Proc. Vth International Workshop, Advances in Signal Processing for Non Destructive Evaluation of Materials
Québec City (Canada),2-4 Aug. 2005. © X. Maldague ed., É. du CAO (2006), ISBN 2-9809199-0-X

Pulsed eddy current empirical modeling

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Abstract

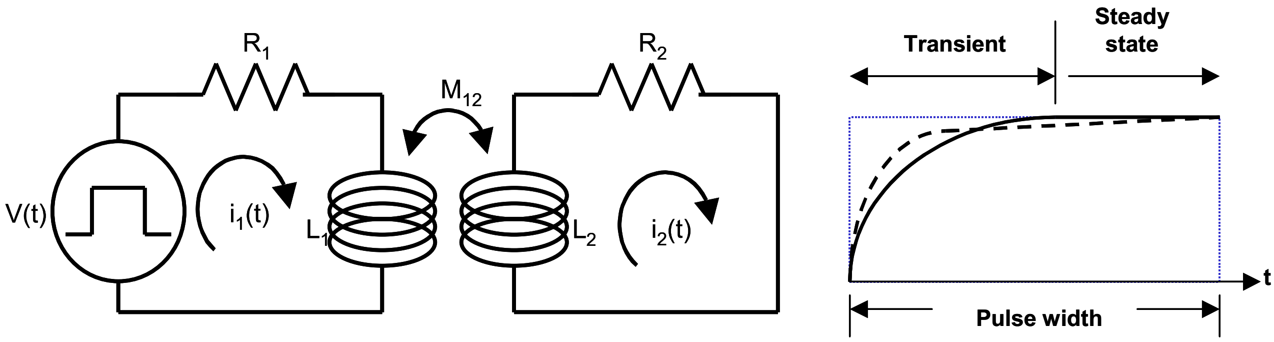
The pulsed eddy current responses to varying material thickness and conductivity are modeled using an electrical circuit ideal transformer non-linear model. The Levenberg-Marquardt algorithm is used to curve fit the experimental signal to the model. The resulting synthetic signal is examined for its ability to preserve experimental signal features such as lift-off point of intersection (LOI), a pulsed eddy current signal feature used successfully in NDE of corrosion, cracks, thickness and conductivity measurements. The method is tested on specimens for its ability to reproduce inspection images using synthetic signals in lieu of experimental signals. This procedure conserves data storage space and allows rapid analysis of images using equations.

Keywords: pulsed eddy current, Levenberg-Marquardt, bootstrap, curve fitting, lift-off

1. Introduction

The signal obtained during conventional eddy current testing ranges from simple curves in the impedance plane diagram to more complex lemniscates. The obtained signal depends on the type of transducer used, the material under inspection, its discontinuities, and the frequency of excitation. There have been many attempts to model conventional eddy current signal patterns using Fourier descriptors [1,2,3]. The objective was to characterize the signal patterns with a few parameters so that signal storage space could be reduced. Some issues were (1) the ability to reproduce the original signal with high fidelity, (2) using a set of parameters capable of providing an intuitive understanding of the physical phenomena, (3) a different and unique set of parameters for different discontinuities, and (4) the ability to allow automatic discontinuity recognition [1]. Dodd and Deeds explored the equivalent work for pulsed eddy current within a general theoretical framework for polynomial approximation [4,5]. This multi-parameter polynomial method allowed material property recognition with good accuracy, providing the proper points were selected in the experimental signal. The ability to reproduce the signal with minimal data storage and high fidelity was not explored. The uniqueness of parameters to the material condition was also not determined.

In this study, the non-linear response of a single absolute coil transducer is fitted to an electrical circuit equivalent model *(figure 1.a)*. The resulting model is then used to simulate pulsed eddy current inspection of simple structures. The Levenberg-Marquardt (LM) curve-fitting algorithm for non-linear models is used [7,8]. The quality of the curve fitting is verified by means of visual observation and residuals comparison of the experimental curve to the synthetic curve generated using the parameters. The parameters' stability to small changes in the data due to noise is tested using a bootstrap algorithm. The model's ability to reproduce with high fidelity the experimental signal is further tested by comparing the lift-off point of intersection (LOI) obtained experimentally and synthetically under various test conditions. It is a requirement of this work that the LOI be retained as it is a pulsed eddy current feature used for evaluation of corrosion, for cracks, and for multi-layer structures inspection [6]. The method is lastly verified by comparing a test article image constructed with experimental signals to that of an image constructed with synthetic data.



(1.a) (1.b)

Figure 1. Circuit equivalent model. (1.a) The transducer is represented by a resistance (R_1) and an inductance (L_1). It is coupled by a mutual inductance (M_{12}) to the specimen represented by a resistance (R_2) and an inductance (L_2). (1.b) Signal when exposed to air (full line) and signal when exposed to a conductor (dotted line).

2. Non-linear curve fitting algorithm

A recognized non-linear least-square fitting method is the LM algorithm [7]. This algorithm uses information in the gradient and Hessian matrices to find an approximate distance and direction to the nearest minimum from the starting values [8]. The LM algorithm requires (i) a model, represented by an equation with n parameters, to which to fit the experimental data, (ii) the n partial derivatives with respect to the n parameters, and (iii) adequate initial guess values. This provides a problem with $n+1$ equations to find n parameters.

2.1 Circuit equivalent model. A circuit equivalent model commonly used to represent the physical eddy current test is the ideal transformer [9]. It provides a first order approximation capable of describing the signal response behaviour. *Figure (1.a)* represents the ideal case under a pulsed excitation, $V(t)$. A resistance (R_1) in series with an inductance (L_1) represents a single coil transducer electrical properties. A resistance (R_2) in series with an inductance (L_2) represents the specimen's electrical properties. The transducer and the specimen are coupled by the mutual inductance (M_{12}) a function of the coupling factor (k). Under this arrangement the current in the transducer, $i_1(t)$, will vary with varying test article conditions. A set of coupled differential equations (1) represents this situation. If the transducer is exposed to air (a non-conductor) equation (1) is simplified by equating R_2, L_2, and M_{12} to zero. The solutions to equations (1) when the transducer is exposed to air or a conductor can be solved using Laplace transforms, the solutions of which are widely available in literature [10].

$$
\begin{aligned}
V(t) &= R_1\, i_1(t) + L_1 \frac{di_1(t)}{dt} - M_{12} \frac{di_2(t)}{dt} \\
0 &= R_2\, i_2(t) + L_2 \frac{di_2(t)}{dt} - M_{12} \frac{di_1(t)}{dt} \\
M_{12} &= L_1 L_2 k^2
\end{aligned}
\qquad (1)
$$

In the case of the transducer in air, the solution for the current in the transducer has the form given by equation (2). In the case of the transducer near a conductor, the solution for the current in the transducer has the form given by equation (3). Given that the current in the transducer is measured indirectly as a voltage across a resistance, or as a voltage through a voltage-follower, and that the entire test set-up electrical properties are not taken into account, the precise formulations in term of resistance and inductance for the parameters *A*, *a*, *B*, *b*, *C*, *c* or *D* are not of interest. Only the general shape of the equations and number of parameters in (2) and (3) are of interest.

$$e(t) = A - Ae^{at}\,,\; a = \ln(1 - e(t)/A)\Big/ t\,. \tag{2}$$

$$f(t) = Be^{bt} + Ce^{ct} + D \tag{3}$$

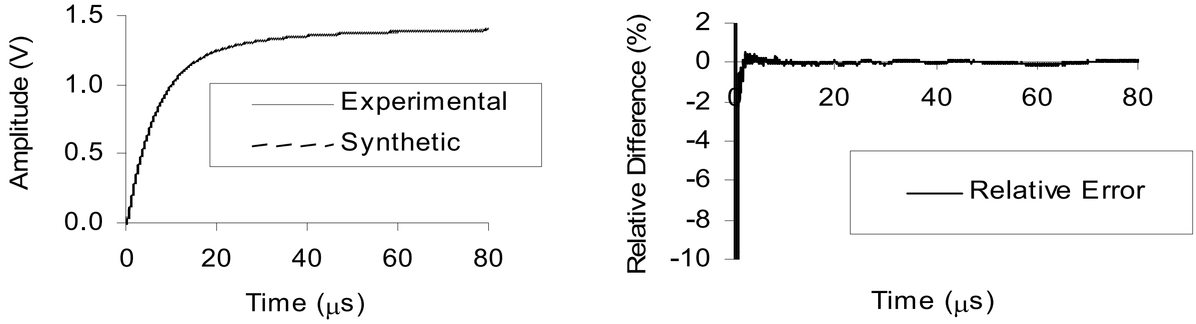
2.2 Associated differential equations. Differentiating equations (3), with respect to the parameters, for the case of the transducer exposed to a conductor provides equations (4). Equations (3) and (4) form the complete set of equations required by the LM algorithm.

$$\begin{array}{lll} \partial f(t)\big/\partial B = e^{bt} & \partial f(t)\big/\partial b = Bte^{bt} & \partial f(t)\big/\partial C = e^{ct} \\ \partial f(t)\big/\partial c = Cte^{ct} & \partial f(t)\big/\partial D = 1 & \end{array} \tag{4}$$

2.3 Initial guess values. With the LM algorithm, choosing adequate initial guess values is a crucial constraint. The starting parameter values need to be as close as possible to final parameters values in order to avoid being trapped in a local minimum or not converging to a solution. This is especially critical when the number of parameters to be fitted is high [7,8]. In this study, two sets of initial guess values are provided. The first set of initial guess values is provided by using the parameters *A* and *a* of the transducer signal in air. The subsequent sets of initial guess values are provided using the parameters *B, b, C, c* and *D* found in a previous curve fitting of the transducer response when exposed to the test article. The rational for this is as follows. It is observed experimentally that the transducer's response when exposed to air is very similar to that of a transducer exposed to a conductive test article *(figure 1.b)*. Yet, equation (2) representing the air exposure situation is relatively simple and its parameters *A* and *a* can be easily estimated. Parameter *A* is estimated as the mean steady state value of the signal. Once parameter *A* is available, equation (2) can be linearized and the value of parameter *a* is found by least-square approximation. The value estimated for *A* as amplitude is then used as the first initial guess for *B*, *C*, and *D* amplitudes. The value of the exponent *a* is used for the exponents *b* and *c* as the first initial guess values in the LM algorithm. The second stage for providing close initial guess values involves simply using the parameters *B*, *b*, *C*, *c*, and *D* from a previous curve fitting. The reason for this being that the parameters found from the previous curve fitting process would always be very close to the new parameters. This allows automatic and rapid curve fitting for a C-scanned specimen. The curve fitting process described above was tried using previously published results with a single coil probe [6]. Sets of titanium (3.5 percent of International Annealed Copper Standard, %IACS), brass (26.5 %IACS), aluminum (59.5 %IACS) and copper (100.1 %IACS) shims and plates of various thickness were used to provide a wide spectrum of thickness and conductivity effects on the probe's response. Conductivity standards (1, 3, 9, 29, 32, 38, 42, 48, 59, 87 and 100 %IACS) were also used to evaluate the effect of conductivity only. The response at design lift-off and three different applied lift-offs were recorded for all test conditions. This experiment provided approximately 130 single coil transducer responses to test the curve fitting process.

3. Results

3.1 Visual and Residuals. A typical direct visual comparison of the experimental responses and the synthetically generated curves is shown in *figure 2.a.* Both curves overlap with excellent fit. However, important, yet minute, differences can still be present but hidden by the scale used. By showing the difference between the experimental and synthetic curves in relative percentage terms, these minute differences are enhanced. The relative residuals for the typical curves are shown in *figure 3.b*. The relative difference between the experimental curve and the synthetic curve did not exceed 0.3% in most regions of the curve. The exceptions always being relatively large discrepancies during the first 2 to 3 µs that represent about 4% of the recorded response. This is discussed later in section 3.5. Overall, the fit between experimental and synthetic curves proved to be excellent.



(2.a) (2.b)

Figure 2. Typical curve fitting results. (2.a) Overlapping experimental and synthetic curves, (2.b) relative difference between experimental and synthetic curves.

3.2 Stability. One issue of concern is the stability of the parameters under varying noise. If small changes in noise induce high variations in parameter values then the parameters may not be reproducible and pattern recognition impossible. To test the parameter stability, it was decided to use a bootstrap algorithm to generate synthetic sets of data from which new synthetic parameters would be obtained [7]. The bootstrap was performed 20 times to obtain a mean parameter and a standard deviation. The curve fitting process was deemed stable if (i) the bootstrap standard deviation was within ± 5% of the mean bootstrap parameter found and if (ii) the mean bootstrap parameter agreed within ± 5% of the original curve fitting parameters found. All parameters were compared against these two criteria. Adherence to the first criterion was evaluated by taking the ratio of the bootstrap standard deviation to its bootstrap mean parameter. Adherence to the second criterion was evaluated using the ratio of the original curve fitting parameter value to the bootstrap parameter value. This was repeated for all parameters and for all signals that were curve fitted. To ease the evaluation, all ratios obtained were plotted in *figure 3*, where the semi-circle represents the ± 5% criteria. It is to be noted that most points fall in a small cluster well inside the semi-cercle. With a few exceptions, the curve fitting process proved to be very stable showing only small variations in the parameter values when subjected to varying noise. This is indicative of a very stable curve fitting process. Again exceptions are observed and are explained in section 3.5.

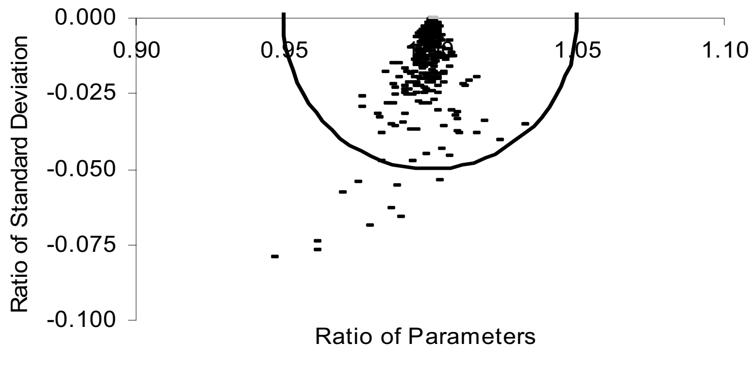
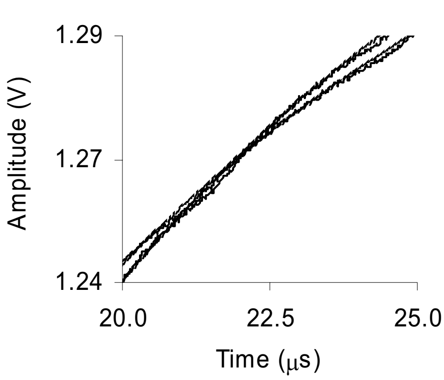
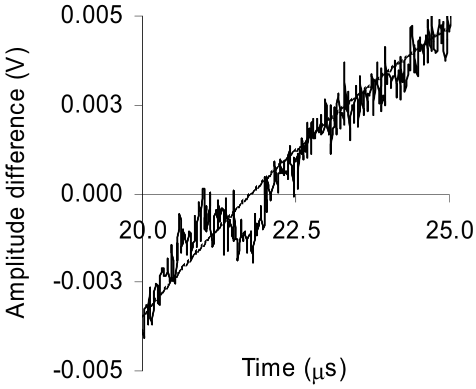


Figure 3. Parameter Stability

3.3 LOI feature. The LOI is a feature used to provide an evaluation independent of lift-off in pulse eddy current. Therefore, it is very important to preserve it in the curve fitting process. *Figure 4.a* shows two typical experimental signals taken at different lift-offs but identical test article condition. Their synthetic curves are also shown. It can be seen intuitively from *figure 4.a*, that the LOI feature is preserved in that all four signals cross each other at the same location. This can be amplified by subtracting the experimental signals one from another and likewise for the synthetic curves. *Figure 4.b* shows that the differences provide the same zero amplitude crossing time near 22 μs. The ability of the curve fitting process to preserve the LOI time-amplitude coordinates was verified for all test conditions and good agreement was obtained.



(4.a)

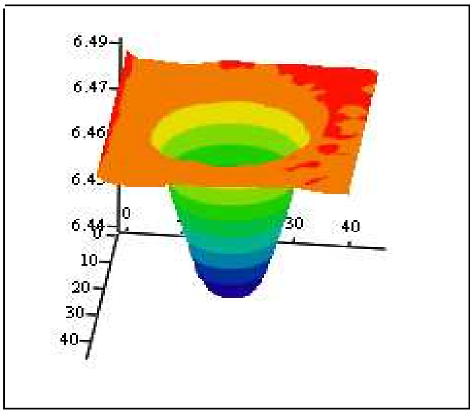


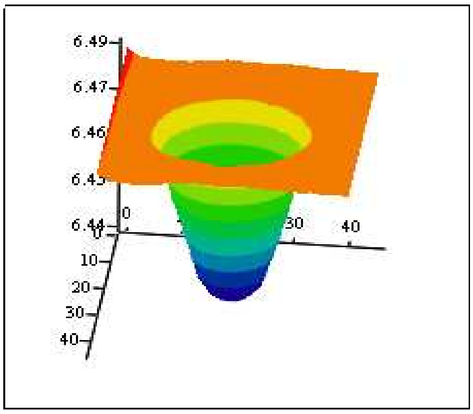
(4.b)

Figure 4. LOI feature preservation. (4.a) close-up view of two experimental signals with varying lift-off only, (4.b) close-up view of the difference between the two signal showing that experimental and synthetic curve subtraction provide the same LOI time.

3.4 Imaging. The curve fitting process was also used tested by comparing an experimental image and a synthetic image. The data was obtained by scanning a 1.025 mm thick aluminum specimen with a 10 mm radius circularly-shaped bottom side material loss of 35%. The top surface was covered with varying layers of non-conducting tape to simulate varying lift-off effects of 0 mm, 0.115 mm and 0.230 mm. The experimental and synthetic amplitudes results at LOI time are shown side by side in *figure 5*. It can be seen that there is no loss of information from the synthetic data.

3.5 Limitations. In sections 3.1, 3.2 and 3.3 exceptions to the general good fit of the process used here were noted. For all signals, the first 2 to 3 μs showed some discrepancy between the experimental data and the synthetic curve. The problem may be due to an impedance mismatch and ringing or the trigger delay between the start of the signal recording and the pulse generation not being fully accounted for. However the early discrepancy did not affect the overall results. The parameter stability and LOI feature preservation also showed some problem areas. Under close examination, the exceptions to the stability and LOI feature preservation were confined to a few samples. It was noticed that for very thin samples or not so thin very low conductivity samples, the experimental signal varied little with varying lift-off or even from the transducer response when exposed to air. Furthermore, noise levels prevented differentiation of some test article variations. For those specimens, the transducer was operating at or beyond its own limitations. Under those circumstances, the curve fitting process still provided an overall good fit but the parameters became unstable.





(X , Y, Experimental) (X , Y, Synthetic)

(5.a) (5.b)

Figure 5: Image reconstruction. The experimental image (5.a) information is preserved in the synthetic image (5.b).

4. Conclusion and future work

A simple process to curve-fit pulsed eddy current signals using the Levenberg-Marquardt algorithm and the ideal transformer model was presented here. The process is capable of providing an excellent fit between experimental and synthetic curves. The curve fitting process is shown to provide stable parameters under noise variation. It has also the ability to preserve key signal features such as the LOI time-amplitude coordinates and the LOI behaviour. The curve fitting process is, however, limited to the operating regime of the transducer. Future work will consist of adapting and testing the curve fitting process with other models of transducer. The synthetic results will also be explored to extract more information from the signal.

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