STRESS DISTRIBUTION CAUSED BY OUT OF PLANE CO-PHASE PERIODICAL CURVING OF TWO NEIGHBORING FIBERS IN A COMPOSITE MATERIAL

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ABSTRACT

In the present paper within the framework of the piecewise homogeneous body model with the use of the exact equations of the three-dimensional equations of the theory of elasticity for anisotropic body, the method proposed in references has been developed for the co-phase periodical curving out of plane of two neighboring fibers in an infinite elastic body . It is assumed that these fibers are located along two parallel planes and each of them has a periodical curving. Moreover, it is assumed that the curving of each fiber is co-phase with the other. At infinity, uniformly distributed normal forces act in the direction of the fibers' location. The numerical investigations have been made for the case where the materials of the fibers and matrix are both isotropic and homogeneous. The normal and shear self-balanced stresses arising as a result of the fiber curving are analyzed. In particular, the influence of the interaction between the fibers on the distribution of these stresses is studied.

Key words: Fibrous composite, Stress distribution, Fiber curving

1. INTRODUCTION

Curvature of fibers in structure of composite materials may occur as a result of design, or as a consequence of some technological process, as mentioned in [1-5]. It is known that this curvature cause to arise self-balanced stresses and the values of these stresses can pass over adhesion resistance values. According to these and various other reasons, it is needed mechanics of composite materials with curved structure. The widely explanation and interpretation of investigations carried out on this subject are given in [1].

However in [1], the concrete investigations for unidirectional composites with curved fibers are made for only low concentration of fibers and in this case, composite material is modeled as an infinite elastic body containing a single periodical curved fiber. For investigate such a problem, a method is developed in [6] and numerical results are obtained. In [7], the stress distribution in an infinite elastic body containing two neighboring fibers is studied while the middle lines of the fibers are located in the same plane and they are curved periodically co-phase. In [8], the problem considered in the paper [7] is investigated for the case where the curving of the fibers is anti-phase with respect to other. In [9], the approach [6-8] is developed for the periodically located row fibers in the infinite matrix and the numerical results on the self-balanced normal and shear stresses acting on the interface are presented for the co-phase curving of the fibers.

In all the studies mentioned above, It was assumed that midlines of the fibres are in the same plane. However, the observation of the cross sections of the unidirected fibrous composites shows that the middle lines of the curved fibers may locate on various planes. In connection with this in the present paper, the stress distribution caused by co-phase periodical curving out of plane of two neighboring fibers in an infinite elastic body is studied. It is assumed that these fibers are located along two parallel planes and each of them has a periodical curving. It is also assumed that the body is loaded by uniformly distributed normal forces with intensity p acting along the fibers (Figure 1) at infinity. The investigations are carried out in the framework of the piecewise homogeneous body model with the use of the exact equations of the three-dimensional linear theory of elasticity. The normal and shear self-balanced stresses arising as a result of the fibers curving are analyzed. In particular, the influence of the interaction between the fibers on the distribution of these stresses is studied. Numerical results related to this interaction are presented.

2. FORMULATION OF THE PROBLEM

For mathematical formulation of the problem, we associate rectilinear $O_k x_{k1} x_{k2} x_{k3}$ and cylindrical $O_k r_k \theta_k z_k$ system of coordinates (Figure 1). Here $k = 1, 2$ is related to the first and second fiber in turn in order.

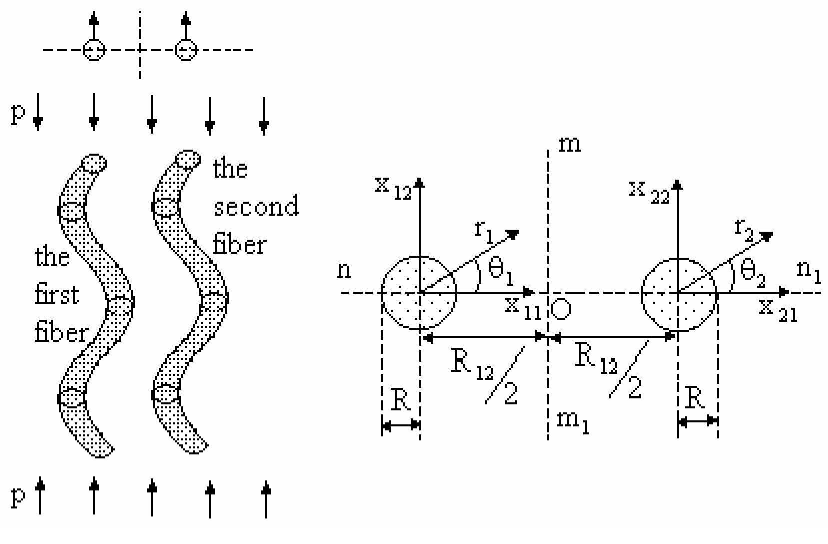

Figure 1. The geometry of the material structure and chosen coordinates

According to Figure 1, between these coordinates we have the following relations

$x_{12} = x_{22},\ x_{13} = x_{23} = x_3,\ r_1 e^{i\theta_1} = R_{12} + r_2 e^{i\theta_2},\quad z_1 = z_2 = z$

(1)

We assume that the equations of middle line of each fiber are given as follows

$x_{12} = L\sin(\frac{2\pi}{\ell} x_{13}),\quad x_{11} = 0$ (for the first fiber),

$x_{22} = L\sin(\frac{2\pi}{\ell} x_{23}),\ x_{21} = 0$ (for the second fiber)

(2)

and the cross-section of each fiber, which is perpendicular to the middle line, is a circle with constant radius R and this is invariant along the entire length of the fiber. Assume that, L (curving amplitude of the fiber) is smaller than ℓ (the length period of the curving); we introduce a small parameter $\varepsilon = L/\ell$, $(0 < \varepsilon << 1)$.

If contact surfaces between the fibers and matrix are denoted by S_1 and S_2, from (2) and from the condition of fiber cross-section, the equations of these surfaces and the components of their normal vectors are derived as follows.

$$r_k = R + \sum_{q=1}^{\infty} \varepsilon^q a_{kq}(\theta_k, t_3)\ ,\ z_k = t_3 + \sum_{q=1}^{\infty} \varepsilon^q b_{kq}(\theta_k, t_3),$$

$$n_{kr} = 1 + \sum_{q=1}^{\infty} \varepsilon^q c_{kq}(\theta_k, t_3)\,,\ n_{k\theta} = \sum_{q=1}^{\infty} \varepsilon^q d_{kq}(\theta_k, t_3)\,,\ n_{kz} = \sum_{q=1}^{\infty} \varepsilon^q f_{kq}(\theta_k, t_3) \qquad (3)$$

t_3 is a parameter and $t_3 \in (-\infty, +\infty)$, the explicit expression of functions $a_{kq}(\theta_k, t_3),\ldots,$ $f_{kq}(\theta_k, t_3)$ in (3) are given in [1].

The values related to the fibers will be denoted by upper indices (21), (22), but those related to the matrix by upper index (1). Assume that the fibers and matrix materials are transversal-isotropic, with symmetry axis Ox_3. Thus, within the fibers and infinite matrix in the cylindrical system of coordinates we write the governing field equations:

$$\frac{\partial \sigma_{rr}^{(k)}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}^{(k)}}{\partial \theta} + \frac{\partial \sigma_{rz}^{(k)}}{\partial z} + \frac{1}{r}\left(\sigma_{rr}^{(k)} - \sigma_{\theta\theta}^{(k)}\right) = 0\,,\ \frac{\partial \sigma_{r\theta}^{(k)}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}^{(k)}}{\partial \theta} + \frac{\partial \sigma_{\theta z}^{(k)}}{\partial z} + \frac{2}{r}\sigma_{r\theta}^{(k)} = 0\,,$$

$$\frac{\partial \sigma_{rz}^{(k)}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta z}^{(k)}}{\partial \theta} + \frac{\partial \sigma_{zz}^{(k)}}{\partial z} + \frac{1}{r}\sigma_{rz}^{(k)} = 0\,,$$

(4)

$$\sigma_{rr}^{(k)} = A_{11}^{(k)}\varepsilon_{rr}^{(k)} + A_{12}^{(k)}\varepsilon_{\theta\theta}^{(k)} + A_{13}^{(k)}\varepsilon_{zz}^{(k)}\,,\ \sigma_{\theta\theta}^{(k)} = A_{12}^{(k)}\varepsilon_{rr}^{(k)} + A_{11}^{(k)}\varepsilon_{\theta\theta}^{(k)} + A_{13}^{(k)}\varepsilon_{zz}^{(k)}\,,$$

$$\sigma_{zz}^{(k)} = A_{13}^{(k)}\varepsilon_{rr}^{(k)} + A_{13}^{(k)}\varepsilon_{\theta\theta}^{(k)} + A_{33}^{(k)}\varepsilon_{zz}^{(k)}\,,\ \sigma_{rz}^{(k)} = 2G_{13}^{(k)}\varepsilon_{rz}^{(k)}\,,$$

$$\sigma_{r\theta}^{(k)} = \left(A_{11}^{(k)} - A_{12}^{(k)}\right) \varepsilon_{r\theta}^{(k)}, \ \sigma_{\theta z}^{(k)} = 2G_{13}^{(k)} \varepsilon_{\theta z}^{(k)}$$

(5)

$$\varepsilon_{rr}^{(k)} = \frac{\partial u_r^{(k)}}{\partial r}, \ \varepsilon_{\theta\theta}^{(k)} = \frac{\partial u_\theta^{(k)}}{r\partial\theta} + \frac{u_r^{(k)}}{r}, \ \varepsilon_{zz}^{(k)} = \frac{\partial u_z^{(k)}}{\partial z}, \ \varepsilon_{r\theta}^{(k)} = \frac{1}{2}\left(\frac{\partial u_r^{(k)}}{r\partial\theta} + \frac{\partial u_\theta^{(k)}}{\partial r} - \frac{u_\theta^{(k)}}{r}\right),$$

$$\varepsilon_{\theta z}^{(k)} = \frac{1}{2}\left(\frac{\partial u_\theta^{(k)}}{\partial z} + \frac{\partial u_z^{(k)}}{r\partial\theta}\right), \quad \varepsilon_{zr}^{(k)} = \frac{1}{2}\left(\frac{\partial u_z^{(k)}}{\partial r} + \frac{\partial u_r^{(k)}}{\partial z}\right).$$

(6)

It is assumed that on the inter-medium surfaces S_k (Figure 1) the completely cohesion conditions are satisfied:

$$\left.\left(\sigma_{rr}^{(2k)} n_{kr} + \sigma_{r\theta}^{(2k)} n_{k\theta} + \sigma_{rz}^{(2k)} n_{kz}\right)\right|_{S_k} = \left.\left(\sigma_{rr}^{(1)} n_{kr} + \sigma_{r\theta}^{(1)} n_{k\theta} + \sigma_{rz}^{(1)} n_{kz}\right)\right|_{S_k},$$

$$\left.\left(\sigma_{r\theta}^{(2k)} n_{kr} + \sigma_{\theta\theta}^{(2k)} n_{k\theta} + \sigma_{z\theta}^{(2k)} n_{kz}\right)\right|_{S_k} = \left.\left(\sigma_{r\theta}^{(1)} n_{kr} + \sigma_{\theta\theta}^{(1)} n_{k\theta} + \sigma_{z\theta}^{(1)} n_{kz}\right)\right|_{S_k},$$

$$\left.\left(\sigma_{rz}^{(2k)} n_{kr} + \sigma_{z\theta}^{(2k)} n_{k\theta} + \sigma_{zz}^{(2k)} n_{kz}\right)\right|_{S_k} = \left.\left(\sigma_{rz}^{(1)} n_{kr} + \sigma_{z\theta}^{(1)} n_{k\theta} + \sigma_{zz}^{(1)} n_{kz}\right)\right|_{S_k},$$

$$\left.u_r^{(2k)}\right|_{S_k} = \left.u_r^{(1)}\right|_{S_k}, \ \left.u_\theta^{(2k)}\right|_{S_k} = \left.u_\theta^{(1)}\right|_{S_k}, \ \left.u_z^{(2k)}\right|_{S_k} = \left.u_z^{(1)}\right|_{S_k}, \ k = 1,2,$$

(7)

In the considered case it is also assumed that the conditions $\sigma_{zz}^{(1)} \xrightarrow[r_k \to \infty]{} p$, $\sigma_{(ij)}^{(1)} \xrightarrow[r_k \to \infty]{} 0$, $(ij) \neq zz$ are satisfied. Thus, with the above-stated, the formulation of the considered problem is exhausted.

3. METHOD OF SOLUTION

For investigation of this problem we use the boundary shape perturbation method developed in [1, 6] according to which the unknown values are presented in series form in ε:

$$\left\{\sigma_{(ij)}^{(m)}; \varepsilon_{(ij)}^{(m)}; u_{(i)}^{(m)}\right\} = \sum_{q=0}^{\infty} \varepsilon^q \left\{\sigma_{(ij)}^{(m),q}; \varepsilon_{(ij)}^{(m),q}; u_{(i)}^{(m),q}\right\},$$

$$(ij) = rr; \theta\theta; zz; r\theta; rz; \theta z, \ (i) = r; \theta; z$$

(8)

From (4), we obtain equations set for each approximation in (8). In this case due to linearity the equations (4)-(6) are satisfied for each approximation separately. Substituting the expression (8) in (5), using (3) and after some mathematical manipulation described in [1, 6], we obtain contact condition satisfied in $r_k = R$ for each approach in (8). We write these conditions belong to the zeroth and first approximations:

The zeroth approximation,

$$\sigma_{(ij)}^{(2k),0} = \sigma_{(ij)}^{(1),0} \; ; \quad u_{(i)}^{(2k),0} = u_{(i)}^{(1),0}$$

(9)

The first approximation,

$$\left[\sigma_{(i)r}\right]_{1,1}^{2k,1} + f_{1k}\left[\frac{\partial \sigma_{(i)r}}{\partial r}\right]_{1,0}^{2k,0} + \varphi_{1k}\left[\frac{\partial \sigma_{(i)r}}{\partial z}\right]_{1,0}^{2k,0} + \gamma_{rk}\left[\sigma_{(i)r}\right]_{1,0}^{2k,0} + \gamma_{\theta k}\left[\sigma_{(i)\theta}\right]_{1,0}^{2k,0} + \gamma_{zk}\left[\sigma_{(i)z}\right]_{1,0}^{2k,0} = 0$$

$$\left[u_{(i)}\right]_{1,1}^{2k,1} + f_{1k}\left[\frac{\partial u_{(i)}}{\partial r}\right]_{1,0}^{2k,0} + \varphi_{1k}\left[\frac{\partial u_{(i)}}{\partial z}\right]_{1,0}^{2k,0} = 0$$

(10)

where $(i) = r, \theta, z$. In (9)-(10) replacing (i) with r, θ and z we obtain explicit form of the corresponding contact conditions in the considered approximations. Moreover in (10) the following notation is used.

$$[\varphi]_{1,s}^{2k,s} = \varphi^{(2k),s} - \varphi^{(1),s} \; ; \; f_{1k} = \delta_k(t_3)\sin\theta_k \; ; \; \varphi_{1k} = -R\delta_k'(t_3)\sin\theta_k \, ,$$

$$\delta_1(t_3) = \delta_1(t_3) = \ell \sin(2\pi t_3/\ell) = \ell \sin(\alpha t_3) \qquad (11)$$

Similar contact conditions are obtained for the subsequent approximations.

Now, we determine the unknown values belong to these approximations. Assume that the materials of each fiber are the same and Poisson coefficients of this material are equal to Poisson coefficient of the matrix material. Thus, for the zeroth approach we obtain:

$$\sigma_{zz}^{(1),0} = p; \; \sigma_{zz}^{(21),0} = \sigma_{zz}^{(22),0} = \frac{E_3^{(2)}}{E_3^{(1)}} p \; ; \; \varepsilon_{zz}^{(21),0} = \varepsilon_{zz}^{(22),0} = \varepsilon_{zz}^{(1),0} = \frac{p}{E_3^{(1)}} \; ; \; z = z_1 = z_2$$

$$u_z^{(21),0} = u_z^{(22),0} = u_z^{(1),0} = \varepsilon_{zz}^{(1),0} z \quad ; \; \sigma_{(ij)}^{(2q),0} = \sigma_{(ij)}^{(1),0} = 0 \quad (ij) = rr, \theta\theta, r\theta, \theta z, rz$$

(12)

In (12) $E_3^{(1)}$ and $E_3^{(2)}$ are the moduli of elasticity of the matrix and fiber materials respectively in the direction of the Ox_3 axis.

The assumption of equality Poisson coefficients of fiber and matrix materials does not have considerable effect to numerical results, as known from [1]. This assumption is accepted just to simplify the solution procedure. Now we consider the determination of the values of the first approximation. We obtain the following contact conditions for this approximation from (10) and (12)

$$[\sigma_{rr}]_{1,1}^{2k,1}=0\,,\ \ [\sigma_{r\theta}]_{1,1}^{2k,1}=0\,,\ \ [\sigma_{rz}]_{1,1}^{2k,1}=2\pi\left(\sigma_{zz}^{(1),0}-\sigma_{zz}^{(2),0}\right)\cos\alpha t_3\sin\theta\,,$$

$$[u_r]_{1,1}^{2k,1}=0\,,\ [u_\theta]_{1,1}^{2k,1}=0\,,\ [u_z]_{1,1}^{2k,1}=0\,.$$

(13)

For determination of the values of the first approximation we use the following representations in cylindrical system of coordinates, which are given in [10].

$$u_r=\frac{1}{r}\frac{\partial}{\partial\theta}\psi-\frac{\partial^2}{\partial r\partial z}\chi\ ;\ u_\theta=-\frac{\partial}{\partial r}\psi-\frac{1}{r}\frac{\partial^2}{\partial r\partial z}\chi;\ u_3=A\left(B\Delta_1+C\frac{\partial^2}{\partial z^2}\right)\chi\,;$$

$$\Delta_1=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\,,$$

(14)

In (14) the functions of ψ,χ satisfy the following equations.

$$\left(\Delta_1+\xi_1{}^2\frac{\partial^2}{\partial z^2}\right)\psi=0\,;\ \left(\Delta_1+\xi_2{}^2\frac{\partial^2}{\partial z^2}\right)\left(\Delta_1+\xi_3{}^2\frac{\partial^2}{\partial z^2}\right)\chi=0$$

(15)

In (14), (15) A, B, C and ξ_i (i=1,2,3) are constants and determined by the corresponding expressions given in [10, 11].

Taking the expression of the contact conditions (13) into account, the solutions of the equations (15) are found as follows:

$$\psi^{(2k)}=\alpha\sin\alpha z\sum_{n=-\infty}^{\infty}C_n^{(2k)}I_n(\xi_1^{(2k)}\alpha r_k)\exp(in\theta_k),$$

$$\chi^{(2k)}=\cos\alpha z\sum_{n=-\infty}^{\infty}\left[A_n^{(2k)}I_n(\xi_2^{(2k)}\alpha r_k)+B_n^{(2k)}I_n(\xi_3^{(2k)}\alpha r_k)\right]\exp(in\theta_k)\,, \qquad (16)$$

$$\psi^{(1)}=\alpha\sin\alpha z\sum_{k=1}^{2}\sum_{n=-\infty}^{\infty}C_n^{(1)k}K_n(\xi_1^{(1)}\alpha r_k)\exp(in\theta_k)\,,$$

$$\chi^{(1)}=\cos\alpha z\sum_{k=1}^{2}\sum_{n=-\infty}^{\infty}\left[A_n^{(1)k}K_n(\xi_2^{(1)}\alpha r_k)+B_n^{(1)k}K_n(\xi_3^{(1)}\alpha r_k)\right]\exp(in\theta_k)\,, \qquad (17)$$

where $\alpha=2\pi/\ell$ and $I_n(x), K_n(x)$ are Bessel functions of a purely imaginary argument and Macdonald functions, in turn. Moreover the unknowns $C_n^{(2k)},...,B_n^{(2k)}$ are the complex constant and satisfy the relations:

$$A_n^{(2k)} = \overline{A_{-n}^{(2k)}},\ B_n^{(2k)} = \overline{B_{-n}^{(2k)}},\quad C_n^{(2k)} = \overline{C_{-n}^{(2k)}},\ \mathrm{Im}\, A_0^{(2k)} = \mathrm{Im}\, B_0^{(2k)} = \mathrm{Im}\, C_0^{(2k)} = 0,$$

$$A_n^{(1)} = \overline{A_{-n}^{(1)}},\ B_n^{(1)} = \overline{B_{-n}^{(1)}},\quad C_n^{(1)} = \overline{C_{-n}^{(1)}},\ \mathrm{Im}\, A_0^{(1)} = \mathrm{Im}\, B_0^{(1)} = \mathrm{Im}\, C_0^{(1)} = 0.$$

(18)

Now we attempt to satisfy the contact condition (13). For this purpose we must represent the expressions (16) and (17) in the k-th (k=1,2) cylindrical coordinate system to satisfy the contact conditions on the k-th fiber-matrix interface S_k. The expressions (16) are already presented in the k-th cylindrical system of coordinates. To make these operations for the expressions (17) we use the summation theorem [12] for the $K_n(x)$ function, which can be written for the case at hand as follows

$$r_m \exp i\theta_m = R_{mn} \exp i\varphi_{mn} + r_n \exp i\theta_n,$$

$$K_\nu(cr_n)\exp i\nu\theta_n = \sum_{k=-\infty}^{\infty} (-1)^\nu I_k(cr_m) K_{\nu-k}(cR_{mn}) \exp[i(\nu-k)\varphi_{mn}] \exp ik\theta_m,$$

$$mn = 12;21;\ \ m;n = 1,2;\ \ r_m < R_{mn};\ \ R_{12} = R_{21};\ \ \varphi_{12} = 0;\ \ \varphi_{21} = \pi \qquad (19)$$

Using (16)-(19) we obtain from (13) an infinite system of algebraic equations with respect to the unknown constants (18). For numerical investigations the infinite system of algebraic equations must be approximated by a finite system. To validate such replacement it must be shown that the determinant of this infinite system of equations is a normal type [13]. As shown in [7], this holds if we use the following inequality

$$R/(R_{12} - 2L) > R/R_{12},\ \ R_{12}R^{-1} > 2 \qquad (20)$$

which means that the fibers do not touch each other.

4. NUMERICAL RESULTS

Assume that the materials of the fibers and of the matrix are isotropic and Young's moduli denote by $E^{(2)}$ (for the fibers) and by $E^{(1)}$ (for the matrix). The Poisson's coefficients we denote by $\nu^{(2)}$ (for the fibers) and by $\nu^{(1)}$ (for the matrix) and suppose that $\nu^{(1)} = \nu^{(2)} = 0.3$. Investigate the distribution of the self-equilibrium shear stresses $\sigma_{n\tau}$, σ_{ne} and normal stress σ_{nn}. According to the corresponding symmetry and periodicity we consider the distribution of these stresses on the S_1 surface only. In the case where $\varepsilon = 0$ (i.e. the curving is absent) the stresses σ_{nn}, $\sigma_{n\tau}$ and σ_{ne} coincide with σ_{rr}, σ_{rz} and $\sigma_{r\theta}$ respectively.

Introduce the parameters $\kappa = 2\pi R/\ell$ and $\rho = R_{12}/R$. Note that the present investigations and the investigations carried out in [6] show that the stress σ_{nn} has its maximum at the point of S_1 which is determined from (3) under $\theta = \pi/2$, $\alpha t_3 = \pi/2$. However the stresses $\sigma_{n\tau}$ and σ_{ne} have their maximum at the vicinity of the point of S_1 corresponding to $\theta = \pi/2$, $\alpha t_3 = 0$ and $\theta = 0$, $\alpha t_3 = \pi/2$ respectively. Below we will consider the values of these stresses at these points.

Below we will also consider the numerical results obtained within the zeroth and first approximations. Because, according to [1], the second and subsequent approximations just correct these results insignificantly in the quantitative sense.

Thus, we consider the graphs given in Figures. 2, 3 and 4, which show the dependencies between σ_{nn}/p (Figure 2), $\sigma_{n\tau}/p$ (Figure 3), σ_{ne}/p (Figure 4) and κ for various ρ under $E^{(2)}/E^{(1)} = 50$ $\varepsilon = 0.015$.

Figures 2 and 3 show that the values of σ_{nn}/p and the absolute values of $\sigma_{n\tau}/p$ increase monotonically with approaching of the fibers to each other. However, it is follows from Figure 4 that the values σ_{ne}/p decrease monotonically with approaching of the fibers to each other (with decreasing ρ). In this case the considerable increasing is observed in the values of the normal stress σ_{nn}.

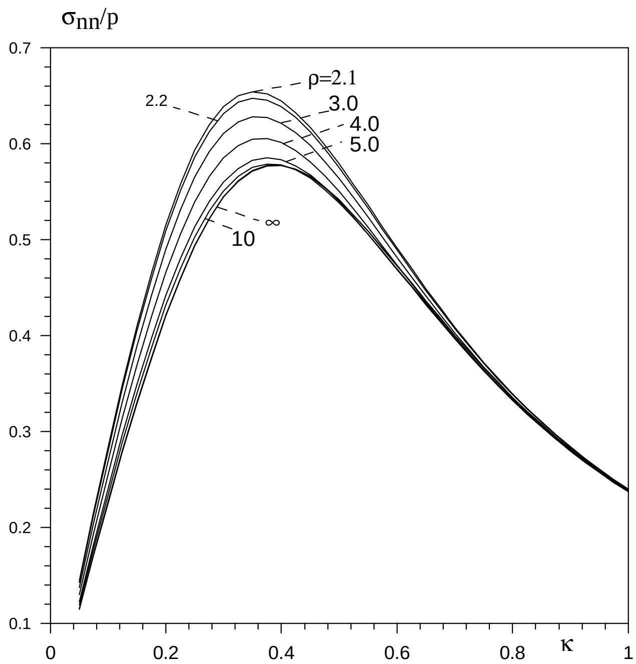

Figure 2. The graphs of the dependencies between σ_{nn}/p and κ for various values of ρ for the case where $E^{(2)}/E^{(1)} = 50$, $\varepsilon = 0.015$.

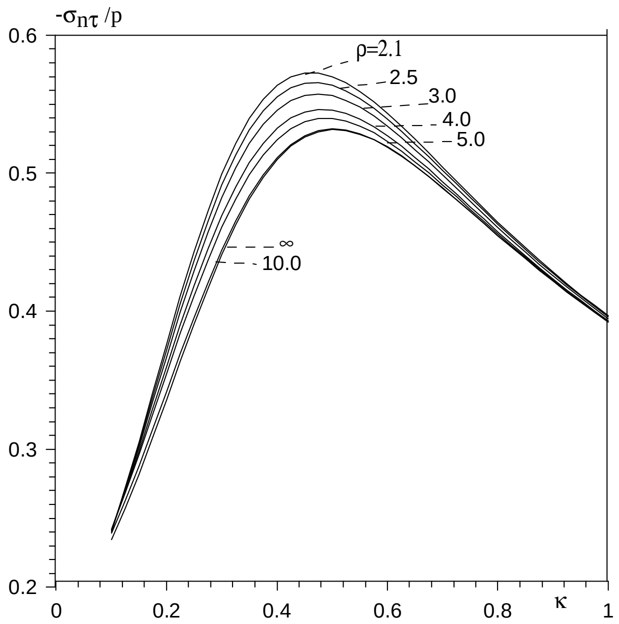

Figure 3. The graphs of the dependencies between $\sigma_{n\tau}/\mathrm{p}$ and κ for various values of ρ for the case where $\mathrm{E}^{(2)}/\mathrm{E}^{(1)} = 50$, $\varepsilon = 0.015$.

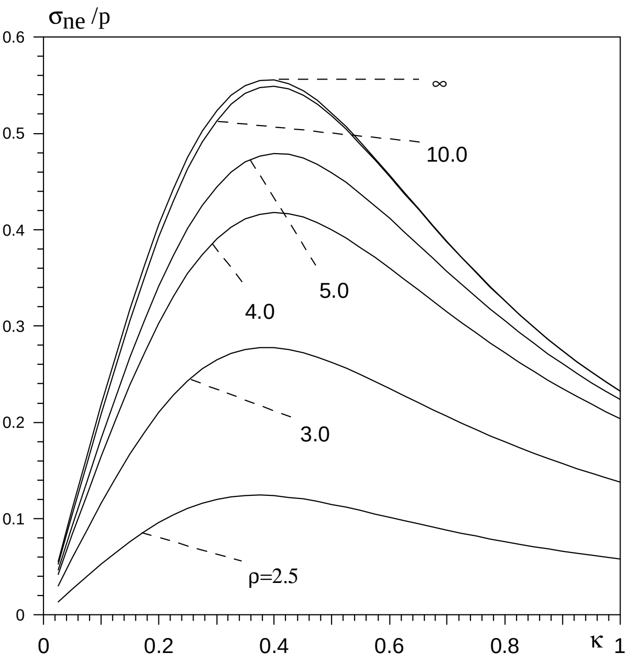

Figure 4. The graphs of the dependencies between σ_{ne}/p and κ for various values of ρ for the case where $\mathrm{E}^{(2)}/\mathrm{E}^{(1)} = 50$, $\varepsilon = 0.015$.

Moreover, it follows from the obtained numerical results that with increasing ρ the values of the considered stresses approach to the corresponding values obtained for the single fiber [6]. At the same time, it is follows from the presented numerical results that the dependencies between the considered stresses and the parameter κ have non-monotone character.

Table 1. The values of σ_{nn}/p, $\sigma_{n\tau}/p$ and σ_{ne}/p for various ρ, $E^{(2)}/E^{(1)}$ and ε.

$\frac{E^{(2)}}{E^{(1)}}$	ε	$\rho = 2.5$			$\rho = 3.0$			$\rho = 4.0$		
		$\frac{\sigma_{nn}}{p}$	$-\frac{\sigma_{n\tau}}{p}$	$\frac{\sigma_{ne}}{p}$	$\frac{\sigma_{nn}}{p}$	$-\frac{\sigma_{n\tau}}{p}$	$\frac{\sigma_{ne}}{p}$	$\frac{\sigma_{nn}}{p}$	$\frac{\sigma_{n\tau}}{p}$	$\frac{\sigma_{ne}}{p}$
10 (κ=0.6)	0.010	0.1031	0.1882	0.0196	0.0992	0.1861	0.0456	0.0957	0.1835	0.0676
	0.015	0.1546	0.2824	0.0295	0.1488	0.2792	0.0684	0.1435	0.2752	0.1014
	0.020	0.2062	0.3765	0.0393	0.1984	0.3723	0.0912	0.1914	0.3670	0.1353
20 (κ=0.5)	0.010	0.1954	0.2498	0.0388	0.1883	0.2462	0.0884	0.1819	0.2413	0.1314
	0.015	0.2932	0.3748	0.0583	0.2825	0.3693	0.1326	0.2729	0.3620	0.1972
	0.020	0.3909	0.4997	0.0777	0.3767	0.4924	0.1769	0.3638	0.4826	0.2629
50 (κ=0.4)	0.010	0.4144	0.3701	0.0824	0.4007	0.3638	0.1850	0.3887	0.3549	0.2784
	0.015	0.6216	0.5552	0.1237	0.6011	0.5457	0.2776	0.5830	0.5324	0.4176
	0.020	0.8288	0.7403	0.1649	0.8015	0.7277	0.3701	0.7774	0.7099	0.5569
100 (κ=0.3)	0.010	0.7165	0.4868	0.1391	0.6917	0.4774	0.3073	0.6688	0.4639	0.4626
	0.015	1.0748	0.7302	0.2086	1.0375	0.7161	0.4610	1.0032	0.6958	0.6939
	0.020	1.4331	0.9736	0.2782	1.3834	0.9549	0.6147	1.3377	0.9278	0.9253

Table 2. The convergence of the numerical results with $E^{(2)}/E^{(1)} = 50$, $\varepsilon = 0.015$, $\rho = 2.5$.

κ	stresses	38	44	50	56	62	68	74	80
0.1	σ_{nn}/p	0.2665	0.2698	0.2698	0.2688	0.2688	0.2691	0.2691	0.2690
	$\sigma_{n\tau}/p$	-0.2399	-0.2404	-0.2404	-0.2404	-0.2404	-0.2404	-0.2404	-0.2404
	σ_{ne}/p	0.0593	0.0563	0.0547	0.0538	0.0533	0.0531	0.0530	0.0529
0.2	σ_{nn}/p	0.4860	0.4915	0.4916	0.4898	0.4898	0.4903	0.4903	0.4902
	$\sigma_{n\tau}/p$	-0.3710	-0.3718	-0.3718	-0.3717	-0.3717	-0.3717	-0.3717	-0.3717
	σ_{ne}/p	0.1066	0.1017	0.0989	0.0974	0.0967	0.0963	0.0961	0.0960

The values of the stresses obtained in the first approximation are given in Table 1 for various $E^{(2)}/E^{(1)}$, ε and ρ. Moreover, Table 2 illustrates the convergence of the numerical results in the first approximation with respect to the number of the selected equations. . The numerical results given in this table are obtained under $E^{(2)}/E^{(1)} = 50$, $\varepsilon = 0.015$, $\rho = 2.5$. It follows from this table that the convergence of the used solution method is adequate.

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