1. Introduction

Structural health monitoring (SHM) is the general process of making an assessment, based on appropriate analyses of in-situ measured data (more generally, features extracted from measured data), about the current ability of a structural component or system to perform its intended design function(s) successfully. Damage prognosis (DP) extends this process by considering how the SHM state assessment, when combined with probabilistic future loading and failure mode models with relevant sources of uncertainty adequately quantified, may be used to forecast remaining useful life (RUL) or similar performance-level variables in a way that facilitates efficient life cycle management. RUL prediction employs physics-based or empirical models to forecast the future state, and fundamental to either model class is the quantification of uncertainty; such uncertainty could emanate from modeling error, measurement error, feature estimation, the operating environment, or other sources.

Fukuzono\cite{1} observed that landslides (where critical earth slippage correlates to “failure” in the current SHM context) were associated with accelerating observations of the inverse rate of change of ground surface velocity. This observation was formalized by Voight\cite{2} into an empirical form known as the Failure Forecast Method (FFM), which has been applied to a number of geophysical and even...
material-level failure mechanisms\cite{3-8}; the universal feature of the model is that the time of “failure” is self-defined by a positive feedback mechanism that leads to (mathematically) an infinite value in the rate-of-change of the observed feature. Fatigue damage is an example of a positive feedback mechanism, as an increase in damage leads to an increase in the rate of damage accumulation; consequently, the fatigue crack growth rate behavior has this characteristic form\cite{8}. Compared to conventional damage assessment methods, the FFM does not rely on assumptions of material properties, geometry, or operating conditions, but rather the observed response of the component. This reduces the number of sources of uncertainty and potentially provides more confident RUL estimates.

This work will conduct a statistical analysis to establish the confidence of fatigue RUL estimates using both an inspection and a more conventional monitoring approach. The analysis is conducted on crack growth data obtained from a fatigue experiment using a standard 316 stainless steel compact tension specimen (Figure 1, left). The values of the variables in Figure 1 (left) are $W=50$ mm, $B=25$ mm, and $a=15.5$ mm, with a max cycle load of 11 kN and a load ratio of 0.1. The crack length is measured using a permanently-installed voltage potential drop system, and these results as a function of load cycle are shown in Figure 1 (right) under continuous measurement vs. periodic NDE inspection. This work will compare the confidence in the RUL predictions made with both methods after the defect is identified. The framework of the analysis can also be used as a tool in real-life applications to assess the confidence in predictions using the rate-based monitoring approach.

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{(left) Specimen geometry; (right) Measured crack growth under two approaches: continuous, in-situ monitoring (blue dots) and periodic inspections (red crosses).}
\end{figure}

\section{2. Monitoring/Inspection Approaches and Comparison Results}

\subsection{2.1 FFM Monitoring Approach}

The FFM model posits that the time rate-of-change $R$ in some feature $\Omega$, e.g., $R=\dot{\Omega}$, obeys the following evolution equation

$$\dot{R} = kR^\alpha,$$

where $k > 0$ and $\alpha > 1$ are constants relevant to the specific physical process. The solution to Eq. (1), assuming that the rate $R$ at the time of failure $t_f$ is $R_f$, is given by

$$R = \left( R_f^{-\alpha} + k(\alpha-1)(t_f-t) \right)^{\frac{1}{1-\alpha}},$$

$$= \left( R_f^{-\alpha} + K(\alpha-1)(N_f-N) \right)^{\frac{1}{1-\alpha}},$$

where time $t$ is transformed to cycle $N$ through a suitable rescaling of $k$. The most common implementation is to consider the inverse of the rate $R$, since this facilitates easier definition of the
failure criterion, i.e., the inverse rate tends to zero or something finite near zero (the rate itself tends to a very large number, or infinity) at the time of failure. Defining the inverse rate $P = R^{-1}$, the solution Eq. (2) could be written

$$\frac{P^{\alpha-1} - P^{\alpha-1}_t}{\alpha - 1} = KN_f - KN.$$  

(3)

If $\alpha$ and $k$ are not known, a maximum likelihood estimation technique could be used\(^9\). In this current work, we will exploit the observation that, based on historical observations in many of the cited studies above regardless of the physical process, $\alpha = 2$, so that Eq. (3) is a simple linear regression; in this case, the regression coefficients obtained from a time/data linear regression (over some given window of time) are given by $\beta_0 = KN_f$ (intercept) and $\beta_1 = -k$ (slope) such that the regression-estimated time to failure is $\hat{N}_f = -\beta_0 / \beta_1$, the negative of the ratio of the intercept to the slope. The “idealized, mathematical” target failure criterion is that $P_f = 0$; by setting it to any positive non-zero amount, a degree of conservatism is introduced into the approach. Of course, in most practical applications, “failure” occurs at a point prior to an infinite data rate-of-change observation, but to be consistent with general implementation in the literature and for the purposes of parametric studies in this paper, we will employ $P_f = 0$ as the failure criterion, which won’t change the basic nature of this study.

Any given regression on a data set represents a “single block” observation over some time interval, which is presumed representative of an ensemble population of regressions over the same time frame under inevitable noise and uncertainty in the data. Thus, the regression coefficient estimates $\hat{P}_0, \hat{P}_1$ are estimates from populations of regression coefficients. For a given linear regression model $\mathbf{P} = \mathbf{T} \hat{\mathbf{P}}$, where $\mathbf{P}$ is the data, $\mathbf{T}$ is the design matrix, and $\mathbf{e}$ is the regression error, it is assumed that the regression process yields errors that are unbiased, uncorrelated Gaussian $\mathbf{e} = N(0, \sigma^{-1})$ under typical central limit theorem assumptions (regardless of the distribution in the rate data, $\mathbf{P}$). Thus, it is known that the regression coefficients themselves have jointly normal distributions $\hat{\mathbf{P}} = N(\hat{\mathbf{P}}, \sigma^2(\mathbf{T}^* \mathbf{T})^{-1})$, $j=0,1$. An unbiased estimate of the population error variance is $\sigma^2 = \|\mathbf{P} - \hat{\mathbf{P}}\| / (n-2)$, where $n$ is the number of data points used in the regression design.

Since our RUL estimate from the regression is given by $\hat{N}_f = -\hat{P}_0 / \hat{P}_1$ (the ratio of our two regression coefficients), a straightforward application of the law of probabilities under change-of-variables leads to a probability density function (PDF) in the RUL as

$$p(\hat{N}_f) = \frac{\sqrt{1 - \rho^2} \sigma_\theta \sigma_\hat{N}_f}{\pi \left( \sigma_1^2 + 2 \rho \sigma_1 \sigma_\hat{N}_f + \sigma_\hat{N}_f^2 \right)} e^{-\frac{(\rho \sigma_\theta + \sigma_\hat{N}_f)^2}{2(1 - \rho^2)}} \left[ e^{\frac{-2 \rho \sigma_\theta \sigma_\hat{N}_f}{\sqrt{1 - 2 \rho^2} \sigma_\theta}} \left( \mu, \sigma, \rho \sigma_\theta, \sigma_\hat{N}_f \right) - \mu, \sigma, \rho \sigma_\theta, \sigma_\hat{N}_f \right]$$

$$+ \frac{\sqrt{2 \pi} \left( \sigma_1^2 + 2 \rho \sigma_1 \sigma_\hat{N}_f + \sigma_\hat{N}_f^2 \right)^{1/2}}{\sqrt{2 \pi} \left( \sigma_1^2 + 2 \rho \sigma_1 \sigma_\hat{N}_f + \sigma_\hat{N}_f^2 \right)} e^{\frac{-2 \rho \sigma_1 \sigma_\hat{N}_f}{\sqrt{1 - 2 \rho^2} \sigma_1}} \left( \mu, \sigma, \rho \sigma_1, \sigma_\hat{N}_f \right)$$

(4)
where erf(*) is the error function, \( \mu_j = \hat{\beta}_j \), \( \sigma_j = \sqrt{\left( P - \tilde{\beta} \right)^\intercal \left( T^\intercal T \right)^{-1}_{j,j} / (n-2) } \), and \( \rho = \left\| P - \tilde{\beta} \right\|^2 \left( T^\intercal T \right)^{-1}_{12} / (\sigma_0 \sigma_1 (n-2)) \) for \( j=0,1 \); the double subscript “12” refers to the row-column selection of the subscripted matrix. Theoretically, since the exact population \( \mu_j \) and \( \sigma_j \) are not known a priori and must be estimated from the data, a sampling distribution for the ratio mean and standard deviation should be derived, but Eq. (4) is a suitable model. It should be noted, however, that the PDF in Eq (4) has no analytically-calculable order statistics, since the tails are too “fat”\cite{10}. However, histograms of Monte Carlo-generated time-of-failure data were compared to Eq. (4) to verify that the distribution of the data is appropriately modeled such that estimating order statistics from the data itself is reasonable\cite{11}.

Thus, the FFM makes an updated prediction every time a block of rate data are obtained. The rate of change in the resistance measurement from the potential drop measurement system is calculated to perform the FFM without converting to crack length measurements as with typical analysis of potential drop measurement results. This is obtained from the slope of the linear regression fit performed on every 5 resistance measurements. The inverse of the rate of change in resistance is then calculated and linear regression is performed on Equation (3) to obtain \( N_f \). Figure 2 shows the results at four different cycle counts (100,000 cycles apart). Data within the red lines are used for a given regression. The dotted black line indicates the actual failure cycle, and the intersection of the solid blue line with the horizontal axis corresponds to the predicted failure cycle.

Figure 2. RUL estimates using the FFM at intervals of 100,000 cycles. The red line indicates the window of inverse rate data used in the regression, the intercept of the blue line with the horizontal axis is the predicted failure cycle, and the dashed black line is the actual failure cycle.
Figure 3 (left) shows the corresponding generated PDFs for $N_f$ at the same 4 cycle intervals as in Figure 2; they represent the transformation (e.g., Equation (4)) of the feature distribution to the failure cycle distribution. One can see the convergence in both accuracy (central tendencies) and reduced dispersion (variance/spread) in time. Even though only 4 such snapshots are shown in Figure 2 and Figure 3 (left), Figure 3 (right) shows the more continuous prediction of the failure cycle every 100 cycles (using the median of the corresponding PDFs as the predictor), which was the minimum amount of new data used to make an updated prediction\textsuperscript{[11]}.

**Figure 3.** (left) The evolution of the PDF of $N_f$ at the same four cycle intervals as Figure 2; (right) The median of the PDF of $N_f$ every 100 cycles as fatigue progresses.

### 2.2 Conventional Inspection Approach

Conventional fatigue-based RUL approaches typically rely on using empirical crack growth laws (assuming sufficient information on operating conditions, material properties, and geometry) after a defect is detected using some appropriate NDE technique. Paris’ Law\textsuperscript{[12]} is the most common of such laws, and integrating Paris’ equation from the initial crack size $a_0$ to the critical crack size $a_f$ gives

$$ N_f = C \left[ \frac{\Delta \sigma(a)}{Y(a)} \right]^{\frac{m}{2}} a_f, $$

where $C$ and $m$ are material constants, $\Delta \sigma$ is the stress range, and $Y(a)$ is a crack-size dependent geometric effect function. Any or all of the parameters in Equation (5) potentially contain sources of uncertainty or error, but only some will be considered in this work. The NDE inspection process itself primarily governs the uncertainty in the crack size $a$ (from initial detection size $a_0$ to present size $a$), and in this work, it is assumed that the NDE technique error is normally distributed with no bias error and a 1 mm standard deviation. Of course, these values greatly depend on the specific NDE architecture and implementation strategy (including calibration error), but this assumption is generally achievable within the current state of the art\textsuperscript{[13]}. The critical crack length $a_f$ is usually estimated using linear elastic fracture mechanics (38 mm in this experiment), and its uncertainty is ignored in the present study. Uncertainty in the material constants is usually high, and a UK standard was used in establishing an uncertainty model for $C$ (normal with $E[\log C]=-25.5$, $\sigma[\log C]=0.264$), but ignoring uncertainty in $m$ ($E[m]=2.88$)\textsuperscript{[14]}. The geometry function and load ratio were also estimated from standards or specified with no uncertainty. The loading uncertainty is modelled as normally-distributed with a 10% of maximum load (3.5 kN) standard error. No parameters are assumed to have correlated uncertainty, but for the purposes of simulating an inspection process to compare it to the FFM uncertainty, these simplifications are sufficient.

The uncertainty models just described were appropriately sampled, and 10,000 trials of a Monte Carlo simulation were conducted to estimate the failure prediction histograms in accordance with Equation
A lognormal distribution was found to fit the histograms very well over time and was used for calculating order statistics.

### 2.3 Comparison of Accuracy and Confidence in the Approaches

A comparison of the two approaches is summarized in Figure 4, where the PDFs are shown at the $10^5$ inspection cycle intervals over which the inspection method was performed. The median predictor from the PDFs are compared in Figure 5 as a function of all available prediction cycles, which, for the FFM approach, is every 100 cycles rather than every $10^5$ cycles. The figures together show how much more quickly the FFM RUL estimates converge to the actual failure cycle; from about 200,000 cycles onward, the FFM prediction is always within 10% of the actual failure cycle. Since the FFM method is conditioned only upon measured data (rather than geometric, material, and loading knowledge, which are required for the inspection process to be converted into prognosis), which presumably should capture all relevant observable variability, the confidence in the RUL is substantially better. As an example, consider RUL prediction at $N=2\times10^5$; assuming a required $3\sigma$ confidence in the integrity of the component (99.7%), the conservative RUL estimation using the inspection and FFM approach would be $N_f=2.8\times10^5$ and $N_f=3.5\times10^5$, respectively. At this same point in time, the estimated median RUL would be $0.8\times10^5$ cycles for the inspection approach and $1.5\times10^5$ cycles for the FFM approach. Thus, since the inspection is only performed every $10^5$ cycles, the component would fail to meet the required threshold of confidence before the next inspection. The conservative RUL made with the FFM approach is much closer to the actual failure cycle. One might argue, confidently, that it would be possible to operate this component safely much more closely to its actual failure time, which translates into operational and economic advantage. Of course, while some of the parameters in this study (such as inspection interval, target confidence, and modelling of the inspection process) are arbitrary, the higher frequency of updates provided by continuous SHM monitoring in the FFM approach clearly demonstrate the improved confidence and accuracy that such a monitoring strategy brings.

![Figure 4. A comparison of inspection (red) vs. continuous monitoring (blue) PDFs at four different times (commensurate with when the inspections were performed). The actual failure occurred at the time indicated by the dashed black line.](image)

Figure 4. A comparison of inspection (red) vs. continuous monitoring (blue) PDFs at four different times (commensurate with when the inspections were performed). The actual failure occurred at the time indicated by the dashed black line.
Figure 5. A comparison of median RUL estimates (from the PDFs) from the inspection (red) vs. continuous monitoring (blue).

Despite these advantages, the choice of $\alpha=2$ for the FFM, implying a linear rate/time relationship, is really most appropriate for Stage II crack growth\cite{15}. Consider Figure 6, where different fractions of the total component life cycle were used to fit both Equation (3), which is the full nonlinear model, and a linearized version of Equation (3), where $\alpha=2$. The linearized model assumed by the FFM implies fatigue accumulation is linear, which is truly primarily only in Stage II crack growth; as the data used to fit these models approaches the end of fatigue life, one may see bigger deviations from the FFM fit and the full Equation (3) fit. In fact, the best fits in the final two sub-plots of Figure 6 yielded $\alpha=2.18$ and $\alpha=2.24$, respectively, while a global fit of all the data yielded $\alpha=2.07$. This results in some potential systematic bias in the predictions using the traditional FFM approach, but since the component in this test spent most of its fatigue life in Stage II, this bias is not detrimentally significant. The bias is present in the results of Figure 5, which shows a consistent non-conservative estimation of the failure cycle beginning around $2\times10^5$ cycles. Of course, the overestimation using the traditional inspection process is much worse (beyond just the greater uncertainty discussed above), but the FFM method also induces bias, which implies that non-stationarity in $\alpha$ should be considered in augmenting the FFM, informed by models that also incorporate Stage I and Stage III growth. To this end, Bayesian parameter estimation strategies could be employed to provide continuous parameter updating (including uncertainty bounds in the parameter estimation) to track the evolving crack growth and possibly inform growth stage characterization.

3. Conclusions

This paper compared a continuous monitoring strategy (implemented by the FFM) with a periodic inspection strategy (implemented in conjunction with Paris’ Law), within the context of predicting a component’s fatigue failure under cyclic loading. Data were fundamentally derived from a voltage potential drop measurement system. The RUL estimates from the continuous monitoring/FFM approach yielded both more accurate predictions as well as substantially reduced uncertainty (increased confidence). This increased confidence is primarily due to the controlled sources of uncertainty in the continuous monitoring/FFM approach, where the only real source stems from the data itself. The inspection RUL estimates require geometric, material, and other information to inform
the prediction process, all of which have uncertainty or error that aggregate to less confidence in final predictions.

It is observed, however, that the monitoring/FFM approach does introduce bias into the estimates, which is likely attributable, as evidenced, to its fundamental assumption of Stage II crack growth. Future efforts will be directed at using model updating/parameter estimation strategies to cope with changing crack evolution (and external environmental influences). It should also be noted that in this study, the loading was generally constant/stationary, and a different strategy that introduces load knowledge would otherwise have to augment the FFM method in that case. Nonetheless, the work strongly suggests that a continuous monitoring strategy, using a permanently installed sensor network and minimal-information analysis such as FFM, provides potentially great opportunities in advancing SHM diagnostics into meaningful prognostics.

**Figure 6.** Best-fit linear (red) and full Eq. (3) (blue) models to different segments of data, from full life cycle (top left) to near end-of-life (bottom right). Each sub-figure has the fraction of the total life used to make the model fits.
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