Instantaneous Frequency

2.1 VMD Theory

When the VMD method is used to obtain IMF components, the variational model is introduced to solve the signal decomposition. The constraints are used to search for the optimal solution of the variational model. The center frequency and bandwidth of every modal function component are iteratively updated to achieve signal decomposition. Finally, the frequency band of the signal is adaptively decomposed to obtain the narrow-band IMF of the predetermined scale. VMD theory overall framework, including the construction and solution of variational models and problems, is a variational problem.

(1) Construction of variational models

The mode function $u_k(t)$ is taken as the Hilbert transform, and every analytic signal is obtained. Then, the central frequency of prediction is $e^{-j \omega_k t}$, and the frequency spectrum of every mode is modulated to the corresponding frequency baseband. Finally, the $L^2$ norm of modulation signal is solved, and the signal bandwidth of every mode is evaluated. The variational problem is the solution of $K$ mode function $u_k(t)$, and the sum of estimation bandwidth of every mode is minimum when constrain condition is the sum of every mode equal to input signal. The signal is decomposed to N IMF classifications. The corresponding variational problem is defined as follows:

1) Every IMF component is taken as the Hilbert transform, and analytic signal can be obtained.

$$\left( \delta(t) + \frac{j}{\pi t} \right) * u_k(t),$$  

where $\delta(t)$ is the Dirichlet function and $u_k(t)$ is K IMF.

2) The analytic signal of estimation central frequency can be changed to baseband by using frequency shift.

$$\left( \delta(t) + \frac{j}{\pi t} \right) * u_k(t) e^{-j \omega_k t},$$

where $\omega_k$ is the central frequency.

3) The $L^2$ norm of module signal is solved, and the bandwidth of every mode is estimated. The variational problem is defined as follows:

$$\min_{\{u_k\}_{k=1}^K} \left\{ \sum_k \left\| \delta(t) + \frac{j}{\pi t} \right\| * u_k(t) e^{-j \omega_k t} \right\|^2,$$

s.t. $\sum_k u_k = f(t)$
(2) Solution of variational problem

To solve the optimal solution of constrain variational problem, the original signal is decomposed into K narrow mode components. The augmented Lagrangian can be expressed as follows:

\[ L\{u_k, \omega_k, \lambda\} = \alpha \sum_k \left\| \partial_{u_k}\left[ \left( \delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega t} \right\|^2 \]

+ \left\| f(t) - \sum_k u_k(t) \right\|^2 + \left\langle \lambda(t), f(t) - \sum_k u_k(t) \right\rangle, \tag{4} \]

where \( \alpha \) is the penalty factor, \( \lambda(t) \) is the Lagrange multiplier, \( \| \|_2 \) is the norm, and \( \langle \cdot \rangle \) is the inner-product operator.

The alternative direction multiplier operator is used. \( \omega_k, u_k, \) and \( \lambda \) are updated, and the augmented Lagrangian can be solved.

2.2 Instantaneous Frequency

Using the VMD method, the original signal \( f(t) \) can be decomposed \( n \) number of IMF:

\[ f(t) = \sum_{k=1}^n u_k(t) \tag{5} \]

The Hilbert transform is performed on IMF. The transform form is expressed as:

\[ H[u_k(t)] = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{u_k(\tau)}{t-\tau} d\tau \tag{6} \]

where \( P \) is Cauchy principal value, \( t \) is time. The analytical signal can be constructed through formula (6) and IMF. The formula is expressed as:

\[ A[u_k(t)] = u_k(t) + iH[u_k(t)] \tag{7} \]

In the signal of equation(7), its amplitude and phase is:

\[ a_k(t) = \sqrt{u_k^2(t) + H^2[u_k(t)]} \tag{8} \]

\[ \theta_k(t) = \arctan \frac{H[u_k(t)]}{u_k(t)} \tag{9} \]

The phase of the signal is derived. Then, the instantaneous frequency of the signal can be obtained:

\[ \omega_k(t) = \frac{1}{2} \frac{d\theta_k(t)}{dt} \tag{10} \]

where \( \omega_k(t) \) is instantaneous frequency.

Then \( f(t) \) can be expressed as following:

\[ f(t) = \text{Re}\left( \sum_{k=1}^n a_k(t) e^{j\theta_k(t)} \right) \tag{11} \]

The formula (11) is expanded and the Hilbert instantaneous frequency is as following:

\[ H(\omega, t) = \text{Re}\left( \sum_{k=1}^n a_k(t) e^{j\theta_k(t)\omega} \right) \tag{12} \]

where \( H_k(t, \omega) \) is Hilbert instantaneous frequency spectrum, \( \omega \) is frequency. Instantaneous frequency spectrum flow chart is shown in figure.1.
Figure 1. Hilbert instantaneous frequency amplitude spectrum solving process

\[ H(\omega, t) \] describes the amplitude of the signal in the entire frequency domain with time and frequency of the transformation. In Fourier analysis, the energy representation at a certain frequency is a Harmonic wave that exists for the entire time. By contrast, in Hilbert’s instantaneous frequency analysis, the frequency energy at some point may indicate its occurrence in the entire time. A frequency wave appears and represents the probability of occurrence.

3. Comparison of VMD, EMD AND EEMD

The synthetic signal \( s \) is overlapped by \( s_1 \) and \( s_2 \), where \( s_1 \) is the sinusoidal periodic function and \( s_2 \) is the segmented sine function (Figure 2a). The function is as follows:

\[
\begin{align*}
\ s_1 &= \sin(20\pi t) \\
\ s_2 &= \begin{cases} 
0.2\sin(300\pi t) & \text{when } 0.05 \leq t \leq 0.1 \\
0.4\sin(100\pi t) & \text{when } 0.15 \leq t \leq 0.25 \\
0 & \text{other}
\end{cases}
\end{align*}
\]

\( s = s_1 + s_2 \)  

(15)

The synthetic signal \( s \) is decomposed by using VMD (Figure 2a). The results of the decomposition are shown in Figure 2 (b) ~ (d).

Figure 2. Synthetic signal and IMF: (a) signal \( s_1 \) (b) the first IMF (c) the second IMF (d) the third IMF

According to the results of Figure 2 (b) - (d), the signal \( s \) is decomposed to obtain three IMFs. The three
IMFs correspond to the three sine functions of signal s. IMF1 corresponds to s1, IMF2 corresponds to s2 in 0.15 ~ 0.25s, and IMF3 corresponds to s2 in 0.05 ~ 0.1s. The results show that VMD can properly separate IMFs from the signals.

In order to study the performance of VMD, the signal s is decomposed by VMD, EMD and EEMD respectively, and the instantaneous spectrum of the signal s is obtained by using the three methods (Figure 3).

![Time-frequency amplitude spectrum](image)

**Figure 3.** Time-frequency amplitude spectrum: (a) time-frequency amplitude spectrum based on EMD (b) time-frequency amplitude spectrum base on EEMD (c) time-frequency amplitude spectrum based on VMD.

In Figure 3, the three methods were used to solve instantaneous frequency. In Figure 3 (a), EMD method only obtained a component at a frequency of 10 Hz, but failed to get components at 50 Hz and 150 Hz frequencies, most especially components at 150Hz frequency. Figure 3 (b), EEMD is able to separate different components in the three frequencies. However, the component at 10Hz frequency has some fluctuations in the 0.15 ~ 0.25s, and the other two components have some bending corresponding to the frequency section. We find that EEMD is able to separate the different components although, the accuracy is not high. Figure 3 (c), the VMD method clearly separates the three different components with accurate frequency for each time period. Thus, the effect of VMD in the elimination of mode aliasing is significantly superior to the other two methods.

During the process of mode decomposition, the endpoint effect is usually a serious problem in EMD since it causes energy to be changed after decomposition. The signals u(i) decomposition generates and IMF energy can be expressed as follows:

$$E = \sqrt{\frac{\sum_{i=1}^{n} u^2(i)}{n}}$$  \hspace{1cm} (16)

where $E$ is IMF energy of n number, $u(i)$ is signal, and $n$ is the sample number of signals.

The energy leakage can be reflected by defining the deviation between the total energy of IMF and the original signal energy. The evaluation index can be expressed as follows:
\[ \xi = \frac{\sqrt{\sum_{i=1}^{k} E_i^2 - E_u}}{E_u} \]  

where \( E_u \) is signal energy, \( E_i \) is IMF energy of \( i \) number, and \( k \) is the total number of IMF.

### Table 1. Evaluation index of energy leakage

<table>
<thead>
<tr>
<th></th>
<th>EMD</th>
<th>EEMD</th>
<th>VMD</th>
</tr>
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<tbody>
<tr>
<td>( \xi )</td>
<td>0.065</td>
<td>0.0125</td>
<td>0.0119</td>
</tr>
</tbody>
</table>

From Table 1, the energy leakage when using VMD is smaller than that when using EMD and EEMD, and the end effect when using VMD is not obvious. Therefore VMD is superior to the other two methods in regard to the endpoint effect.

**4. Time Frequency Analysis of GPR Signal**

**4.1 Numerical Simulation Signal of GPR**

We used a detection of three-layer concrete pavement as an example. For the GPR forward sketch, see Figure 4(a). The model measures 2.5 m \( \times \) 0.45 m; the uppermost layer is air, the middle layer is concrete, and the bottom layer is dry sand. There is one incline facet between concrete and dry sand. The depth of the concrete is between 0.15m and 0.25m. The permittivity and conductivity of the concrete are 6 and 0.005 S/m respectively and the permittivity and conductivity of dry sand are 3 and 0.0001 S/m respectively. During the simulation of GPR with finite difference time domain (FDTD), the computational grid size is 0.0025m, the length of the time window is 12ns, and absorbing boundary is unsplit field perfectly matched layer (UPML). The central frequency of the transceiver antenna is at 900MHz, the transceiver antenna distance is 0.025 m, and the work mode is self-excitation and self-receiving. From the left 0.1m position to the right 2.3m at 0.02m intervals, a total of 115 trace data were obtained. The scanner result of the simulation is shown in Figure 4(b).

![Simulation model of GPR](image)

**Figure 4.** Simulation model of GPR: (a) forward geometry model (b) simulation signal of GPR

From Figure 4 (b), each IMF can be decomposed using VMD, and the time-frequency spectrum can be obtained using the Hilbert transform. Figure 5 shows time-frequency spectrum slices of simulation signal of GPR at 1.0 and 1.2GHz frequencies.
Figure 5. Sliced time-frequency amplitude spectrum based on VMD: (a) time-frequency amplitude spectrum at 1.0GHz (b) time-frequency amplitude spectrum at 1.2GHz

From Figure 5, the slice of the time-frequency spectrum at 1.0 GHz clearly shows the ground-coupled direct wave (Figure 5 (a)), and the slice of the time-frequency spectrum at 1.2 GHz clearly shows the reflected wave between the concrete and the dry sand interface (Figure 5 (b). It can be seen that the time-frequency spectrum separates the different GPR components based on VMD.

In order to study further the effects of VMD on GPR signals, we used EMD and EEMD to calculate the time-frequency spectrum of GPR signal simulation, respectively. Figure 6 shows the time-frequency spectrum at 1.0GHz and 1.2GHz based on EMD, and Figure 7 shows the time-frequency spectrum at 1.0GHz and 1.2GHz based on EEMD.

Figure 6. Sliced time-frequency spectrum based on EMD: (a) time-frequency amplitude spectrum at 1.0GHz (b) time-frequency amplitude spectrum at 1.2GHz

Figure 7. Sliced time-frequency spectrum based on EEMD: (a) time-frequency amplitude spectrum at 1.0GHz (b) time-frequency spectrum at 1.2GHz

From Figure 6, the time-frequency spectrum slice at 1.0GHz has a weak performance on the direct wave (Figure 6 (a)). However, the time-frequency spectrum slice at 1.2GHz does not show a reflected wave.
on the interface between the concrete and sand (Figure 6 (b)), but there is still direct wave information. The time-frequency spectrum obtained using EMD is not clear for the separation of the different components of radar signal.

Figure 7 (a) shows the time-frequency spectrum slices at 1.0 GHz based on EEMD. The time-frequency spectrum slice at 1.0 GHz shows direct waves. However, the time-frequency spectrum at 1.2 GHz displays a reflection at the interface of concrete and sand (Figure 7 (b)), but the effect is not obvious. There are direct waves and other information involved in Figure 7 (b). EEMD can’t decompose each of the frequency components of the simulation GPR signal. From the three methods of solving time-frequency spectrum analysis, VMD is better than the other two methods in separating different components of the GPR signal.

4.2 The Analysis of Real Exploration GPR Signal

The Yunnan Province of China plans to build a highway through a mining area, and the project required a geological survey. The detection device used was a GPR (GSSI SIR-3000). The choice central frequency of the antenna was 100MHz. The length of the detection section was 15.0 m, and acquisition space interval was 0.050 m. The cross section had 300 trace records, and each trace contained 400 sampling points. Figure 8(a) shows the survey records of the GPR.

Figure 8. Real world GPR data and frequency spectrum: (a) real world GPR record (b) time-frequency amplitude spectrum at 80kHz

From the original records of the field (Figure 8 (a)), the mining area is not clear. The detection data is processed using VMD in order to test VMD’s effect. The time-frequency spectrum of the detection signal is obtained using the proposed method. Figure 8 (b) shows the time-frequency spectrum slice of the signal at 80kHz.

From Figure 8 (b), it can be seen that there are obvious anomalies located in survey line 4.5m, which are obviously different from other regions. The region with anomalies is highlighted in Figure 8 (b). It is a verified fact that there were caverns at the 4.5m location of the survey line after drilling. The evidence proves that the proposed method has significant applicability for processing GPR signals.

5. Conclusions

Based on the VMD and the Hilbert transform theory, the VMD technique was introduced into GPR signal analysis, and the instantaneous frequency analysis method of the GPR signal was also established. The analysis results showed that the proposed method has better outcomes than the traditional methods. Finally, the method was applied to a real engineering exploration. The conclusions are as follows:

1) The VMD method better overcomes the influences of mode aliasing and endpoint effects as compared to other traditional methods.

2) The time-frequency spectrum of the GPR signal can be obtained using VMD. VMD separates the IMF of the GPR signal and has obvious advantages compared to traditional methods.

3) Based on the time-frequency spectrum of the GPR signal based on VMD, the processing method can highlight abnormal areas and give good applicability to discovery anomalies.
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