Zero-Frequency Mode and Its Application in Nondestructive Testing

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ABSTRACT

When a wave propagates through an elastic solid with quadratic nonlinearity, nonlinear acoustic components including second harmonic higher-order harmonics subharmonics and zero-frequency mode (static displacement) are induced. And when a longitudinal wave propagates through a linearly elastic solid with distributed micro-cracks, cracks “breathing” or “clapping” are excited under cyclic compression and tension. During the tensile cycles, the crack might be open and the crack faces are traction-free. During the compressive cycles, the crack might be closed and the crack faces are in contact. Such tension and compression asymmetry causes acoustic nonlinear harmonics including second harmonic, higher-order harmonics and zero-frequency mode too. Theoretically, we have confirmed that, the signal of zero-frequency mode is stronger than that of the traditional nonlinear harmonics, and there is no need for phase-velocity matching to generate the zero-frequency mode for Lamb waves. The zero-frequency mode induced by distributed contact cracks during longitudinal wave propagation has been also investigated. The acoustic nonlinearity parameter measured by zero-frequency mode is strong and linearly proportional to the propagation distance (or crack density). Signals were numerically detected and analysed for waves propagating in an aluminium material, which verifies theoretical predictions. Zero-frequency mode is of great use in damage monitoring or nondestructive testing, based on the techniques of nonlinear ultrasonic waves.

1. Introduction

An effective and reliable inspection technique for continuous evaluating and monitoring of early-stage nonlinearities in materials is necessary for engineering parts. Among all kinds of nondestructive methods studied for efficient damage detection and evaluation, ultrasonic method is the most useful one and has been widely exploited for many decades[1-4]. Traditional ultrasonic inspection method is bottomed on linear theory, and the characterizing parameters are the velocity attenuation or transmission and reflection coefficients of the ultrasonic wave. However conventional linear ultrasonic evaluation method is not sensitive to the micro-damages or micro-plastic deformation, and the smallest crack that current linear ultrasonic can detect is just about 1 mm; this drawback causes application limitations that fail to take any preventive actions[5-7].

When a high intensity ultrasonic wave passes through a nonlinear medium the waveform distorts as the wave progresses[8]. A linear solid medium may become a nonlinear one gradually in the process of fatigue damage[9, 10], radiation damage[11], hardening[12] and thermal aging[13], as the microstructural features occupied by micro-voids multi-poles micro-cracks[14] dislocation[15-17] persistent slip bands[18] precipitation characteristics[19] and so on. When a finite amplitude monochromatic sinusoidal ultrasonic wave interrogates the nonlinear solid, the initial waveform distorts and thus the higher-order harmonics waves are generated. Overcome the limitations of the linear ultrasonic method, the nonlinear ultrasonic technique which measures the higher harmonics generated by material intrinsic nonlinearity has undergoing a rapid development in recent years. Among these experiment or theoretical studies, only

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bulk waves are used, and a useful acoustic nonlinearity parameter $\beta$ presenting the material nonlinearity is established by theoretical models$^{[8, 20, 21]}$. However, bulk wave propagates with a great energy loss and couldn’t be used as an efficient large-scale inspection approach. Guided waves such as Lamb waves combine the large-scale inspection ranges and the sensitivity of nonlinear parameters can be used for long-range inspection to interrogate large shell- and plate-like structures. However, the inspection experiments of nonlinear Lamb waves might not very accurate due to their dispersive and multi-mode; the corresponding theory studies may not entirely clear and even not consistent with each other due to the mathematical complexity of the problem$^{[22-28]}$. Fortunately, all theory studies reach agreement on that phase velocity matching and non-zero power flux are two necessary conditions for cumulative second harmonic generation. Mode pairs which satisfy these two conditions such as S1-S2, S2-S4, and A2-S4 were used to conduct experiment and numerically study. And tensile plasticity-driven damage$^{[29]}$, material nonlinearity$^{[30]}$, thermal fatigue$^{[31]}$, creep damage$^{[32]}$ and fatigue damage$^{[33]}$ are effective characterized by these experiments. However, most of the existing work in the literature focuses on the second harmonic$^{[1-33]}$. In fact, the propagation of an ultrasonic wave through a nonlinear material not only leads to generation of the harmonics of the original wave but also a static displacement component (also called direct current component or zero-frequency mode)$^{[34-42]}$. Narasimha et al$^{[39, 40]}$, Jacob et al$^{[41]}$, and Nagy et al$^{[42]}$ indicated that the zero-frequency mode is vary linearly with the propagation distance. Their excellent research are mainly confine to longitudinal acoustic waves. Later, Sun, X., et al$^{[43]}$ and Wan, X., et al$^{[44]}$ had shown that the signal of zero-frequency mode is stronger than that of the traditional nonlinear harmonics for Lamb waves (guided waves). In addition, unlike second harmonics, there is no need for phase-velocity matching to the zero-frequency mode accumulation. Thus, the existence of zero-frequency mode may be of great significance (comparing with the second harmonic) in the field of non-destructive evaluation and structural health monitoring of early-stage material nonlinearity based on the ultrasonic (Lamb) waves. And, only quadratic nonlinearity is considered in these work.

For quasi-brittle materials, however, micro-cracks are prevalent. Considerable experimental evidence, e.g.$^{[45-49]}$, has shown that ultrasonic waves do interact with micro-cracks in a nonlinear fashion, and measuring the acoustic nonlinearity could be a potential tool for nondestructively characterizing the degree of damage induced by micro-cracks. Extensive work has been done to model the nonlinear interactions between ultrasonic waves and micro-cracks. There are three major approaches, the hysterisis model, the bi-linear stiffness model, and the rough surface contact model. The hysterisis model is a phenomenological approach where it is assumed that the stress-strain relationship constitutes a hysterisis loop, i.e., the stress-strain curve follows different nonlinear paths under tensile and compression and forms a loop. Such nonlinear and hysteresis stress-strain relationship induces energy dissipation as well as acoustic nonlinearity$^{[50-53]}$. The bi-linear stiffness model is based on the fact that crack faces are open under tension and closed under compression, thus leading to a tension-compression asymmetry in the effective modulus of the cracked solid, which generates acoustic nonlinearity$^{[54, 55]}$. The rough surface contact model assumes that the cracks faces are rough (with asperities) so that, when in contact, these asperities are subjected to plastic deformation which yields acoustic nonlinearity$^{[56-58]}$. A recent review paper$^{[59]}$ on this topic gives more details of these three different general approaches. However, theses reported literatures are mainly focus on the second harmonic (generated by micro-cracks). And so far, there are few studies focusing on micro-cracks induced zero-frequency.

As the application of zero-frequency mode in nondestructive testing of early-stage material nonlinearity (weak material nonlinearity) is reported in our previous papers$^{[60, 61]}$. With the limitation in space, in this paper, we focus on demonstrating that zero-frequency mode also can be generated by micro-cracks (theoretically and numerically). The amplitude ratio, i.e., $\beta_i = A_i / A_1$, $(i = 0, 2)$, is defined to measure the bi-linear stiffness crack induced acoustic nonlinearity. The amplitude ratio of zero-frequency is much stronger than that of the second harmonic. In addition, the amplitude ratio of zero-frequency is increase linearly with the crack density. Thus, zero-frequency mode is of great use in material early stage damage monitoring or nondestructive testing, based on the techniques of nonlinear ultrasonic waves.

2. Zero-frequency mode generation for micro-cracks (longitudinal wave)

2.1 Theory
The physical nature of the crack-related nonlinearity can be considered as two simple models of a contact type interface. The first model was suggested by Richardson. This model (bi-linear stiffness model) is based on the fact that crack faces are open under tension and closed under compression, thus leading to a tension-compression asymmetry in the effective modulus of the cracked solid, which generates acoustic nonlinearity. The second model of an crack interface was a contact between two rough elastic surfaces in contact. The rough surface contact model assumes that the cracks faces are rough (with asperities) so that, when in contact, these asperities are subjected to plastic deformation which yields acoustic nonlinearity.

2.1.1 Case I: Bi-linear stiffness model

Following Zhao et al., consider wave propagation in an isotropic and linearly elastic solid with Young’s modulus $E$ and Poisson’s ratio $\nu$. Assuming that the solid contains randomly distributed and randomly oriented micro-cracks. For a longitudinal plane wave propagating along the $x$-direction, the displacement equation of wave motion given by

$$L[u] = \frac{\partial^2 u}{\partial t^2} - c_L^2 \frac{\partial^2 u}{\partial x^2} = -\gamma_c c_L^2 f_c[u] - \gamma_t c_L^2 f_t[u]$$

(1)

where $c_L = \sqrt{E/\rho}$ is the longitudinal velocity, $\rho$ is the mass density of the uncracked solid, $\gamma_t \ll 1$ and $\gamma_c \ll 1$, and

$$f_c[u] = \frac{\partial}{\partial x} \left( H \left( \frac{\partial u}{\partial x} \right) \right) = H \left( \frac{\partial u}{\partial x} \right) \frac{\partial^2 u}{\partial x^2}, \quad f_t[u] = \frac{\partial}{\partial x} \left( H \left( -\frac{\partial u}{\partial x} \right) \right) = H \left( -\frac{\partial u}{\partial x} \right) \frac{\partial^2 u}{\partial x^2}.$$  

(2)

Consider wave motion in the half space $x > 0$ induced by a prescribed boundary excitation at $x = 0$,

$$u(0, t) = A_t \sin(\omega t) \quad \text{and} \quad u(x, 0) = 0 \quad \text{for} \quad t \leq 0$$

(3)

Solutions to Equations (1)–(3) are obtained as

$$u = -\frac{\gamma_t A_t}{2} \left\{ \frac{\omega \rho}{2c_L} \left[ \cos(\omega(t - \frac{x}{c_L})) + \cos(\omega(t + \frac{x}{c_L})) \right] + H(\cos(\omega \frac{x}{c_L}) \sin(\omega \frac{x}{c_L}) - \pi) \left[ \cos(\omega \frac{x}{c_L}) - \cos(\omega \frac{x}{c_L}) \right] \right\}$$

$$+ \frac{\gamma_c A_c}{2} \left\{ \frac{\omega \rho}{2c_L} \left[ \cos(\omega(t - \frac{x}{c_L})) + \cos(\omega(t + \frac{x}{c_L})) \right] - H(\cos(\omega \frac{x}{c_L}) \sin(\omega \frac{x}{c_L}) - \pi) \left[ \cos(\omega \frac{x}{c_L}) + \cos(\omega \frac{x}{c_L}) \right] \right\}$$

(4)

To obtain the amplitude of the second harmonic component in (4), the Fourier transform can be used,

$$a = \frac{\omega}{\pi} \int_0^{2\pi/\omega} u \cos(2\omega t) dt = \frac{(\gamma_t - \gamma_c) A_t \omega}{3\pi c_L} \cos(2\omega \frac{x}{c_L})$$

(5)

$$b = \frac{\omega}{\pi} \int_0^{2\pi/\omega} u \sin(2\omega t) dt = \frac{(\gamma_t - \gamma_c) A_t \omega}{3\pi c_L} \sin(2\omega \frac{x}{c_L})$$

(6)

To obtain the amplitude of the zero-frequency mode component in (4), the Fourier transform can be used again,

$$u_0 = \frac{\omega}{\pi} \int_0^{2\pi/\omega} u dt = \frac{(\gamma_t - \gamma_c) A_c \frac{\omega x}{\pi c_L}}{\pi c_L} + (\gamma_c - \gamma_c) A_c (\pi + \frac{1}{2} \tan(\omega \frac{x}{c_L})) \left[ \cos(\omega \frac{x}{c_L}) - \cos(\omega \frac{x}{c_L}) \right]$$

(7)

Thus, one may write

$$u = u_0 + u_1 + u_2 + \cdots = u_0 + A_t \sin(\omega(t - x/c_L)) + A_c \sin(2\omega(t - x/c_L + \phi_2)) + \cdots$$

(8)

where $\phi_2$ is a phase angle independent of $t$ and $x$,

$$A_2 = \sqrt{a^2 + b^2} = \frac{(\gamma_t - \gamma_c) A_t \omega}{3\pi c_L}$$

(9)

is the amplitude of the second harmonic, and the symbol $\cdots$ indicates higher harmonic terms. Equations (7) and (9) show that $A_2$ ($A_0$) is linearly related to $A_t$. This is different from the second harmonic (zero-frequency mode) generated by quadratic nonlinearity in the solid, where $A_2$ ($A_0$) is proportional to the
square of $A_1$. For simplicity, note that only the first term of Equation (7) are considered here as an approximation when integer number signal periods be received. And the expressions $\beta_0 \propto A_0 / A_1$ and $\beta_2 \propto A_2 / A_1$ can be adopted to measure the bi-linear stiffness crack induced acoustic nonlinearity, where $A_0$, $A_1$ and $A_2$ are the measured amplitude of zero-frequency mode fundamental wave and second harmonic respectively. The amplitude of the zero-frequency mode is about three times larger than that of the second harmonic, and the acoustic nonlinearity parameter $\beta_i = A_i / A_1$, $i \in [0, 2]$ increase linearly with the wave propagation distance ($x$). As indicated in reference[63], the acoustic nonlinearity parameter $\beta$ is scaled by the parameter $\gamma = \gamma_c - \gamma_s$ which represents the tension and compression asymmetry in the elastic modulus. And $\gamma$ is a function of the crack density, frequency and the coefficient of friction. Here we focus on crack density only. Hence, the acoustic nonlinearity parameter $\beta_i = A_i / A_1$, $i \in [0, 2]$ increase linearly with crack density.

2.2.1 Case II: Rough surface contact model

2.2.2 Theory

In this case the stress-strain relationship around the crack can be defined by the relationship:

$$\sigma = E(\epsilon - \frac{1}{2} \beta \epsilon^2)$$  \hspace{1cm} (10)

Where $\beta$ is the dimensionless acoustic nonlinearity parameter. Substituting $\epsilon = \partial u / \partial x$ and Equation (10) into the equation of motion $\partial \sigma / \partial x = \rho \partial^2 u / \partial t^2$ leads to equation:

$$\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = -\beta \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x^2}$$  \hspace{1cm} (11)

where $x$ is the Lagrangian coordinate describing the location of the material particle in the initial ($t = 0$) state. At any given time $t$, the displacement of the particle $x$ from the initial configuration is denoted by $u(x, t)$. Following Qu[37] and consider a half-space defined by $x \geq 0$, and for boundary condition $u(0, t) = U \sin(\omega t)$ and considering $|\beta U k^2 x| \ll 1$, the solution of Equation (11) has the form

$$u(x, t) = U \sin[\omega(t - \frac{x}{c})] + u_0(x, t) + u_2(x, t)$$

$$u_0(x, t) = -\left(\beta U^2 \omega^2 / 8c^2\right)x \quad \text{and} \quad u_2(x, t) = \left(\beta U^2 \omega^2 / 8c^2\right)x \cos\left[2\omega(t - \frac{x}{c})\right]$$  \hspace{1cm} (12)

where $u_0(x, t)$ and $u_2(x, t)$ denote the zero-frequency mode and the second harmonic mode respectively. Equation (12) shows that the amplitude of the zero-frequency mode at a given point is proportional to the distance between this point and the signal excitation point. Although this coefficient of the two wave modes is equal, the energy carried by the zero-frequency mode is two times larger than that of the second harmonic via integral operation. If higher harmonics generated in this process are considered, more energy would flow into the zero-frequency mode while less energy would flow to the second harmonic. Based on the above analysis, we can conclude that the signal intensity of the zero-frequency mode should be stronger than that of the second harmonic. In this paper, we disregard the shape of the propagating zero-frequency mode (static displacement pulse) and mainly focus on the amplitude of the zero-frequency mode. Indicated by Equation (12), the expressions $\beta_0 \propto A_0 / A_1^2$ and $\beta_2 \propto A_2 / A_1^2$ can be adopted to measure the Rough surface contact crack induced acoustic nonlinearity, where $A_0$, $A_1$ and $A_2$ are the measured amplitude of zero-frequency mode fundamental wave and second harmonic respectively.

3. Simulation and model

Here only case I (case II can be classified as weak material nonlinearity) is simulated and the simulation model is the same as the one we established in literature [63]. With the limitation in space, the detail of
the model is not presented here and more detail can be found in literature [63]. The numerical simulations are performed using the finite element method (FEM) for the case of two-dimensional plane strain deformation. The commercial FEM software ABAQUS is used for this purpose. To this end, a two-dimensional FEM model is constructed using the four-node plane strain (CPE4R) elements. The dimension of the simulation cell is $L \times L$. It contains N micro-cracks. For numerical expediency, all the cracks have the same length of 2a. They are randomly distributed spatially, and their orientations are also random. A dimensionless wavenumber $\eta = \omega a / c_l$ is also defined in literature [63].

The material parameters used in the FEM analysis are taken from a typical polycrystalline aluminium. They are $\rho = 2700 \text{kg/m}^3$, $E = 7 \times 10^{10} \text{Pa}$ and $\nu = 0.33$, which yield a longitudinal phase velocity of $c_L = 6198 \text{m/s}$. Linear elastic constitutive law is used in the FEM simulations. In the simulation, a displacement excitation $u(x,t) = A_1 \sin(\omega t)$ is prescribed on the left edge of the $L \times L$ FEM model. Fig. 1 shows the waveform and frequency spectrum of wave propagation in the cracked solid with $\eta = 0.1258$, micro-crack density $c = 0.0125$ and coefficient of friction $\mu = 0.3$. Solid and dashed lines are from the model predictions, while the symbols are from the FEM simulations. It is seen that model predictions yield good agreement with the FEM simulations. And the validation of our model is verified.

![Figure 1. Waveform (a) and frequency spectrum (b) of wave propagation in a cracked solid with $\eta = 0.1258$, $c = 0.0125$ and $\mu = 0.3$.](image)

![Figure 2. Acoustic nonlinearity parameter $\beta_i = A_i / A_1$, $(i = 0, 2)$ versus crack density $c$ (a) and wave propagation distance (b) for $\eta = 0.1258$ and $\mu = 0.3$.](image)

Fig. 2(a) shows the acoustic nonlinearity parameter $\beta_i = A_i / A_1$, $(i = 0, 2)$ versus crack density. It is seen that the acoustic nonlinearity parameter $\beta_i = A_i / A_1$, $(i = 0, 2)$ increases linearly with increasing crack density, as predicted by the micromechanics models. The acoustic nonlinearity parameter of zero-frequency $\beta_0$ is larger than the acoustic nonlinearity parameter of second harmonic $\beta_2$. And $\beta_0$ increases with a larger slope. Fig. 2(b) shows the acoustic nonlinearity parameter versus wave
propagation distance. Similarly, it is seen that the acoustic nonlinearity parameter \( \beta_i = A_i / A_1 \) \((i = 0, 2)\) increases linearly with increasing wave propagation distance, as predicted by the micromechanics models. The acoustic nonlinearity parameter of zero-frequency \( \beta_0 \) is larger than the acoustic nonlinearity parameter of second harmonic \( \beta_2 \). And \( \beta_0 \) increases with a larger slope too. Predictions from micromechanics models agree quite well with the FEM results.

4. Crack localization (longitudinal wave)

4.1 Theory

Figure 3. Two-dimensional 6 mm thick beam model with distributed cracks (6 mm width locating at 90 mm from the left end of the model) used for longitudinal wave propagation model based on Abaqus.

Figure 3 shows the schematic illustration of crack localization using longitudinal wave. The longitudinal wave propagating in an aluminium plate along x-axis direction is excited at the left end of the beam. For simplicity, the plate with thickness 1 mm could be modeled in two dimensions. The model should be long enough (276 mm) to ensure that the received signals are not affected by boundary reflections. The excitation function can be described as \( x(t) = A \sin(2 \pi ft) \), where \( A (=0.0001 \text{ mm}) \) is the amplitude of tone burst, \( f (=1.25 \text{ MHz}) \) is the frequency in MHz, \( t \) is the end time of the loading. The width of our crack zone is 6 mm. The beam model thickness is 6 mm. The left boundary of the crack zone is located with a distance 90 mm from the left end of the model. Six line sensors (S13, S12, S11, S21, S22, S23) are considered with a distance 30 mm, 60 mm, 90 mm, 96 mm, 126 mm and 156 mm from the left end of the model, where the excitation signal actuated, as illustrated in Figure 3 (b).

As illustrated in Figure 3 (a), we can find the crack zone left boundary localization by using the reflected compressional waves. Note that these reflected waves is different from those generated by macro-cracks. The former are mainly composed of zero-frequency fundamental frequency and higher harmonics, and the latter are dominated by fundamental frequency. Set the distance of the crack zone left boundary from the model left end is \( l \), Set the distance of a sensor from the model left end is \( x_i \), then we can write

\[
2l - x_i = v_l t
\]

where \( v_l \) denote the longitudinal wave phase velocity respectively, \( t \) is the signal arriving time. So we have the crack zone left boundary location

\[
l = (v_l t + x_i) / 2
\]

4.2 Simulation Results and Discussion

4.2.1 Zero-frequency mode is sensitive to micro-crack damage

Here, two models are adopted. One model without crack considers the material nonlinearity only (no crack model). Similar to [65], the Landau and Lifshitz model of hyper-elasticity was adopted by a user subroutine VUMAT, as generally used for material definition. The material properties are shown in Table 1. The other model considers material nonlinearity and crack both (crack model).

<table>
<thead>
<tr>
<th>Table 1. Material properties of Al</th>
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<tbody>
<tr>
<td>( \lambda ) (GPa)</td>
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<tr>
<td>2704</td>
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Figure 4. Signal received at S22 (the wave propagation distance is 126 mm), (a) time domain and (b) frequency domain, the amplitude ratio of zero-frequency is 0.1286 and the amplitude ratio of second harmonic is 0.05203

Figure 4 ((a) time domain and (b) frequency domain) shows the received signal when longitudinal wave propagates through 126 mm. As indicated in Figure 4 (b), both zero-frequency and second harmonic are generated at the amplitude ratio of 0.1286 and 0.05203 respectively. We computed the amplitude ratios of the received signals from the five line signal receiving points for these two models, and then we plotted the results in Figure 5 to compare and analyse for convenience. As illustrated in Figure 5, the curve marked by triangle (circle) represents the acoustic nonlinearity parameter versus wave propagation distance for crack model (no crack model). The blue (black) curve denotes the acoustic nonlinearity parameter versus wave propagation distance for the second harmonic (zero-frequency). It can be seen that the acoustic nonlinearity increase with the wave propagation distance for zero-frequency and second harmonic both. The acoustic nonlinearity for zero-frequency is stronger and increases with a larger slope. Besides, the acoustic nonlinearity increases remarkably when wave propagation through the crack zone. Compared with those without cracks, the acoustic nonlinearity indicated by zero-frequency (second harmonic) for crack model have an approximate 300% (200%) increase when wave propagation through the crack zone. Thus, zero-frequency can be used to evaluate micro-crack material damage, or, is capable of characterizing the occurring of crack initiation, and it is more sensitive to micro-crack damage than second harmonic.

Figure 5. Acoustic nonlinearity parameter. $\beta$, a measure of material nonlinearity, plotted as a function of propagation distance for zero-frequency mode ($\beta_0$ the black curve) and the second harmonic ($\beta_2$ the blue curve).

4.2.2 Crack damage localization

Here, in order to study the acoustic nonlinearity induced by crack only, another two models are adopted. Two models consider ideal elastic material properties which are shown in Table 2. One model considers cracks and the other model has no crack. The no crack model is a reference model and the corresponding acoustic nonlinearity is zero. Signal received at a sensor for the crack model is subtracted by the corresponding
Table 2. Material properties of Al

<table>
<thead>
<tr>
<th>ρ (kg/m³)</th>
<th>λ (GPa)</th>
<th>μ (GPa)</th>
<th>A (GPa)</th>
<th>B (GPa)</th>
<th>C (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2704</td>
<td>70.3</td>
<td>26.96</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 6. Signals (subtracted by the signals of the corresponding reference no crack model) received at different wave propagation distance in time domain, a S11 b S12 c S13 d S21 e S22 and f S23 (longitudinal component).

signal received at the same location sensor of the reference model. Figure 6 shows the simulation results. Figure 6 (a) (b) (c) (d) (e) and (f) represent the longitudinal wave signals (subtracted by the reference model) received at S11 S12 S13 S21 S22 and S23 respectively. These results show that the time-domain shape of the zero-frequency generated by a longitudinal wave propagation through crack zone has a flat-top shape. And this phenomena is similar to that found by Qu et al. [36-38]. Note that in their excellent work, zero-frequency is generated by quadratic nonlinearity which consists with the material properties shown in Table 1. Here, the zero-frequency is merely generated by micro-cracks. The first pulse in Figure 6 (a) (b) and (c) are the reflected wave induced by crack zone, and the second pulse in these figures are the wave reflected the second time by the left end. The first pulse in Figure 6 (d) is the crack zone transmission wave, and the second incomplete pulse is induced by the left end. Figure 6 (e) and (f) are contain the crack zone transmission wave only, and the wave reflected by the left end has no time enough to arrive. Here, we focus on the reflected wave and transmission wave caused by crack zone. As mentioned earlier, the flat-top shape is represents zero-frequency. The mixing wave of fundamental wave and the second harmonic hang at the peak of the zero-frequency flat-top shape.

We compute the acoustic nonlinearity parameter of these signals and show them in Table 3. Several conclusions can be drawn. Firstly, the acoustic nonlinearity parameter of reflected wave is much larger than that of the transmission wave. Secondly, the acoustic nonlinearity parameter remain unchanged during wave propagation both for reflected and transmission wave (elastic model). Thirdly, the acoustic nonlinearity parameter of zero-frequency is much larger than that of second harmonic. Note that when we consider the two models adopted in section 4.2.1 (hyper-elastic model with weak material nonlinearity), with the same data processing method, the computing corresponding acoustic nonlinearity parameter are the same as those shown in Table 3.

Table 3. Acoustic nonlinearity parameter for crack (reflected and transmission wave)

<table>
<thead>
<tr>
<th></th>
<th>S11</th>
<th>S12</th>
<th>S13</th>
<th>S21</th>
<th>S22</th>
<th>S23</th>
</tr>
</thead>
<tbody>
<tr>
<td>β₀</td>
<td>13.7292</td>
<td>13.5574</td>
<td>13.7667</td>
<td>0.1274</td>
<td>0.1286</td>
<td>0.1280</td>
</tr>
<tr>
<td>β₂</td>
<td>0.3185</td>
<td>0.3097</td>
<td>0.3190</td>
<td>0.0515</td>
<td>0.05203</td>
<td>0.0534</td>
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As mentioned in section 4.1, the crack zone left boundary can be localized through Equation (14). The distance between the model left end and the sensor $x_i$ can be measured easily. The value of $x_i$ in Figure 6 (a) (b) and (c) is 90 mm 60 mm and 30 mm respectively. The blue dotted line located the estimated time value (estimated through wavefront and marked by the corresponding blue number) and the red dotted line located the theoretical time value (marked by the corresponding red number). These two dotted lines coincide with each other means that this method is effective to locate the left boundary of the crack zone. The corresponding zero-frequency arriving time $t$ is $1.45 \times 10^{-5}$ s, $1.94 \times 10^{-5}$ s and $2.42 \times 10^{-5}$ s respectively. The phase velocity of zero-frequency is equal to the longitudinal wave velocity, so the value of $v_i$ is 6197.8 m/s. Introduce these data into Equation (14), we calculate the location of the crack zone left boundary $l$ and show them in Table 4. It can be seen that the calculation value of $l$ is equal to its exact value for S11 S12 and S13 respectively.

<table>
<thead>
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<th>Table 4. Crack zone left boundary localization.</th>
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<tr>
<td>$l$/mm</td>
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<td>l/mm</td>
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5. Conclusions
The signal of the zero-frequency mode is stronger than that of the traditional nonlinear harmonics for ultrasonic waves. And for Lamb waves, unlike the second harmonic, phase-velocity matching is not required for the zero-frequency mode accumulation [37, 39, 60, 61, 66, 67].

In this paper, we theoretically and numerically demonstrate that zero-frequency mode also can be generated by micro-cracks. The amplitude ratio, i.e., $\beta_i = A_i / A_i$, $(i = 0, 2)$ is defined to measure the bi-linear stiffness crack induced acoustic nonlinearity. The amplitude ratio of zero-frequency is much stronger than that of the second harmonic and is increase linearly with the crack density. Compared with those without cracks, the acoustic nonlinearity indicated by zero-frequency (second harmonic) for crack model have an approximate 300% (200%) increase when wave propagation through the crack zone. The amplitude ratio of zero-frequency and second harmonic generated by the crack remain unchanged during the wave propagation through none crack zone. The crack zone left boundary can be precise localized through reflected longitudinal zero-frequency. Anyway, the zero-frequency mode can be used as an evaluation index for micro-cracks induced material early stage damage. And it is more effective and sensitive than second harmonic.

It is obvious that zero-frequency mode is of great use and application in damage monitoring or nondestructive testing, based on the techniques of nonlinear ultrasonic waves. And further experimental studies are going to be done in our future work.

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