# The *Puck* theory of failure in laminates in the context of the new guideline VDI 2014 Part 3

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## ABSTRACT

The new guideline VDI 2014 Part 3 has been released for printing in September 2006. In addition to a condensed presentation of general topics which are essential for the calculation of fibre reinforced plastics (FRP) components such as the modelling of various lamina types, the laminate analysis by means of netting theory and Classical Laminate Theory, elastic stability analyses, joints and cyclically loaded laminates the application of the new physically based 'action plane' fracture criteria of *Puck* is comprehensively described in the guideline. The *Puck* theory of failure in laminates is both physically correct and easy to use in design practice. It is based on the Coulomb-Mohr hypothesis of brittle fracture. This paper gives some interesting background information on this theory.

## 1. The new guideline VDI 2014 Part 3

Frequently, the development of components made from fibre reinforced plastics (FRP) takes unacceptably long time and the development costs are too high. The great variety in the selection of materials and laminate construction offers on the one hand quite a number of possible solutions, but on the other hand it increases the effort required to find an 'optimized' component design. In addition to this it is rather difficult for an engineer active in practice to become aware of the progress in research and to assess its relevance and practical applicability. Therefore, work has been going on since many years on a VDI (Verein Deutscher Ingenieure) guideline with the aim to facilitate the design work for FRP components, by providing a compilation of proven methods for FRP laminate and component design.

The value of the meaningful terminology and symbols well adapted for FRP, which is provided in the guideline, is not to be underestimated. After some more general considerations on methods for the component design and information on available computer programs an extensive chapter dealing with the modelling of laminates follows. According to the recognized need in design practice macro-mechanical and phenomenological models are used throughout. However they cover both in quantitative and qualitative manner the necessary physical basis. The smallest computational element is the single lamina within the laminate, being reinforced by random mat, woven fabric or uni-directionally (UD) oriented fibres. Within the guideline, which comprises around 160 pages, one also finds advices for the design and analysis of several types of joints, such as bolted, loop and bonded joints. In order to account for the risk of failure due to the loss of elastic stability of thin walled FRP components, the state of the art buckling formulae are given. Cyclically loaded laminates are treated both in the chapter "lamina-by-lamina" fracture analysis and in a separate paragraph for design advices.

## 2. The *Puck* theory of failure in laminates in the guideline VDI 2014 Part 3

A large part (about 50 pages) of the guideline [VDI06] is dedicated to the laminate failure theory of Puck. There is still considerable uncertainty among designers on how to assess the confusing amount of failure theories and degradation models for FRP. Apparently, even the World Wide Failure Exercise (WWFE) [Hin04] could not remove the uncertainty in this field. The alarmingly high divergences between the results of different theories for identical problems produced by the participants of the WWFE may have even increased the scepticism of practitioners. The computational results of the different theories have been assessed by the organizers of the WWFE only with regard to a comparison with the provided test results, but in some cases these are incomplete and a little ambiguous. This is especially true for the results on 'isolated' UD-test specimen (see for instance Figure 10 to 12 in [Puc02]) which are used for the judgement of failure criteria for interfibre failure, see also [Kno07]. For the ranking of the theories there were no considerations on the physical background of the theories. In order to counteract the feeling of uncertainty of practitioners, the topic 'strength analysis' has been treated in a somewhat unusual way in VDI 2014 Part 3 insofar as not only the procedures are described but, in some sections rather comprehensive explanations are given on the physical and mathematical background of the theory.

Puck started his work on successive failure processes in GRP laminates in about 1955, when he and a few other enthusiastic members of the Darmstadt Technical University began to develop, build and fly the first sailplane with an all glass fibre/epoxy wing featuring a low drag laminar profile. In 1959, he became a member of the composite materials research group at the German Plastics Institute in Darmstadt. Before he left in 1969 for GRP industry he published two papers [Puc69a; Puc69b] in which he described already the general procedure of a non-linear lamina-by-lamina fracture analysis of laminates. Puck was obviously the first author, who has published the idea that fibre failure (FF) and interfibre failure (IFF) should be distinguished and theoretically treated by separate and independent failure criteria [Puc69a]. This was at that time a first, but decisive, step on the way to physically based failure criteria.

For a physically based theoretical treatment of the successive failure process in laminates, the Puck theory supplies at least 4 essential topics:

1) Non-linear stress and strain analysis before IFF
2) Physically-based action plane related fracture criteria for IFF and FF
3) A continuous degradation after the onset of IFF
4) Considerations on total failure of a laminate

## 3. Phenomenological observations on brittle fracture of UD-composites

On a unidirectional composite element, there are 6 different types of stresses which can produce fracture, see Figure 1:

- $\sigma_{\parallel}$-tension and $\sigma_{\parallel}$-compression
- $\sigma_{\perp}$-tension and $\sigma_{\perp}$-compression
- $\tau_{\perp\perp}$-shear
- $\tau_{\perp\parallel}$-shear

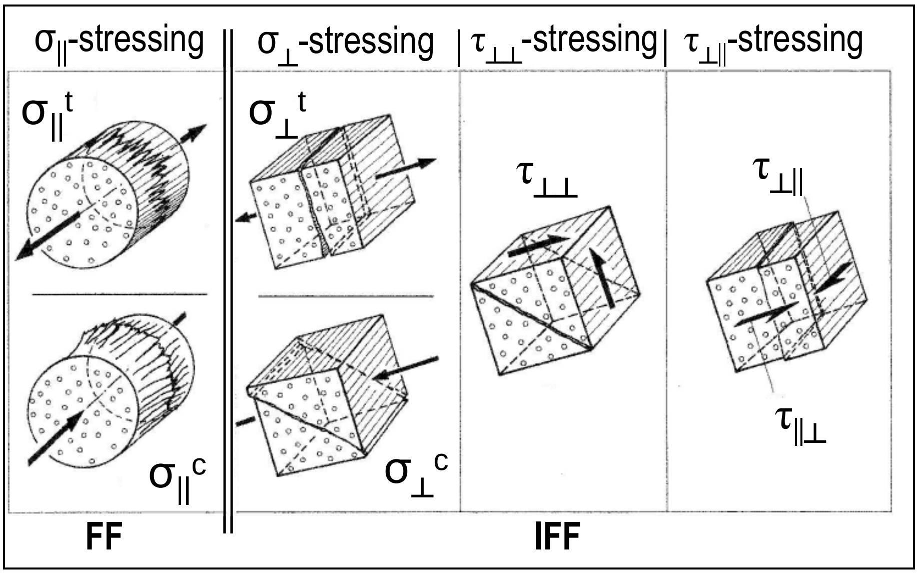


Figure 1 The basic **stressings** (stressing means type of stress) of a UD-composite element
$\sigma_{||}$ stressing is responsible for fibre failure (FF), while $\sigma_{\perp}$, $\tau_{\perp\perp}$, $\tau_{\perp||}$ stressings cause interfibre failure (IFF).
Also shown are the planes in which brittle fracture occurs ($\tau_{||\perp}$ can not produce a fracture of its action plane, because fibres would have to be sheared off. Fracture occurs much easier on the parallel to fibre action plane in which $\tau_{\perp||}$ with the same magnitude as $\tau_{||\perp}$ is acting).

Remark: According to the terminology used in VDI 2014 Part 3 symbols for tensile stresses, stressings and strengths are denominated by the superscript $^t$ and those for compressive stresses, stressings and strengths by the superscript $^c$.

In order to characterize certain **types of stress** *Puck* introduces the concept of **'stressing'**. Consider for instance a $\sigma_{\perp}$-stressing, it is quite unimportant whether we are dealing with a stress $\sigma_2$ or $\sigma_3$ stress or with a stress $\sigma_n$ as shown in Figure 2; the decisive feature is that it is acting transverse ($\perp$) to the fibre direction.

In accordance with the 6 stressings shown in Figure 1, also 6 corresponding strengths exist and they are designated as follows:

$$R_{||}^{t}, R_{||}^{c}, R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\perp}, R_{\perp||}\,.$$

It is important to note that the widely used term 'strength' means the experimentally determined ultimate load divided by the cross section of the plane, which is oriented perpendicularly to the applied stress. This is valid for a tensile or compressive load. If a shear strength has to be determined the shear load at failure is divided by the area of the plane in which the shear load was acting. However in case of brittle fracture behaviour of the material this plane is in most cases not identical with the fracture plane. When determining a strength value, there is normally no attention paid to the failure mode of the material which can be for instance extensive yielding, crushing or brittle fracture or whatever and also not to the orientation of a fracture plane, if such a plane would appear.

The fracture planes which appear when a UD-composite is loaded by a given stressing until fracture is reached, are also shown in Figure 1. Such fractures can always be observed when GFRP or CFRP with a non-flexibilized thermoset matrix are tested.

With transverse compression $\sigma_{\perp}^{c}$ for instance an oblique fracture plane occurs. From this observation we conclude that the fracture is produced mainly by shear. Also with transverse/transverse shear $\tau_{\perp\perp}$-stressing the fracture plane is inclined by about 45° against the action plane of $\tau_{\perp\perp}$. Obviously the appearing fracture is a cleavage fracture caused by the maximum principal (tensile) stress which acts on a plane inclined by 45° against the action plane of $\tau_{\perp\perp}$.

These observations lead to the following conclusion: Uniaxial transverse compressive stressing $\sigma_{\perp}^{c}$ causes fracture in a plane which does not coincide with the action plane of $\sigma_{\perp}^{c}$. Also pure $\tau_{\perp\perp}$ causes fracture in a plane which does not have the same direction as the action plane of $\tau_{\perp\perp}$. Only the tensile $\sigma_{\perp}^{t}$-stressing and the $\tau_{\perp\|}$-stressing produce a fracture in their action planes, i.e. their action plane is also a fracture plane.

## 4. Failure hypotheses based on the experimental observations

The observations mentioned above lead to the conclusion that we are dealing with typical **brittle fracture behaviour**. Therefore, for composites of this kind which are intrinsically brittle it does not make sense to adopt criteria similar to the well-known '*von Mises* yield criterion' for ductile metals. It seems much more reasonable to step back to the roots of brittle fracture theory, which means to the ideas of Coulomb and Mohr. It was around 1900 that Otto Mohr came up with his surprisingly simple fracture hypothesis:

**'The stresses on the fracture plane are decisive for fracture'.**

This hypothesis is very easy to understand but it is not as easy to handle, because *the location of the fracture plane is not known a priori*. The position of the fracture plane has to be determined first, before the fracture stresses can be calculated using a suitable brittle fracture criterion which, according to the above hypothesis, has to be based on the stresses on the fracture plane.

The idea of using the concept of Mohr was originally recommended by *Hashin* in his well-known 1980 paper [Has80] but he did not further pursue this idea. *Puck* tackled the task of modifying the Coulomb-Mohr Theory for the application on UD-composites in 1992.

*Puck's* fracture hypothesis adapted to a UD-composite element reads as follows:

**The normal stress $\sigma_n$ and the shear stress $\tau_{nt}$ and $\tau_{n1}$ on the fracture plane** (see Figure 2) **are decisive for Interfibre Fracture (IFF).**

**A tensile stress $\sigma_n$ supports the fracture, while in contrast a compressive stress 'makes the material stronger'.** In other words: A compressive $\sigma_n$ impedes the IFF brought about by the shear stresses $\tau_{nt}$ and $\tau_{n1}$.

When comparing Figure 2 and Figure 1, it can be seen that $\sigma_n$ causes a normal stressing (tensile or compressive), $\tau_{nt}$ causes a $\tau_{\perp\perp}$-stressing and $\tau_{n1}$ a $\tau_{\perp\|}$-stressing.

One is obviously confronted now with the following situation:

**It is not possible to formulate a physically based fracture criterion using the stresses $\sigma_2$, $\sigma_3$, $\tau_{23}$, $\tau_{31}$, $\tau_{21}$ which are related to the normally used natural axes of the UD-composite, because a brittle fracture hypothesis for the combined action of these stresses does not exist.**

According to *Coulomb-Mohr*, the brittle fracture criterion for IFF of UD-composites has to be formulated with the 3 stresses $\sigma_n$, $\tau_{nt}$, $\tau_{n1}$, which are acting on the fracture plane. (The given 5 stresses $\sigma_2$, $\sigma_3$, $\tau_{23}$, $\tau_{31}$, $\tau_{21}$ reduce to a set of only 3 stresses $\sigma_n$, $\tau_{nt}$, $\tau_{n1}$ which are relevant for IFF).

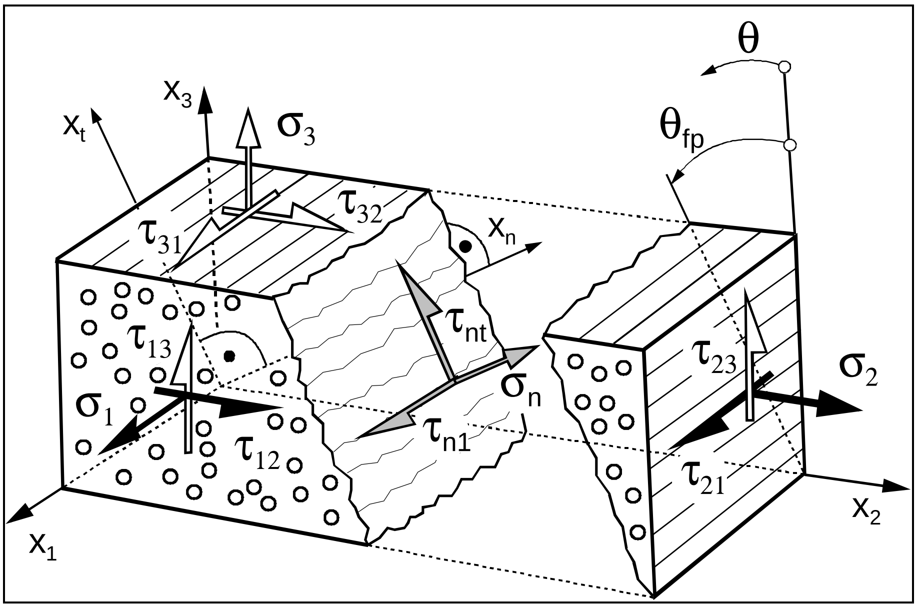


Figure 2 Stresses on a UD-composite element. Stresses $\sigma_1$, $\sigma_2$, $\sigma_3$, $\tau_{23}$ (= $\tau_{32}$), $\tau_{31}$ (= $\tau_{13}$), $\tau_{21}$ (= $\tau_{12}$) act on planes perpendicular to the natural axis $x_1$, $x_2$, $x_3$. The stresses $\sigma_n$, $\tau_{nt}$, $\tau_{n1}$, which govern the brittle fracture act on the fracture plane which is perpendicular to the $x_n$ axis and inclined against the the thickness direction $x_3$ by the 'fracture angle' $\theta_{fp}$.

## 5. Development of physically based fracture criteria and their application

For completeness of the following set of fracture conditions, the maximum stress failure conditions for fibre failure (FF) [Puc69a] are shown first.

**FF:**

$$\frac{\sigma_1}{R_\parallel^t} = 1 \quad for\ \sigma_1 > 0 \tag{1a}$$

$$\frac{\sigma_1}{\left(-R_\parallel^c\right)} = 1 \quad for\ \sigma_1 < 0 \tag{1b}$$

From now on we shall concentrate solely on *Puck's* theory for Interfibre Fracture (IFF).

The 3 stresses $\sigma_n$, $\tau_{nt}$, $\tau_{n1}$ on the fracture plane which are relevant for brittle fracture, are proportional to the given 5 stresses $\sigma_2$, $\sigma_3$, $\tau_{23}$, $\tau_{31}$, $\tau_{21}$ according to the stress transformation in equation (2):

$$\begin{Bmatrix} \sigma_n\left(\theta_{fp}\right) \\ \tau_{nt}\left(\theta_{fp}\right) \\ \tau_{n1}\left(\theta_{fp}\right) \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc & 0 & 0 \\ -sc & sc & \left(c^2 - s^2\right) & 0 & 0 \\ 0 & 0 & 0 & s & c \end{bmatrix} \begin{Bmatrix} \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{21} \end{Bmatrix} \tag{2}$$

$$where\ c = \cos\theta_{fp}\ and\ s = \sin\theta_{fp}$$

With the so called 'Mohr-stresses' $\sigma_n(\theta_{fp})$, $\tau_{nt}(\theta_{fp})$, $\tau_{n1}(\theta_{fp})$ the following brittle-fracture-conditions have been formulated by Puck [Puc96; Puc02; Kno07]. According to the IFF hypothesis only 2 different fracture conditions for IFF are required:

**IFF:**

$$\left(\frac{\tau_{nt}}{R_{\perp\perp}^{A}}\right)^2+\left(\frac{\tau_{n1}}{R_{\perp\parallel}^{A}}\right)^2+(1-c)\left(\frac{\sigma_n}{R_{\perp}^{t\,A}}\right)^2+c\left(\frac{\sigma_n}{R_{\perp}^{t\,A}}\right)=1 \quad for\,\sigma_n\geq 0 \tag{3a}$$

$$\left(\frac{\tau_{nt}}{R_{\perp\perp}^{A}-p_{\perp\perp}^{c}\cdot\sigma_n}\right)^2+\left(\frac{\tau_{n1}}{R_{\perp\parallel}^{A}-p_{\perp\parallel}^{c}\cdot\sigma_n}\right)^2=1 \quad for\,\sigma_n<0 \tag{3b}$$ *

The constant c in equation (3a) controls the slope of the contour line of the body described by equation (3a) at the point, where it reaches the ($\tau_{nt}$, $\tau_{n1}$)-plane at $\sigma_n = 0$.

Since the above IFF conditions are modified Coulomb-Mohr criteria, one has to be very careful when establishing the 'strength' values which have to be used in these fracture conditions.

From the foregoing it is to conclude that global fracture criteria can be formulated with the stresses $\sigma_2$, $\sigma_3$, $\tau_{23}$, $\tau_{31}$, $\tau_{21}$ and the corresponding strengths $R_{\perp}^{t}, R_{\perp}^{c}, R_{\perp\perp}, R_{\perp\parallel}$ from uniaxial tests or tests with pure shear respectively.

By contrast, IFF-criteria which are physically based on a modified Mohr-Coulomb hypothesis have to be formulated with 'Mohr-stresses' $\sigma_n(\theta_{fp})$, $\tau_{nt}(\theta_{fp})$, $\tau_{n1}(\theta_{fp})$ and 'fracture resistances $R^A$ of the fibre-parallel action plane'. Corresponding to the 3 stressings $\sigma_{\perp}^{t}$, $\tau_{\perp\parallel}$, $\tau_{\perp\perp}$ which can, in the most general case, be involved in bringing about the IFF, 3 corresponding 'fracture resistances $R^A$ of the fibre-parallel action plane' have to be used, namely:

$$R_{\perp}^{t\,A}, R_{\perp\perp}^{A}, R_{\perp\parallel}^{A}$$ (the superscript A means action plane).

The following definition is valid:

**The fracture resistance of an action plane is the maximum resistance, with which the action plane can resist its own fracture caused by a uniaxial $\sigma_{\perp}^{t}$ -stressing or a pure $\tau_{\perp\perp}$-stressing or $\tau_{\perp\parallel}$-stressing respectively.**

Important advice: The *action plane fracture resistance* $R_{\perp\perp}^{A}$ must not be mixed up with the *strength* $R_{\perp\perp}$! The value for $R_{\perp\perp}^{A}$ can not be determined from a pure $\tau_{\perp\perp}$-test, instead $R_{\perp\perp}^{A}$ must be *calculated* from the result of a transverse compression test [Puc02; Kno07]. The reason is given in Figure 1: If one deals with intrinsically brittle material a pure $\tau_{\perp\perp}$-stressing never produces a fracture in the action plane of $\tau_{\perp\perp}$, and therefore the measured strength (i.e. the transverse/transverse-shear load divided by the area of the plane in which the shear load is applied) would not deliver the value of $R_{\perp\perp}^{A}$ but instead the transverse/transverse shear strength $R_{\perp\perp}$.

With regard to a compressive stress $\sigma_n$, it is evident that the latter can certainly not produce a fracture in its own action plane, which would mean a separation of material in a plane which

* In the literature on Puck's fracture criteria some minor modifications of equation (3b) are to be found, which mainly serve for mathematical simplification. The above form of Equation (3b) is shown here because it demonstrates best the physical background of the strength increasing compressive stress $\sigma_n$.

is perpendicular to the direction of the acting compressive stress. In an abstract mathematical sense this could be expressed by $R_{\perp}^{cA} = \infty$ . In other words: We can forget $R_{\perp}^{cA}$ .

Very remarkable and favourable is the fact that the interfibre fracture behaviour of every arbitrary state of stress – may it be unidirectional or 5-dimensional – is governed by only one of the two equations, (3a) or (3b). Which one of the two equations has to be applied depends on the answer to the following question: Is $\sigma_n$ a tensile stress, or is $\sigma_n$ a compressive stress? It has been found that this discrimination is necessary, because tensile $\sigma_n$ helps to produce fracture while compressive $\sigma_n$ 'makes the material stronger'. This last fact can easily be seen in the IFF equation (3b): Here the fracture producing shear stresses $\tau_{nt}$, $\tau_{n1}$ are in the numerator but the 'strengthening' stress $\sigma_n$ appears in the denominator. The numerical value of a compressive $\sigma_n$ is negative, therefore in the denominators of equation (3b) the amount of $-p_{\perp\perp}^{c} \cdot \sigma_n$ and $-p_{\perp\parallel} \cdot \sigma_n$ is positive and added to the intrinsic fracture resistances of the material $R_{\perp\perp}^{A}$ or $R_{\perp\parallel}^{A}$ respectively.

In order to calculate the magnitude of the fracture stresses at IFF the vector of the existing stresses $\sigma_n(\theta_{fp})$, $\tau_{nt}(\theta_{fp})$, $\tau_{n1}(\theta_{fp})$ must be increased (stretched) by a stretch factor $f_S$ (see Appendix) in order to achieve fracture. Because of the linear relationship shown in equation (2) between the stresses $\sigma_n(\theta_{fp})$, $\tau_{nt}(\theta_{fp})$, $\tau_{n1}(\theta_{fp})$ and the stresses $\sigma_{2,}$ $\sigma_{3,}$ $\tau_{23,}$ $\tau_{31,}$ $\tau_{21}$ the stretch factor $f_S$ found for $\sigma_n(\theta_{fp})$, $\tau_{nt}(\theta_{fp})$, $\tau_{n1}(\theta_{fp})$ is also valid for the stresses $\sigma_{2,}$ $\sigma_{3,}$ $\tau_{23,}$ $\tau_{31,}$ $\tau_{21}$.

## 6. The master fracture body

The fracture body in ($\sigma_n$, $\tau_{nt}$, $\tau_{n1}$)-stress space described by the two IFF-equations (3a) and (3b) (one for tensile $\sigma_n$ and one for compressive $\sigma_n$) is called the **master-fracture body**, it is shown in Figure 3. Its surface envelopes all ($\sigma_n$, $\tau_{nt}$, $\tau_{n1}$)-stress vectors which the UD-composite material can withstand without fracture. Or in other words: The outer surface of the master fracture body consists of all points representing combinations of stresses $\sigma_n$, $\tau_{nt}$, $\tau_{n1}$ (Figure 2) which by their combined action produce an interfibre fracture (IFF) of the UD-composite.

The problem which arises is the following: Until the 'fracture angle' $\theta_{fp}$ has not been determined, the direction of the fracture vector in ($\sigma_n$, $\tau_{nt}$, $\tau_{n1}$)-space, i.e. within the master fracture body, is also not known. How to find the fracture angle $\theta_{fp}$? The key for solving this problem is the knowledge that the fracture will occur on the plane parallel to the fibres, on which the risk of fracture under the ($\sigma_n$, $\tau_{nt}$, $\tau_{n1}$)-stress combination under consideration has its maximum. Since the risk of fracture is proportional to the so called **stress exposure ratio $f_E$** (see Appendix), the fracture plane is the one with the highest stress exposure $f_E$ due to the stress combination acting on the plane under consideration. Both a numerical and an analytical method for searching the fracture angle is well established and for instance also described in detail in the new guideline VDI 2014 Part 3 [VDI06].

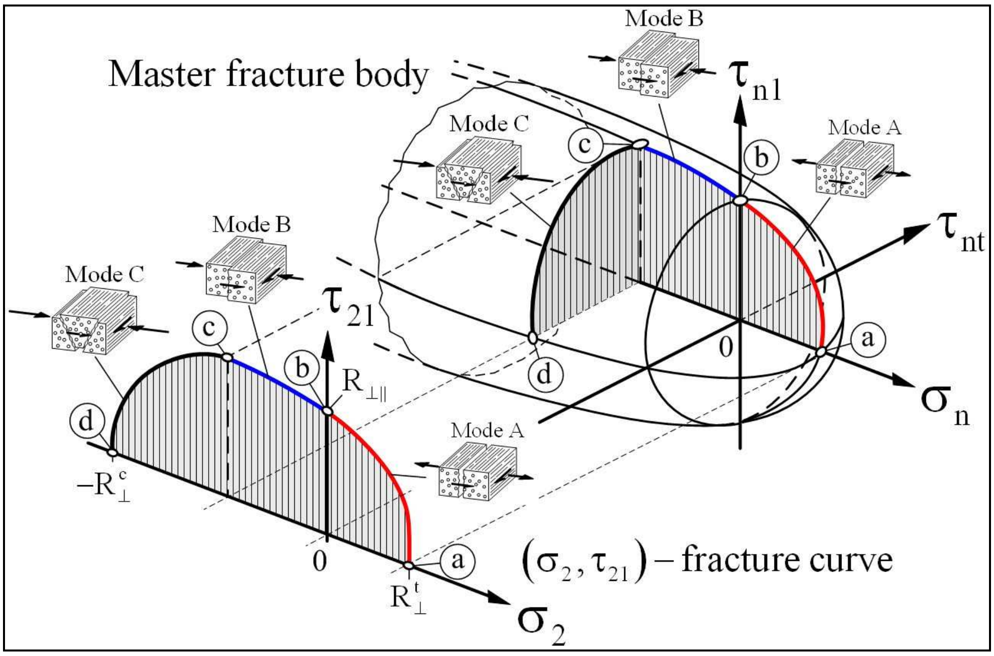


Figure 3 The upper right part of the figure shows the master fracture body in ($\sigma_n$, $\tau_{nt}$, $\tau_{n1}$)-stress space. The master fracture body governs the strength against brittle interfibre fracture caused by any combination of stresses $\sigma_2$, $\sigma_3$, $\tau_{23}$, $\tau_{31}$, $\tau_{21}$. The master fracture body is open towards negative $\sigma_n$, because compressive $\sigma_n$ cannot cause fracture of its action plane.
As a well known example the fracture curve for combined action of $\sigma_2$ and $\tau_{21}$ in the range from $\sigma_2 = R_{\perp}^{t}$ to $\sigma_2 = -R_{\perp}^{c}$ (= negative compressive strength) is shown on the surface of the master fracture body. In the lower left part of Figure 3, the usual presentation of a ($\sigma_2$, $\tau_{21}$)-fracture curve in a ($\sigma_2$, $\tau_{21}$)-plane can be found.
The curve sections (a) → (b) are identical on the master fracture body and on the ($\sigma_2$, $\tau_{21}$)-curve because the fracture angle is $\theta_{fp} = 0°$. In this case $\sigma_n = \sigma_2$ and $\tau_{n1} = \tau_{21}$ is valid while $\tau_{nt} = 0$, see equation (2). The same applies to the curve section between (b) and (c). Surprisingly the stress $\sigma_n$ on the fracture plane keeps a constant value between point (c) and point (d), that means in the whole range of 'oblique fractures' with $\theta_{fp} \neq 0°$. At point (c) the fracture angle $\theta_{fp}$ is still 0°, from there to the point (d) it increases up to $|\theta_{fp}| = 50°$ to 55°. More about the highly interesting features of this figure can be found in the literature [Puc96; Puc98; Puc02; Kno07].

## 7. Fracture modes

As soon as on this physical basis the ($\sigma_n$, $\tau_{nt}$, $\tau_{n1}$)-stress combination at IFF is found, we can also step back into the stress space of the given stresses i.e. into the ($\sigma_2$, $\sigma_3$, $\tau_{23}$, $\tau_{31}$, $\tau_{21}$)-stress space by means of the stretching factor $f_S$ (see Appendix). The result of this is graphically shown in Figure 3 on hand of the chosen example of a ($\sigma_2$, $\tau_{21}$)-combination.

*Puck* distinguishes between the **different fracture modes**, according to the **stress combinations which appear on the detected fracture plane.**

In Figure 3 between points (a) and (b) we can see a Mode A, which corresponds to a combined action of tensile $\sigma_n^t$ (which produces $\sigma_{\perp}^{t}$) and $\tau_{n1}$ (which produces $\tau_{\perp\parallel}$-stressing) on the fracture plane.

Mode B between points (b) and (c) corresponds to a combined action of compressive $\sigma_n^c$ (□ $\sigma_{\perp}^{c}$) and $\tau_{n1}$ (□ $\tau_{\perp\parallel}$).

Mode C between points (c) and (d) is a most general case, that is a combination of compressive $\sigma_n$ with $\tau_{nt}$ and $\tau_{n1}$ on the fracture plane which corresponds to a combined ($\sigma_{\perp}^{c}, \tau_{\perp\perp}, \tau_{\perp\parallel}$)-stressing.

A finding which is very important **for practice** is that in the section (c) to (d) of the ($\sigma_2$, $\tau_{21}$)-curve, where inclined fracture planes occur, these can lead to disastrous splitting effects ('wedge effect') of the laminate. There are unique photographs of such oblique fractures which have caused a sudden total failure of a prototype tubular spring [Puc96; Puc02]. All these important findings are also addressed in VDI 2014 Part 3.

Figure 4 shows an example of the resulting 3-dimensional fracture body for a ($\sigma_2$, $\sigma_3$, $\tau_{21}$)-stress combination which has been found by means of the master fracture body in Figure 3. The marked sub-surfaces belong to the appearing fracture modes. Fracture bodies of this kind have very little similarity with the smooth ellipsoids which we know for instance from the Tsai-Wu failure criterion.

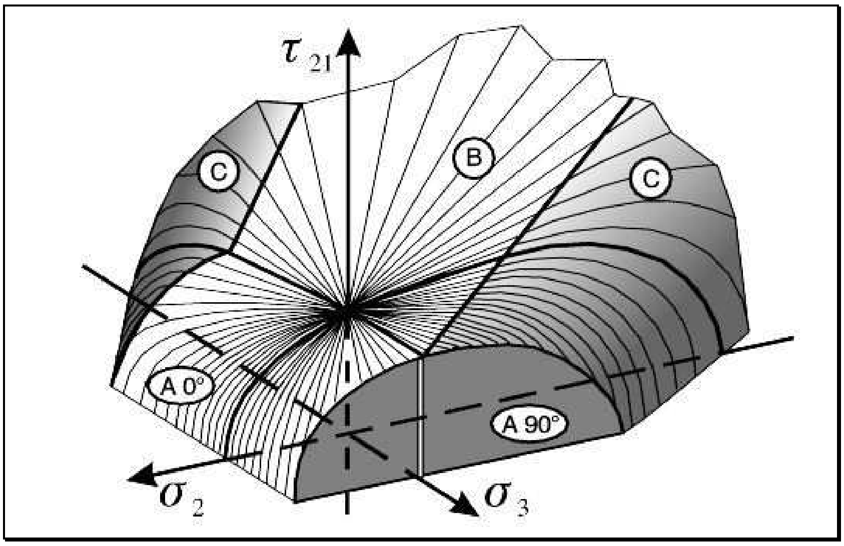


Figure 4 A 3-dimensional fracture body for ($\sigma_2$, $\sigma_3$, $\tau_{21}$)-stress combination is shown [Puc96]. It is the final result of the calculation of all possible fracture stress combinations using the master fracture body, described by equations (3a) and (3b). The total surface of the ($\sigma_2$, $\sigma_3$, $\tau_{21}$)-fracture body is divided into sub-surfaces which are marked by (A), (B) and (C). These letters symbolize the different fracture modes which are to be expected. The symbol (A 0°) marks a surface where under a combination of $\sigma_2^t$ tension and $\tau_{21}$ shear, a fracture angle $\theta_{fp}$ = 0° appears. The area marked (A 90°) marks the region in which everywhere $\theta_{fp}$ = 90°, because the fracture is caused by tensile $\sigma_3$ alone. There is no interaction between $\sigma_3$ and $\tau_{21}$, because these two stresses do not act on a common action plane.

The sharp corners and edges of the fracture body shown in Figure 4 are not to be expected in reality. They will certainly be 'rounded' due to effects of microdamage and probabilistic. A physically based method suitable for incorporating these effects into *Puck's* fracture theory is explained in the extensive Appendix of [VDI06].

## 8. Advise on a calculation software

A calculation software, which considers the non-linear behaviour of the lamina and the *Puck* theory of failure for the prediction of the strength of laminates as described in this paper is under development at the Department 'Konstruktiver Leichtbau und Bauweisen (KLuB)', headed by Prof. Schürmann, of Darmstadt Technical University. The program is named AlfaLam.nl, Advanced layerwise failure analysis of Laminates.nonlinear. The program is based on MS EXCEL and can be downloaded soon from the homepage of KLub (http://www.klub.tu-darmstadt.de). The actual direct link to KLub-downloads is: http://www.klub.tu-darmstadt.de/forschung/download.php.

## CONCLUSION

Very realistic calculation results concerning deformation and strength of laminated FRP components can be achieved by the application of the *Puck* theory of failure in laminates, which basically distinguishes between IFF and FF in UD-composites. The theory as a whole has been significantly improved during the last decade by *Puck's* development of the physically based 'action plane related' criteria for the IFF and is now comprehensively published in German and English in the new guideline VDI 2014 Part 3. The *Puck* theory of fracture in UD-composites is most probably the theory which is best verified by experiments. This is documented for instance in [Kno07].


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## Appendix: Stress exposure ratio and stretch factor

Numerous algorithms in *Puck's* action plane related fracture theory are based on the variable, which is called the **stress exposure** ratio or for short stress exposure. Its meaning is illustrated in Figure 5, again – as an example – a ($\sigma_2$, $\tau_{21}$)-stress is used for visualization.

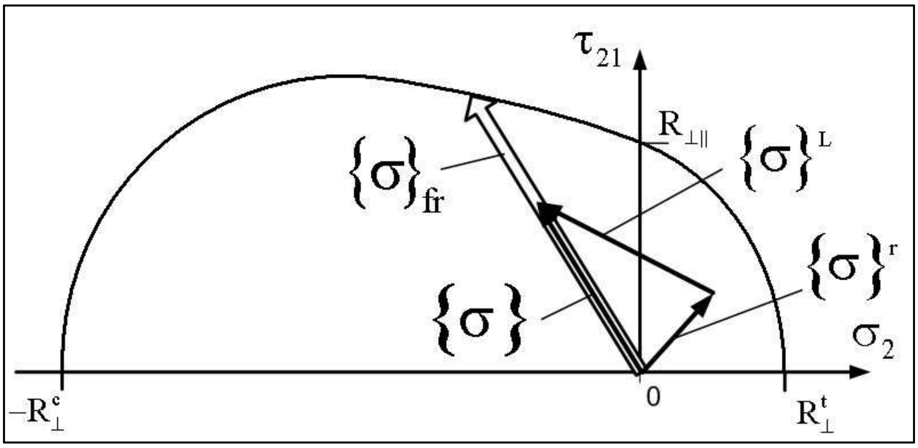


Figure 5 Illustration of the meaning of the stress exposure (ratio) using a ($\sigma_2$, $\tau_{21}$)-stress combination. In general the vector $\{\sigma\}$ of the acting stresses is composed of (constant) residual stresses $\{\sigma\}^r$ and (variable) load dependent stresses $\{\sigma\}^L$.

The stress exposure is defined as the length of the vector $\{\sigma\}$ representing the acting stress combination divided by the length of the fracture vector $\{\sigma\}_{fr}$, which has the same direction as $\{\sigma\}$ (that means proportional stretching of all components of $\{\sigma\}$):

$$f_E = \frac{|\{\sigma\}|}{|\{\sigma\}_{fr}|} \tag{4}$$

The stress exposure $f_E$ is a **quantitative** measure for the risk of fracture, because there is a **linear** relationship between $f_E$ and the acting stresses. If the stress exposure becomes $f_E = 1$, the fracture stress is reached. For instance, calculating $f_E = 1$ for a ($\sigma_n$, $\tau_{nt}$, $\tau_{n1}$)-stress state means, that the vector ($\sigma_n$, $\tau_{nt}$, $\tau_{n1}$) just gets in contact with the surface of the fracture master body.

The reciprocal of the stress exposure $f_E$ is the so called stretch factor $f_S$:

$$f_S = \frac{1}{f_E} = \frac{|\{\sigma\}_{fr}|}{|\{\sigma\}|} \tag{5}$$

The stress exposure $f_E$ is absolutely essential when searching for the fracture plane. The fracture plane is simply the action plane with the highest risk of fracture, i.e. with the highest stress exposure $f_E$ calculated by taking into account all stresses of the action plane, resulting from residual and load dependent stresses. In practice the fracture plane can be found by solving the following extremum problem: the angle $\theta = \theta_{fp}$ has to be found for which

$$f_E\left(\theta_{fp}\right) = f_{E\max} \quad (6a)$$

$$\frac{d f_E(\theta)}{d\theta} = 0 \quad (6b)$$

is fulfilled.

Remark: Fracture conditions are very often written as functions F of the acting stresses containing first and second order terms. It would lead to a wrong result when setting $dF/d\theta = 0$ for searching the fracture plane. In this case the stress exposure $f_E$ has to be calculated from the following equation (7) before applying the equations (6a) and (6b).

With ΣL as the sum of the linear terms and ΣQ as the sum of the quadratic terms $f_E$ follows from:

$$f_E = \frac{1}{2}\left(\sum L + \sqrt{\left(\sum L\right)^2 + 4\sum Q}\right) \quad (7)$$

Experience shows that the stress exposure $f_E$ is a very versatile and easy to use tool for the description of different fracture states both due to IFF and FF. In Puck's lamina-by-lamina fracture theory, the stress exposure $f_E$ **after** IFF is for instance used as the variable which governs the stiffness degradation of the laminate caused by the increasing number of IFF cracks in the lamina. In this case $f_E$ is the only variable which allows to mathematically express the physically based combined action of several stresses, such as $\sigma_1$, $\sigma_2$, $\tau_{21}$. This is explained in more detail in [Puc98; Puc02; Kno07; VDI06].

In a recent paper [Puc07], Puck makes use of the stress exposure $f_E$ for establishing a physically-based model for the mathematical treatment of the non-linear ($\sigma_2^c$, $\varepsilon_2$)- and ($\tau_{21}$, $\gamma_{21}$)-stress/strain behaviour of UD-composites **before** IFF, which is assumed to be caused by micro damage. The idea is to take the stress exposure as a measure for the micro damage due to $\sigma_\perp^c$ - and $\tau_{\perp\parallel}$-stressings and their interactions. This makes it possible to treat the interaction of $\tau_{21}$ on the strain $\varepsilon_2$ and of $\sigma_2^c$ on the shear strain $\gamma_{12}$ analytically.