

Highlights of the updated version of Part 3 of VDI Guideline 2014 (Development of Fibre-Reinforced Plastics components, Analysis)*

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Summary

Part 3 of guideline VDI 2014 provides in concise form the information required by the current state of the technology for dimensioning components made of fibre-reinforced plastics (FRP).

In a number of sections of the recently updated version of Part 3, certain questions are addressed which have on the one hand arisen from the results of research conducted over the last decade and which have on the other hand already thoroughly demonstrated their practical use and their correctness. This particularly concerns the strength analysis of laminates with the aid of Puck's new action-plane fracture criteria. The self-contained presentation of these physically justified fracture criteria and also the provision of all required formulae, not to mention the special problems treated in detail in Annexes A1.1, A1.2 and A2, has resulted in VDI 2014 Part 3 assuming an importance for the practical aspects of FRP component development which, in the case of the special field of 'FRP strength', is currently not offered by any other publication.

The carefully chosen terminology and symbols should foster better communications in the engineering sciences - even in an international context - especially as the guideline is published with the German and English texts in parallel.

1. Preliminary comments

Part 3 of VDI 2014 is a somewhat unusual guideline in that it contains very recent findings in the field of the failure analysis of fibre-reinforced plastics, findings which are nevertheless well-founded scientifically. This is a specialist area which is very important in the development of FRP components but one which is not well enough known in engineering practice. However, the holding of the 'World-wide Failure Exercise' (WWFE) in the 1990s provided a global stimulus to the revision of fracture theories for FRPs. A final comparison of the results was then published by the initiators of the exercise in the so-called Part C in 2004 and this is included in the omnibus volume of the exercise [Hin04].

Participation in the exercise called for knowledge of the basic principles combined with computer-aided analytical methods, this being secured by the collaboration of Puck and Schürmann. In comparison with all of the tasks solved, Puck/Schürmann achieved a very good rating within the context of the World-wide Failure Exercise. Of decisive importance to this success was the necessity - recognized by Puck as early as 1969 ([Puc69a] and [Puc69b]) - of using separate fracture criteria for fibre fracture and for interfibre fracture. Furthermore the use of analytical approaches entirely based on physical concepts proved to be the reason behind Puck/Schürmann's good performance in the exercise.

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As regards the first-class rating given to Puck and Schürmann's contribution in the final assessment of the World-wide Failure Exercise [Hin04] it should still be borne in mind that the utterly new capabilities of Puck's action-plane fracture criteria, such as specifying the angle of the fracture plane and the fracture mode, were not even foreseeable when the exercise tasks were formulated at the beginning of the 1990s. For this reason, requirements of this kind were not even asked for in the exercise and this meant that it was not possible to deploy the full potential of the new theory.

Unfortunately, the initiators of the World-wide Failure Exercise did not in the end decide in favour of making a final recommendation to the specialists in the field that a fracture theory should be used. In the light of this, guideline VDI 2014 Part 3 now takes on the role of a precursor in that priority is clearly given to the new fracture criteria of Puck.

The Committee seized a unique opportunity to incorporate into Part 3 the complete new FRP fracture theory of Puck. In addition to making a general presentation of the subject matter of Part 3, the present contribution will therefore be primarily concerned with a number of selected topics from the new fracture theory of Puck.

2. Terminology and important concepts in Part 3

Naturally every effort was made in Part 3 to take international usage into due consideration in the nomenclature and the symbols used. However it was appropriate in certain cases to eschew international usage and use considerably more rational designations such as have been introduced in the more recent publications in the German language. One example of this is the subscripts used here for stressings (this is an expression for characterising types of stresses), elasticity variables and strengths, which are more accommodating to the underlying fundamental physical concepts (Fig. 1). In these cases guideline VDI 2014 Part 3 does not employ the usual subscripts of 1 (parallel to the fibre) and 2 (vertical to the fibre) but instead the subscripts \parallel and \perp for stressings. For stresses with a fixed direction with respect to the natural axes x_1 , x_2 and x_3 the subscripts 1, 2 and 3 continue to be used.

Although strictly speaking nine stresses (three normal stresses $\sigma_1, \sigma_2, \sigma_3$ and six shear stresses $\tau_{23} = \tau_{32}, \tau_{13} = \tau_{31}, \tau_{12} = \tau_{21}$) must be distinguished in a UD volume unit, as regards the type of stress and strength only five stressings can be distinguished. Using the subscripts \parallel and \perp , these five are the stressings $\sigma_{\parallel}, \sigma_{\perp}, \tau_{\perp\perp}, \tau_{\parallel\perp}, \tau_{\perp\parallel}$. In the case of the shear stresses τ , in accordance with international usage the first subscript indicates the direction of the plane normal with respect to the plane upon which the shear stress is acting. The second subscript indicates the direction of the shear force from the stress under consideration.

Accordingly a shear stressing $\tau_{\perp\parallel}$ acts on a plane (called the 'action plane') whose normal is vertical with respect to the fibre orientation - in other words, the action plane of $\tau_{\perp\parallel}$ is a plane running parallel to the fibres. This plane has a relatively low fracture resistance to shear fracture. In contrast, the shear stress $\tau_{\parallel\perp}$ acts by definition on a plane running transversely through the fibres. More than half of this cross-section consists of fibre cross-sections. To bring about a shear fracture in this plane the shear stress $\tau_{\parallel\perp}$ would have to shear through all of these fibres. The fracture resistance $R_{\parallel\perp}^A$ (where **A** stands for **action plane**) is accordingly a great deal higher than the fracture resistance $R_{\perp\parallel}^A$ of the parallel-to-fibre plane in which $\tau_{\perp\parallel}$ is acting. As regards the absolute value of the stresses and stressings, not only is $\tau_{12} = \tau_{21}$ but also $\tau_{\parallel\perp} = \tau_{\perp\parallel}$. For this reason fracture will always occur in the parallel-to-fibre plane which has the lower fracture resistance $R_{\perp\parallel}^A$. Thinking in terms of 'fracture resistances on the action plane', which is

based on the brittle fracture theory of Mohr/Coulomb, permits some entirely new insights into the fracture behaviour of FRP.

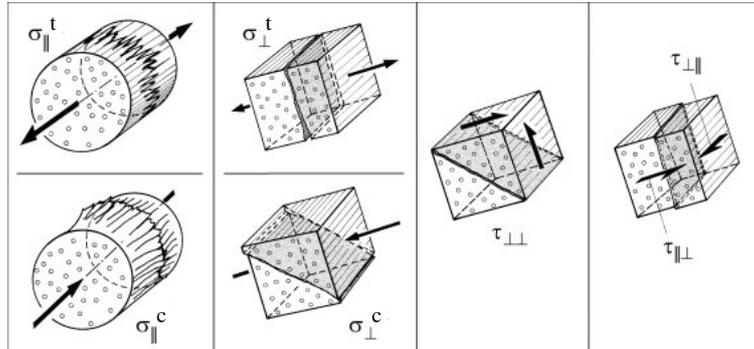


Fig. 1. Possible stressings in a UD composite. The $\sigma_{||}$ stressing causes fibre fracture while the

σ_{\perp} , $\tau_{\perp\perp}$, $\tau_{||\perp}$ stressings cause interfibre fracture. With compressive stressing σ_{\perp}^c an ‘oblique’ shear fracture occurs while with pure $\tau_{\perp\perp}$ stressing there is an ‘oblique’ tensile fracture [Puc96].

The subscripts $||$ and \perp used with stressings and strengths make it extraordinarily easier to understand the action-plane-related fracture criteria developed by Puck. The case is similar with those relations relating to elasticity. With a uniaxial stress on a UD element in the direction of its thickness we often find $\varepsilon_3 = \sigma_3 / E_2$, which at first sight does not exactly seem reasonable. In contrast, $\varepsilon_3 = \sigma_3 / E_{\perp}$ vividly expresses the transverse isotropy which is present, whereby $E_2 = E_3 = E_{\perp}$.

These and similar physical interrelationships are made transparent in the new strength criteria thanks to an illuminating system of subscripts. It has also been adopted consistently in guideline VDI 2014 Part 3.

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<i>Die deutsche Version dieser Richtlinie ist verbindlich.</i>		<i>No guarantee can be given with respect to the English translation. The German version of this guideline shall be taken as authoritative.</i>

Fig. 2. Title page of guideline VDI 2014 Part 3, German/English

3. Contents of Part 3

Part 3 has the following main sections and sub-sections:

- 0 Preliminary note
- 1 Scope
- 2 Abbreviations, terminology, symbols, superscripts and subscripts
- 3 Analytical procedure

3.1	General comments, 3.2 Design philosophy, 3.3 Computer programs
4	Modelling the lamina
4.1	General comments, 4.2 Two-dimensionally loaded lamina, 4.3 Laminae subject to three-dimensional loading, 4.4 Influence of loading type and duration, 4.5 Strength criteria, 4.6 Application of calculation programs
5	Modelling the laminate
5.1	Determining stresses and strains, 5.2 Lamina-by-lamina fracture analysis, 5.3 Application of lamina-by-lamina fracture analysis for different types of loading, 5.4 Cyclically loaded laminates, 5.5 Application of calculation programs
6	Analysis of FRP components
6.1	General comments, 6.2 Stability analyses, 6.3 Joints
7	Bibliography
8	Annex

It can be seen from the table of contents that all aspects important in check calculations for a FRP component are dealt with.

In Section 4 *Modelling the lamina* the fundamentals of modelling the lamina according to the elasticity theory are concisely presented while Section 5 *Modelling the laminate* does the same for the laminate by means of CLT. This means the guideline can be used as a work of reference and as a collection of formulae which uses the correct German and English terminology.

The sections of Part 3 which are briefly described below may be termed its 'highlights' since they are not only a straightforward collection of formulae but are also suitable for tackling the physical fundamentals in more depth. For this reason they are of great use not only to beginners but also to advanced designers and strength analysts working with FRP components.

5.1.1 Netting theory

Netting theory postulates that only the fibres transmit forces and that the fibres alone create the equilibrium with the force flows resulting from external loads. Today it is often unjustly regarded as outdated and therefore without value. Notwithstanding this it remains a fast and dependable method for carrying out preliminary design of laminates which takes proper account of loads. Use of this method ensures a 'rational' choice of the directions and fibre weights of the supporting fibre structure (Fig. 3). 'Robust' laminates can be identified by the fact that basically the fibre structure would be able to absorb the load on its own - precisely as is assumed in netting theory.

Due to the necessity of reaching a required critical speed, fast-running torsion shafts are often designed with a relatively small winding angle. A widely held but nevertheless erroneous opinion is that it is essential to select a winding angle of $\pm 45^\circ$ for the 'fibre-appropriate' structure of a torsion shaft. Netting theory, however, shows that the torsional moment can, for example, be absorbed 'fibre-appropriately' even with a $\pm 15^\circ$ fibre structure (see Fig. 3).

Guideline VDI 2014 Part 3 provides not only the elementary relationships for calculating fibre quantities but also optimisation rules derived from netting theory.

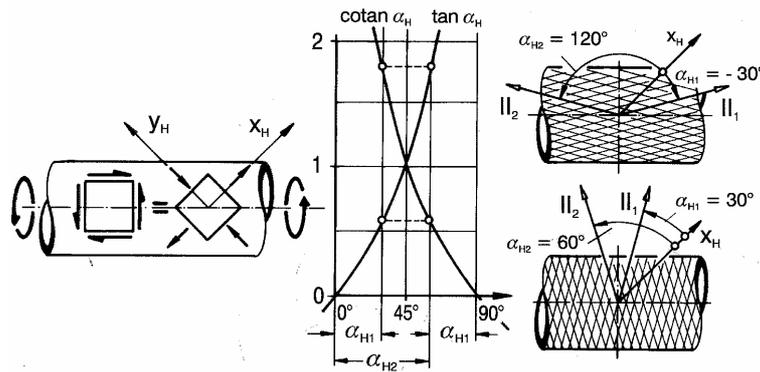


Fig. 3. Designing a torsion shaft with the aid of netting theory for winding angles not equal to $\omega = 45^\circ$. Torsion shafts with a winding angle $\omega = 15^\circ$ or $\omega = 75^\circ$ are capable of bearing loads according to netting theory (in other words, the external loads can be transferred by the fibre structure). The amount of fibre required for $\omega = 15^\circ$ and $\omega = 75^\circ$ is identical but is twice as much as is the case with a torsion tube with $\omega = 45^\circ$ (from [Puc96])

5.1.3 Interlaminar stresses (ILS)

This section deals not only quantitatively but also qualitatively with the most important types of interlaminar stresses, such as ILS at the free edges of the component, ILS between the laminae and last but not least ILS resulting from IFF. Common misunderstandings are cleared away - such as, for example, those concerning the effectiveness of 'soft' inserts between the laminae - while on the other hand promoting increased awareness of interlaminar stresses at the end of cracks caused by IFF and which can under certain circumstances be responsible for a drastic reduction in fatigue strength.

5.4 Cyclically loaded laminates

In contrast to earlier versions of Part 3, not only was a detailed presentation of the phenomenology of fatigue failure included but also information about carrying out an approximate fatigue strength calculation (see also [GL03]).

6.2 Stability analyses

This section provides not only a description of the phenomenology of buckling in FRP columns, plates and shells but also virtually all of the formulae required for carrying out a linear buckling analysis for FRP components. These are for the most part based on the laminate stiffnesses and compliances given in Section 5.1.2 such as, for example, the formula for critical shear flow in the case of torsion of an orthotropic cylinder with a symmetrical laminate ($R =$ radius, $l =$ length of the cylinder):

$$n_{xycr} = \frac{21.75}{2\pi} \cdot \frac{D_{22}^{5/8}}{a_{11}^{3/8} R^{3/4} l^{1/2}}$$

In addition, there is discussion of the application of the finite element method for carrying out linear buckling analyses and for the more complex but more realistic determinations of the critical load on thin-walled structures with the aid of geometrically non-linear analyses.

6.3 Joints

This section contains design advice and descriptions of the failure modes of loop, bolted and bonded joints as well as the set of formulae (and diagrams) required for a first assessment of these types of joint in FRP components.

4. The new fracture criteria of Puck and their application

One special highlight of guideline VDI 2014 Part 3 is the content of and the way in which the sections (examined in somewhat more detail below) are presented, this representing a compilation of the new, physically justified fracture criteria and their application which until now has not been available in this form anywhere else.

- 4.5 Strength criteria of the lamina
- 5.2 Lamina-by-lamina fracture analysis
- 5.3 Application of lamina-by-lamina fracture analysis for different types of loading
- 5.5.3 Calculation of the stretch factor of the load-determined stresses when residual stresses are also present
- Annex A1 Inclusion of stresses not acting on the fracture plane in the action-plane-related interfibre fracture criteria described in Section 4.5
 - A1.1 Inclusion of stresses $\sigma_n(\theta), \tau_{nt}(\theta), \tau_{n1}(\theta)$ which act on parallel-to-fibre planes but not on the fracture plane; and
 - A1.2 Inclusion of a parallel-to-fibre stress σ_1 in the action-plane IFF conditions
- Annex A2 Calculation of the stretch factor f_S^L of the load-determined stresses when residual stresses are also present.

For the sake of clarity the following sections are numbered in the same way as in Part 3.

4.5 Strength criteria of the UD lamina

Following an introductory treatment of the phenomenology of the fracture event in FRP, the main topic is a presentation of Puck's new, physically justified, action-plane IFF criteria. As early as 1969 Puck had no doubt recognized the necessity of distinguishing between IFF and FF and since that time has applied this in the strength analysis of FRP ([Puc69a], [Puc69b]). Even in HSB, in addition to a FF condition, there is also a (σ_2, τ_{21}) fracture curve for IFF but the latter is not provided with any more detailed physical rationale. The new fracture criteria are thus a decisive step forward in that they can provide a comprehensive physical explanation of the events in the interfibre fracture occurring on parallel-to-fibre planes.

Stimulated by a study published by Hashin in 1980 [Has80], Puck formulated in the last decade a self-contained, physically justified new fracture theory for the IFF on the basis of O. Mohr's brittle fracture criteria [Moh00] which date from around 1900. Puck's theory can explain the IFF event considerably better than previous theories and can above all provide information about what really happens during the fracture. In this way we can find out which fractures are relatively 'harmless' and which fractures could be extremely dangerous. The most important point as regards inclusion in the VDI guideline is however that the new fracture theory is tailored to the requirements and possibilities of design practice.

The basic hypothesis in Mohr's brittle fracture theory is:

“The failure limit of a (brittle-fracturing) material is determined by the stresses σ and τ acting on the fracture plane”.

If this basic idea is transferred to the transversally isotropic UD FRP, the following hypothesis emerges for IFF:

IFF is determined by the transverse stress σ_n and the shear stresses τ_{nt} and τ_{n1} acting on the fracture plane. In Fig. 4 (σ_n represents a σ_{\perp} stressing, while τ_{nt} represents a $\tau_{\perp\perp}$ stressing and τ_{n1} a $\tau_{\perp\parallel}$ stressing).

If the stress σ_n is a tensile stress, it facilitates fracture creation. If σ_n is a compressive stress, it makes it more difficult. This leads to a considerable deviation of the (σ_2, τ_{21}) failure curve from the closed ellipse form such as results from the old, not physically justified criteria such as those of Tsai or Hoffmann, for example. The form of the (σ_2, τ_{21}) failure curve (Fig. 5) which is predicted by the new physical approach was repeatedly confirmed experimentally in a major German research project conducted from 1994 to 1997 [Cun97].

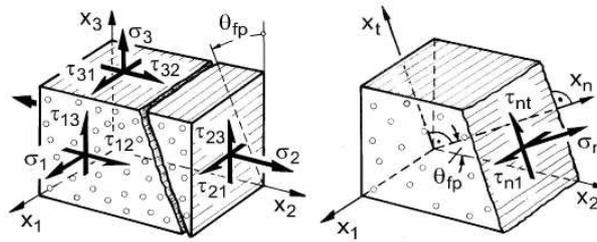


Fig. 4. Stresses in the demarcation planes vertical to x_1 , x_2 , x_3 and in an inclined parallel-to-fibre fracture plane of a UD element. θ_{fp} is the so-called ‘fracture angle’ of the fracture plane.

What is unfamiliar or unusual in the new fracture criteria is that, according to the fracture hypothesis, the only stresses made responsible for the fracture are those which act on the same plane (or to put it another way: which have a common action plane). For this reason they are termed *action-plane* fracture criteria. Since, on the one hand, stresses which have a *common action plane* help (or hinder) fracture, it is, on the other hand, also the fracture resistances of *the action plane* which determine the magnitude of the sustainable stresses and not the commonly used strength values. This calls for a fundamental rethink on the part of the user and Part 3 of guideline VDI 2014 provides valuable assistance in this regard.

A fracture resistance R^A is defined as follows: a *fracture resistance of the action plane* is that resistance which an action plane opposes to *its* fracture as the result of an individual stressing σ_{\perp}^t or $\tau_{\perp\perp}$ or $\tau_{\perp\parallel}$ acting *in it*. Caution: a closer examination will reveal that $R_{\perp}^{At} = R_{\perp}^t$ and $R_{\perp\parallel}^A = R_{\perp\parallel}$ but $R_{\perp\perp}^A \neq R_{\perp\perp}$.

As students we learn that when a concrete prism is subjected to the compression test the result is a shear fracture at $\pm 45^\circ$. A somewhat similar result would be expected with the transverse compression test of UD FRP. If the above hypothesis - that the influence of transverse compressive stress σ_n on the fracture plane is to make fracture more difficult - is correct, we would however expect an angle of the fracture plane of $\pm 50^\circ$ to $\pm 55^\circ$. The test series carried out in the course of the work behind the dissertations of Huybrechts [Huy96] and Kopp [Kop00] confirm this in an impressive manner (see Fig. 5(b)). This meant that it was possible to obtain an excellent verification of the most important basis of the action-plane IFF criteria.

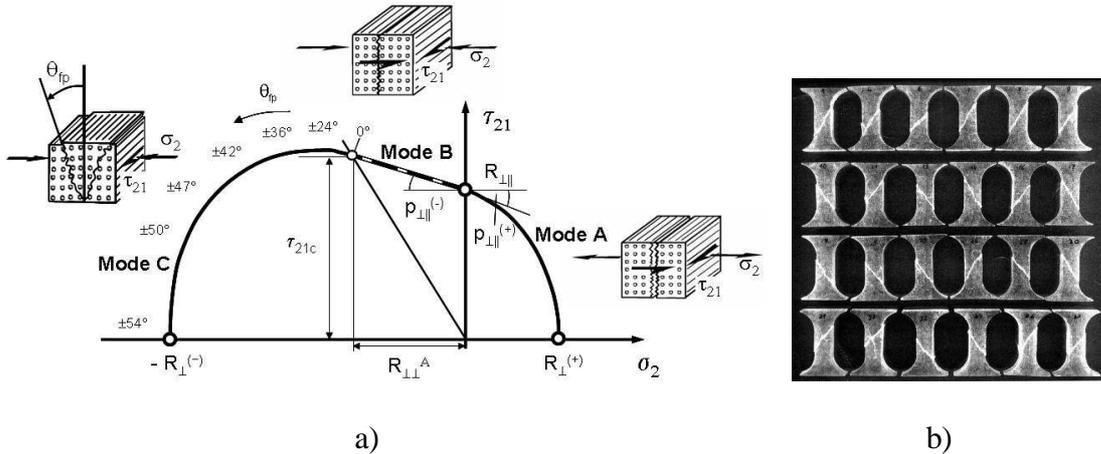


Fig. 5. (a) Fracture modes on the (σ_2, τ_{21}) fracture curve. Fracture modes A and B lead to 'straight' IFF ($\theta_{fp} = 0^\circ$) while with mode C, oblique IFF occurs ($\theta_{fp} = 0^\circ$ to $\pm 54^\circ$); (b) experimental result for pure transverse compression, $|\theta_{fp}| = 54^\circ \pm 3^\circ$ [Huy96]

Recent research results from the IKV in Aachen [Sch05] suggest that the *action-plane* fracture criteria even provide a good description of the interfibre fracture event in UD laminae with a thermoplastic matrix under quasi-static loading at room temperature.

Puck's practical experience in component design provides an example relating to the question as to HOW an IFF can lead to component failure: this is the sudden total failure observed in prototypes of FRP tubular torque springs exposed to high levels of static and cyclic loading resulting from an IFF. Here the oblique IFF in the outer lamina with a transverse compressive stress which is not normally regarded as hazardous results in the formation of oblique fracture surfaces on parallel-to-fibre fracture planes. These cause an absolutely intolerable, abrupt total failure of the spring (Fig. 6) by the appearing wedge effect.

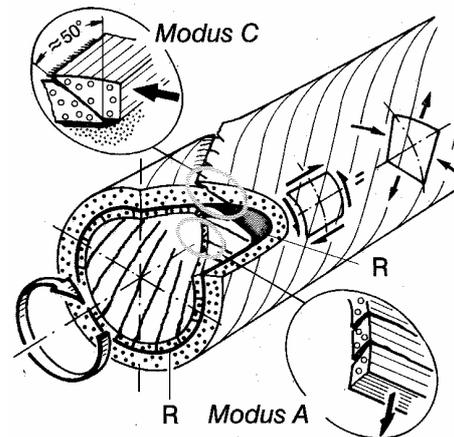


Fig. 6. IFF modes A and C as exemplified in an early prototype of a thick-walled torsion spring which was to be optimised for a long service life [Puc96], [Puc02a]

IFF mode C, which has just been described and which occurs in laminae with high transverse compression loading, can, in laminae embedded in the laminate, lead to the composite being forced open and can initiate delamination. As a result of the compressive force still acting on the fracture surfaces even after the fracture, the wedge-shaped IFF also results in a strong breaking-up effect acting vertically to the laminar plane.

Further experimental confirmation for Puck's fracture theory emerged in particular during work on dissertations at the Institute of Plastics Processing (IKV) of the RWTH Aachen: this was on the basis of measurements of the angle of the fracture plane during IFF in test

specimens and torsion shafts [Huy96], [Kop00]. Other studies include the dissertations of Knops [Kno03] and Fischer [Fis03].

In Section 4.5 of Part 3 the reader will find the formulae for the action-plane IFF criteria for treating not only the (σ_2, τ_{21}) stress state just described but also for the general three-dimensional stress state. As has already been frequently stated by Puck, in the context of application of action-plane fracture criteria to determine fracture stresses, Hashin saw the necessity of having to first calculate the angle of the fracture plane as an insurmountable obstacle to the introduction of this kind of criteria into design praxis. Since then the situation has very much improved as regards computer capacity and the availability of generally accessible basic computing programs (such as Microsoft Excel) and there is now no problem in carrying out a numerical iterative search for the fracture plane angle as part of routine design work. But for the plane $(\sigma_1, \sigma_2, \tau_{21})$ stress state which is dominant in many components, Puck did manage to find an analytic solution for the angle of the fracture plane by introducing a ‘parameter coupling’ [Puc02a]:

$$\cos \theta_{fp} = \sqrt{\frac{1}{2(1+p_{\perp\perp}^c)} \left[\left(\frac{R_{\perp\perp}^A}{R_{\perp\parallel}} \right)^2 \cdot \left(\frac{\tau_{21}}{\sigma_2} \right)^2 + 1 \right]}$$

The stress σ_1 does not have any influence on the angle of the fracture plane. This means the course of the fracture curve can be dealt with using closed formulae, which means that the stress exposure can be calculated directly. The following formulae (45) to (47) from Part 3 are used for this:

For Mode A ($\sigma_2 \geq 0$):

$$f_E = \frac{1}{R_{\perp\parallel}} \left(\sqrt{\left(\frac{R_{\perp\parallel}}{R_{\perp}^t} - p_{\perp\parallel}^t \right)^2 \sigma_2^2 + \tau_{21}^2} + p_{\perp\parallel}^t \sigma_2 \right)$$

For Mode B:

$$f_E = \frac{1}{R_{\perp\parallel}} \left(\sqrt{\tau_{21}^2 + (p_{\perp\parallel}^c \sigma_2)^2} + p_{\perp\parallel}^c \sigma_2 \right)$$

For Mode C:

$$f_E = \frac{\tau_{21}^2}{4(R_{\perp\parallel} + p_{\perp\parallel}^c R_{\perp\perp}^A)^2} \cdot \frac{R_{\perp}^c}{(-\sigma_2)} + \frac{(-\sigma_2)}{R_{\perp}^c}.$$

The variables $p_{\perp\parallel}^t$ and $p_{\perp\parallel}^c$ which appear in these formulae are so-called inclination parameters [Puc02b].

The stress exposure f_E is mentioned here for the first time in this connection. It is by definition the ratio of the length of the vector of the stresses $\{\sigma\}$ which are present and the length of the vector $\{\sigma\}_{fr}$ of the stresses leading to fracture. This is shown in Fig. 7 in the example of a (σ_2, τ_{21}) stress combination consisting of load-determined stresses and residual stresses. The reciprocal of the *stress exposure* f_E is termed the *stretch factor* f_S .

weakening of the matrix caused by this damage on the micro scale be taken into account even in the fracture resistances of the action plane which are of decisive importance to IFF.

For this reason a method already mentioned in [Puc96] is presented in Appendix A1.2. This method results in a very simple (as regards computing) expansion of the IFF criteria in order to cover the influence of σ_1 , and one which is also on a well-founded physical basis. Fig. 8 shows a (σ_1, σ_2) fracture curve for IFF and the influence of parameters s and m , as proposed in A1.2. The curve for weakening (in this case taking the example of the σ_2 stress which results in fracture) is described by an ellipse. The 's' variable marks the beginning of the 'weakening' while 'm' indicates the maximum weakening (this corresponds to the minimum weakening factor η_w , where w stands for 'weakening').

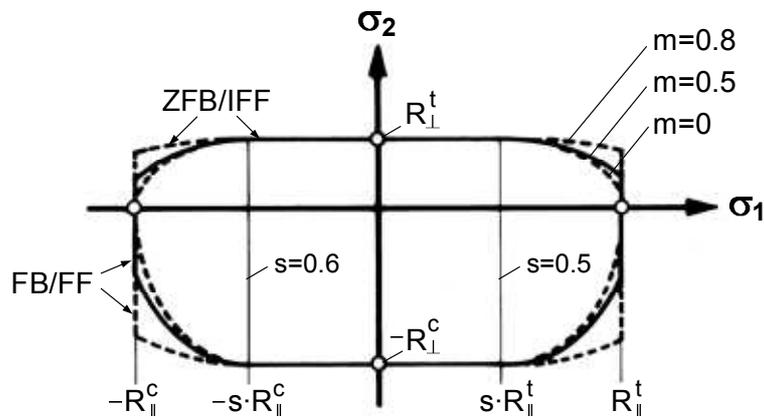


Fig. 8. Shape of a (σ_1, σ_2) fracture curve for IFF with weakening influence of σ_1 as a function of parameters s and m [Kai04]

The analytical method for including the influence of σ_1 (described in Appendix A1.2) has already been included in the laminate analysis program ESACOMP. The formalism selected permits a very flexible adaptation to test results although there is currently still a lack of such results.

As has already been stated, Part 3 also provides the formulae for evaluation of IFF in the case of generally 3D stress states. This possibility in IFF analysis and evaluation will certainly become important in the future - for example, in the field of load introduction. For both (σ_2, τ_{31}) and (σ_2, σ_3) stress combinations a strict application of the pure Mohr hypothesis would result in straight lines fracture curves in the first quadrants of (σ_2, τ_{31}) and (σ_2, σ_3) which would meet in sharp corners (Fig. 9).

Reflections about taking into account the influences of probabilistic aspects and microdamage (reflections which had already been mentioned in [Puc96]) have been fully worked out for publication in Appendix A1.1. On account of the unusual subject-matter, Part 3 in this case provides not only the formal framework required for an analytical treatment but also detailed assistance in understanding the corresponding interrelationships.

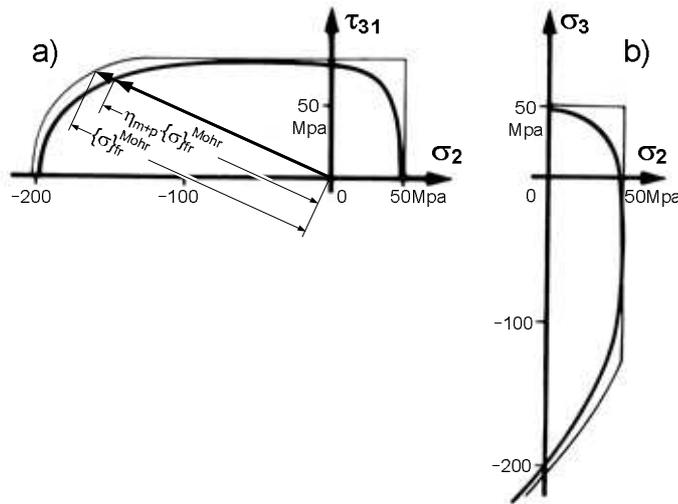


Fig. 9. (σ_2, σ_3) and (σ_2, τ_{31}) fracture curves calculated by the method given in Appendix A1.1 of VDI 2014 Part 3 for including the effects of probabilistic aspects and microdamage

The basic idea is that the calculation of the stress exposure $f_E(\theta)$ (which needs to be carried out anyway when finding the fracture angle in the case of three-dimensional stress states) should be exploited in that range of the intersection angle from $\theta = -90^\circ$ to $\theta = +90^\circ$. The ‘roundness’ of the stress exposure curve $f_E(\theta)$ serves as a criterion for a reduction in the stresses leading to fracture due to the effects of probabilistic aspects and microdamage. The fracture curves obtained on the basis of these reflections are shown in Fig. 9. Special note should be taken of the rounded course of the curve in the first quadrant of the (σ_2, σ_3) and (σ_2, τ_{31}) fracture curve.

Annex A2 Calculation of the stretch factor f_s^L of the load-determined stresses when residual stresses are also present.

As can be seen from Fig. 7, the stress exposure f_E does not tell us anything about how far the load-determined stress can be increased (when constant residual stresses are also present) before IFF would occur. In the field of industrial applications of FRP, however, the mandatory codes of practice, e.g. [DNV00] for the design of FRP drive shafts and couplings in marine propulsion units, for example, do among other things provide theoretical checks for those factors by which load-determined stresses can permissibly be increased before IFF occurs.

This question is dealt with in Section 5.5.3. The so-called stretch factor f_s^L of the *load-determined stresses* while fixed residual stresses are also present is defined (Fig. 10). Appendix A2 provides the complete set of formulae and the calculation sequence together with the necessary background information on calculating f_s^L , this taking into account a weakening influence of σ_1 . Even this section, which is of prime importance to the designer, has not been published in this form anywhere except in guideline VDI 2014 Part 3.

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