Pulsed IR Thermography applied to a two-layer system

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Abstract. Pulsed Infrared Thermography is a tool utilized more and more to measure thermal parameters. Sharing the same basic principles of Photothermal Techniques it extends its applicability thanks to its imaging capabilities. A data reduction transforms temperature data as a function of time in a log-log space whose 2\text{nd} logarithmic derivative presents extrema that allow estimating diffusivity or thickness of a slab and of a two-layer system as well.

1 Introduction

Infrared Thermography (IRT) is a tool utilized more and more to measure thermal parameters and/or the geometry of defects inside materials or in their boundaries [1,2]. Possible fields of applications are: buildings for energetic evaluations [3,4], material science to evaluate ageing under stressing conditions [5], corrosion in pipelines [6].

Sharing the same basic principles of Photothermal Techniques (PT), IRT extends the IR Radiometry applications thanks to its imaging capabilities. Contrary to PT community, where thermal waves are widely used to actively stimulates materials under test, in IRT community pulsed techniques are dominant. The classical pulsed PT to measure thermal diffusivity of a homogeneous slab is the Laser Flash technique originated by Parker's work [7]. The ratio of thermal diffusivity by the squared thickness of the plate can be estimated either by measuring times of specific events in the thermal diffusion process after the pulse, or by fitting data by the analytical model of the thermal process. More recently, a data reduction technique of pulsed IRT data has been introduced [8], that transforms temperature data as a function of time in a log-log space. With this representation, data can be fitted by simple polynomial functions describing the deviation from a straight line of -0.5 slope, that is the behaviour of a semi-infinite body. It has been recognized that the 2\text{nd} logarithmic derivative of the log-log data presents a maximum in correspondence of a time that allows estimating the ratio between diffusivity and the squared thickness of the slab [9, 10]. The log-log representation has been further applied in the case of a two-layer system [11].

2 Flash technique in reflection mode

The classical experimental configuration for the Laser Flash technique [7] is done in the so called transmission mode, i.e., one side of a slab is heated by a short pulse of light while the temperature rise is measured on the other side. In some experimental situations the back side is not accessible, e.g. on in-situ experiments of pipes or in many cases when thermal NDT must be carried out, and therefore the reflection mode must be utilized. The solution of the heat conduction problem on one side of a slab of thickness \(L\), heated by a pulse and considering negligible the exchange with the environment is given by the following equation:

\[
T(t) - T_0 = T_\infty \left[ 1 + 2 \sum_{n=1}^{\infty} e^{-n^2 \alpha t/F_0} \right]
\]

where \(T_\infty = Q/(\rho c L)\) (being \(Q\) the energy of the pulse, \(\rho\) the density and \(c\) the specific heat), \(T_0\) the initial temperature and \(F_0 = \alpha t/L^2\) the Fourier number (\(\alpha\) is the thermal diffusivity and \(t\) is time). Figure 1a shows the experimental scheme in reflection mode, while in Fig. 1b the temperature behaviour of the front and back (rear) side of the slab is represented.
2.1 Theta functional relation

It is worth noting how eq. (1) can be transformed in a perfectly equivalent form by means of the functional relation of the theta function [12,13]:

$$\theta(x) = \frac{1}{\sqrt{x}} \theta \left( \frac{1}{x} \right)$$

Equation (2) transforms eq. (1) in the following:

$$T(t) - T_0 = T_a \frac{1}{\sqrt{\pi \tau_F}} \left[ 1 + 2 \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-n^2} \right]$$

where $x = \pi / \tau_F$. One can recognize that the first term of eq. (3), before the square brackets, represents the solution of the semi-infinite body heated by a pulse. The function inside the square brackets modulates the behaviour of the semi-infinite body by forcing the solution to be that of a slab. The function in square brackets approaches asymptotically $\sqrt{\pi \tau_F}$ as $\tau_F \to \infty$, i.e. $t \to \infty$, leading $T(t) \to T_\infty$, as in eq. (1). Figure 2a and 2b shows the behaviour of the two functions composing eq. (3), assuming $T_\infty = 1$. 

Figure 2a: equation (3) is plotted (continuous line) together with the two factor functions: the one representing the semi-infinite body solution (dash-dot line) and the infinite series representing a modulation of the semi-infinite behaviour (dashed line).

Figure 2b: same as in figure 2a but represented in log-log scale. The point where $T_\infty = 1$ (in general $T_\infty = T_a$) corresponds to $\tau_F = 1/\pi$. That corresponds also to the maximum of the second derivative of eq. (3) in log-log scale.
In Fig. 2b, where a log-log representation is used, the noticeable point \( F_0 = \frac{1}{\pi} \) is displayed, that allows to evaluate the thermal diffusivity. This point is more easily identified by looking at the maximum of the \( 2_{nd} \) logarithmic derivative of eq. (3) [9]. That can be easily computed once a suitable polynomial fitting of the logarithm of temperature vs. the logarithm of time has been computed [8] on the experimental data.

### 2.2 Estimation from experimental data

Figure 3a shows the experimental data obtained on a slab of AISI 304 heated on one side by a laser pulse of duration 1 ms. The temperature is obtained by recording a sequence of images with a thermographic camera with a sampling frequency of 500 Hz. Superimposed on the same figure is the fitting function obtained by non-linear least squares fitting of the data [4]. In Fig. 3b the \( 1_{st} \) and \( 2_{nd} \) logarithmic derivative is represented. It is obtained after fitting the experimental data with a polynomial function of \( 9_{th} \) degree. The maximum of the \( 2_{nd} \) derivative identify a special value of time that allows determining the ratio between thermal diffusivity and the squared thickness of the slab. The results in the two cases of non-linear least squares fitting and identification of the \( 2_{nd} \) derivative of the log-log data is presented in Table 1. The estimation of thermal diffusivity is obtained after the measurement of the thickness of the slab \( (L=0.001474 \text{ m}) \) and the values in the two cases are different of less than 7 % (the precision of the classical Laser Flash technique is around 5 %).

<table>
<thead>
<tr>
<th>Method</th>
<th>( \log(t^*) )</th>
<th>( t^* )</th>
<th>( \alpha/L^2 )</th>
<th>( \alpha [\text{m}^2\text{s}^{-1}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLF</td>
<td>n.a.</td>
<td>n.a.</td>
<td>1.78</td>
<td>( 3.87\times10^{-6} )</td>
</tr>
<tr>
<td>LOG-LOG</td>
<td>-1.79</td>
<td>0.1670</td>
<td>( (\pi\times0.1670)^{-1} )</td>
<td>( 4.14\times10^{-6} )</td>
</tr>
</tbody>
</table>

It is worth comparing the residual after the fitting in the two cases. This is shown in Figure 4a and 4b for the NLF and LOG-LOG respectively. In the first case the difference between the eq. (3), evaluated with the optimum fitting parameters, and the data is represented, while in the second the difference between the logarithm of temperature and the \( 9_{th} \) degree polynomial fitting function is shown.
The residual in case of the NLF shows a clear trend (they are not uniformly distributed in time around zero). This is evidence of a non perfect adherence of the model to the experiment. On the other hand the LOG-LOG procedure is more flexible in following the data thank to the greater number of degrees of freedom (3 for NLF against 10 for LOG-LOG) and the final residual behaves apparently much better. Nonetheless this greater flexibility can lead to a small modification of the position of the maximum of the 2\textsuperscript{nd} derivative. This can affect the final value of the diffusivity estimation.

3 The Two-Layer system

Equation (3) is a particular specification of a more general solution obtained for a system composed by a finite thickness layer on a semi-infinite body of different thermophysical properties. The solution of the heat conduction problem on the surface heated by a pulse and negligible heat exchange with the environment is given by:

$$T(t) - T_0 = T_e \frac{1}{\sqrt{\pi F_0}} \left[ 1 + 2 \sum_{n=1}^{\infty} 1^{n-1} \exp \frac{-n^2 \Gamma}{F_0} \right]$$

(4)

with $\Gamma = (e_c - e_s)/(e_c + e_s)$ the reflection coefficient, ranging from -1 to +1 and accounting for the mismatch between the effusivity of the coating layer $e_c$ and that of the substrate $e_s$. Figure 5a shows a sketch of the experimental lay-out while Fig. 5b represents the solution of eq. (4) for different values of $\Gamma$ plotting the logarithm of temperature vs logarithm of Fourier number.
Equation (4) becomes eq. (3) when \( \Gamma = 1 \), i.e. when the effusivity of the substrate is negligible, as it is the case of the slab where the substrate is made of air or vacuum. When \( \Gamma = 0 \), i.e. the effusivities of the coating and the substrate are the same, the function in square brackets becomes equal to 1 and the eq. (4) is reduced to the solution of the semi-infinite body.

3.1 Computation of 1\(_{st}\) and 2\(_{nd}\) logarithmic derivative

From eq. (4) it is possible to compute the expected behaviour of the 2\(_{nd}\) logarithmic derivative that allows to evaluate the diffusivity of the coating layer by identification of its maximum value, in analogy with the procedure depicted for the slab. Unfortunately the position of the maximum changes with the value of \( \Gamma \) and it becomes a minimum for negative values of \( \Gamma \). Therefore one must know the value of \( \Gamma \) to identify the value of \( Fo \) at which the maximum (minimum) happens. One possible strategy is that of evaluating \( \Gamma \) by analyzing the 1\(_{st}\) logarithmic derivative as shown in Fig. 6a and then select the right curve in Fig. 6b to identify the maximum (minimum) position.

![Figure 6a: 1\(_{st}\) logarithmic derivative of the solution of eq. (4) for different values of \( \Gamma \).](image)

![Figure 6b: 2\(_{nd}\) logarithmic derivative of the solution of eq. (4) for different values of \( \Gamma \).](image)

3.2 Estimation from experimental data

Figure 7a shows the experimental data obtained on a slab of marble on which a superficial layer of around 100 \( \mu m \) has been transformed in gypsum by sulfation. The surface is heated by a laser pulse of duration 1 ms. The temperature is obtained by recording a sequence of images with a thermographic camera with a sampling frequency of 750 Hz. Superimposed on the same figure is the 9\(_{th}\) degree polynomial fitting function obtained by linear fitting the data transformed in log-log space. Figure 7b shows the experimental 1\(_{st}\) logarithmic derivative with indicated the estimated value of \( \Gamma \) that allows to find the value of \( Fo \) in the maximum of the 2\(_{nd}\) derivative. The results of non-linear least squares fitting and identification of the 2\(_{nd}\) derivative of the log-log data is presented in Table 2. In this case we are interested to the evaluation of the thickness of the gypsum layer assuming as known its diffusivity value.

Table 2: estimation of the thickness of gypsum layer due to sulfation on a marble slab by non-linear least squares fitting (NLF) and by maximum identification of the 2nd logarithmic derivative (LOG-LOG). The diffusivity of gypsum is assumed equal to \( \alpha = 3.2 \cdot 10^{-7} \).

<table>
<thead>
<tr>
<th></th>
<th>( \log(t^*) )</th>
<th>( t^* )</th>
<th>( \Gamma )</th>
<th>( \alpha L^2 )</th>
<th>( L [\mu m] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLF</td>
<td>n.a.</td>
<td>n.a.</td>
<td>-0.46</td>
<td>28.4</td>
<td>106</td>
</tr>
<tr>
<td>LOG-LOG</td>
<td>-4.2</td>
<td>0.015</td>
<td>-0.45</td>
<td>30.0 (( Fo^*=0.45 ))</td>
<td>103</td>
</tr>
</tbody>
</table>
4 Conclusions

Pulsed Infrared Thermography has been utilized to measure thermal parameters of a slab and a two-layer system. A data reduction that transforms temperature data as a function of time in a log-log space has been demonstrated to be able estimating diffusivity or thickness by using extrema of the 2nd logarithmic derivative. The results have been compared to those obtained by Non Linear Least Square fitting procedure.

Acknowledgments

Authors are indebted with dr. Fabrizio Clarelli of IAC-CNR for furnishing the marble sulfated samples and for carrying out the experiments.

References